The Postulates

What is a postulate?

1. suggest or assume the existence, fact, or truth of (something) as a basis for reasoning, discussion, or belief.
   "a theory postulated by a respected scientist"
   synonyms: suggest, advance, posit, hypothesize, propose, assume
2. (in ecclesiastical law) nominate or elect (someone) to an ecclesiastical office subject to the sanction of a higher authority.

How do you know it's correct? DATA!

See here for an example of impeccable logic.

The Postulates (the Rules of the Game)

1. Each physical, measurable quantity, A, has a corresponding operator, $\hat{A}$, that satisfies the eigenvalue equation $\hat{A}\ \phi = a\phi$ and measuring that quantity yields the eigenvalues of $\hat{A}$. The 'intensity' is proportional to $|\Psi|^2$ and is interpreted as a probability.

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2. Measurement of the observable A leaves the system in a state that is an eigenfunction of $\hat{A}$.
3. The state of a system is represented by a wave function Ψ that is continuous, differentiable and contains all possible information about the system. The 'intensity' is proportional to $|\Psi|^2$ and is interpreted as a probability. The average value of any observable A is $\langle A \rangle = \int_{all\ space} \Psi^* \hat{A}\, \Psi d\vec{r}$.

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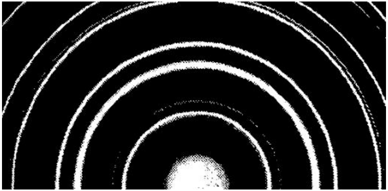
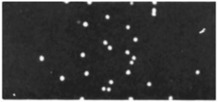
4. The time and spatial dependence of $\Psi(x,t)$ is determined by the time dependent Schroedinger equation.

$$i\hbar \frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) \qquad \mu \equiv \text{reduced mass.}$$

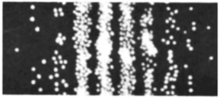


Electron diffraction by gold

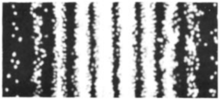
The square of the magnitude of the wave function $|\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t)$ is the probability density.



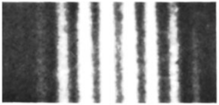
(a) After 28 electrons



(b) After 1 000 electrons



(c) After 10 000 electrons

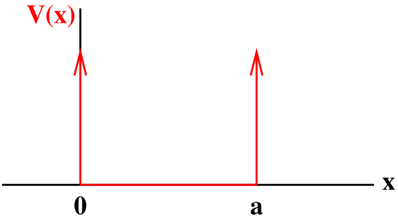


(d) Two-slit electron pattern

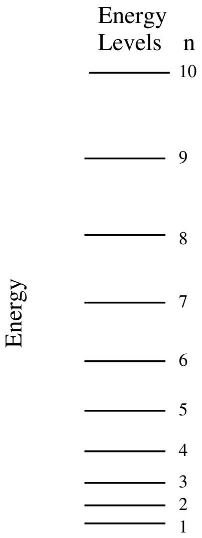
Apply the Rules: A Particle in a Box

Consider the infinite rectangular well potential shown in the figure below.

- What is the time-independent Schroedinger equation for this potential?
- What is the general solution to the previous question?
- What are the boundary conditions the solution must satisfy?
- What is the particular solution for this potential?
- What is the energy of the particular solution?



The Infinite Rectangular Well Potential - Energy Levels



Some Math

The solutions of the Schroedinger equation form a Hilbert space.

1. They are linear, *i.e.* superposition/interference is built in.
   - If a is a constant and $\phi(x)$ is an element of the space, then so is $a\phi(x)$.
   - If $\phi_1(x)$ and $\phi_2(x)$ are elements, then so is $\phi_1(x) + \phi_2(x)$.
2. An inner product is defined and all elements have a norm.

$$\langle \phi_n | \psi \rangle = \int_{-\infty}^{\infty} \phi_n^* \psi \; dx \qquad \text{and} \qquad \langle \phi_n | \phi_n \rangle = \int_{-\infty}^{\infty} \phi_n^* \phi_n dx = 1$$

3. The solutions are complete.

$$|\psi\rangle = \sum_{n=0}^{\infty} b_n |\phi_n\rangle$$

4. The solutions are orthonormal so $\langle \phi_n | \phi_{n'} \rangle = \delta_{n,n'}$.

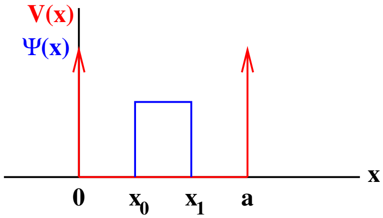
The operators are Hermitian - their eigenvalues are real.

Apply the Rules More: A Particle in a Box

Consider the infinite rectangular well potential shown in the figure below with an initial wave packet defined in the following way.

$$\Psi(x,0) = \frac{1}{\sqrt{d}} \quad x_0 < x < x_1 \quad \text{and} \quad d = x_1 - x_0$$
$$= 0 \quad \text{otherwise}$$

- What possible values are obtained in an energy measurement?
- What eigenfunctions contribute to this wave packet and what are their probabilities?
- What will many measurements of the energy give?



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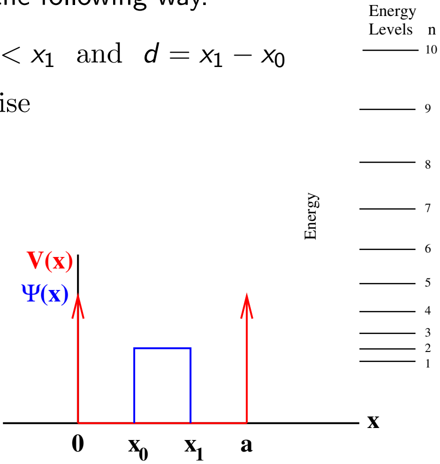


L'Hôpital's Rule

If

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0 \qquad \text{or} \qquad \lim_{x \to c} f(x) = \lim_{x \to c} g(x) = \pm\infty$$

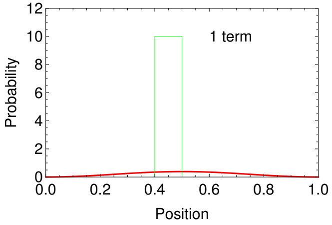
and

$$\lim_{x \to c} \frac{f'(x)}{g'(x)}$$

exists, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)} \qquad .$$

Truncated Fourier Series - 1



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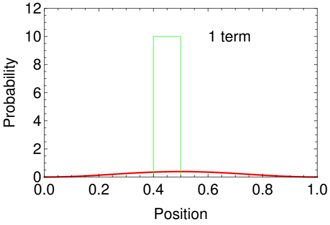

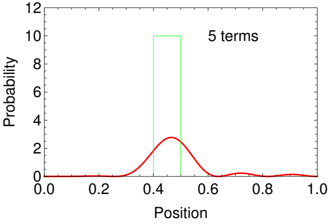

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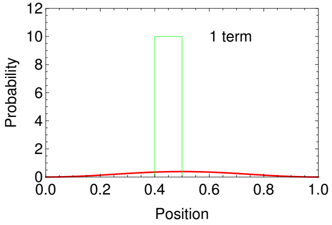

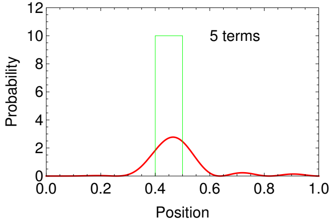

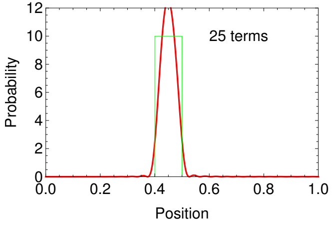

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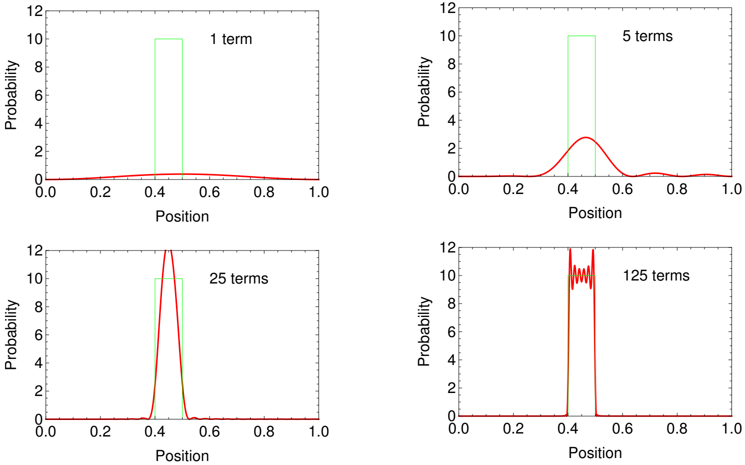
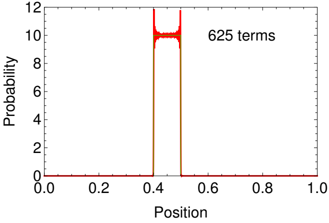


Truncated Fourier Series - 2



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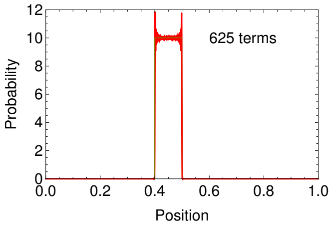
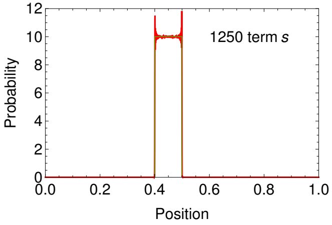
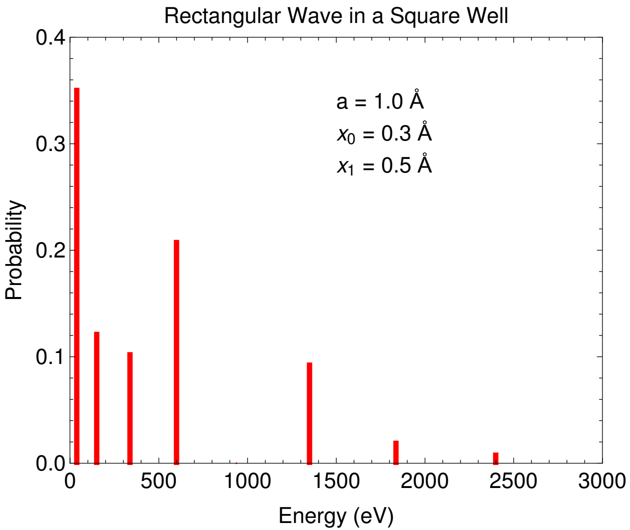




Probabilities of Different Final States



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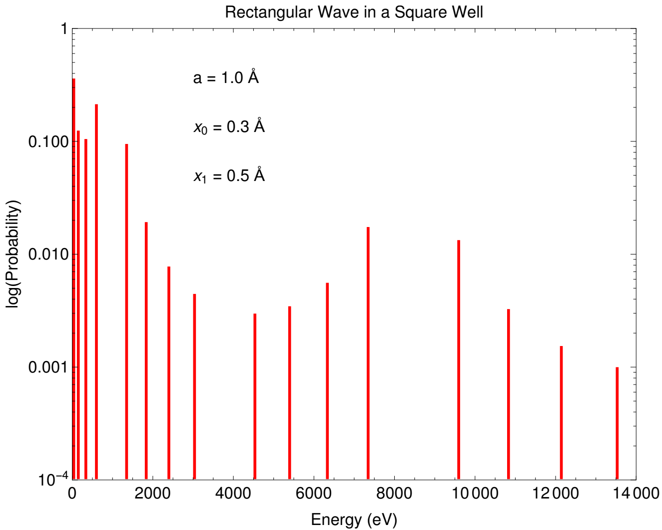
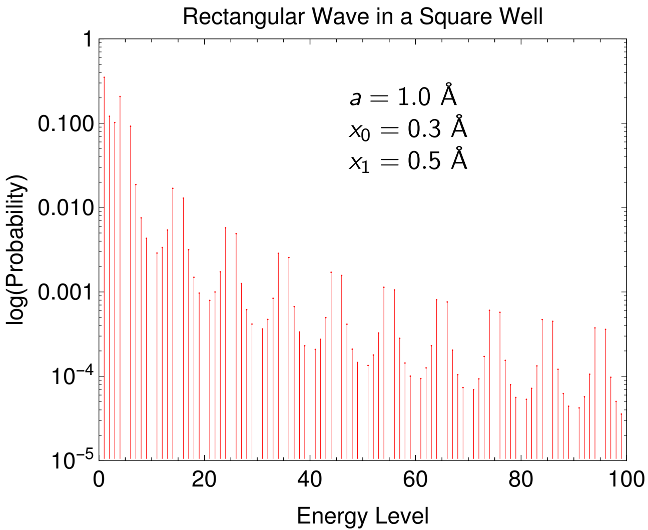
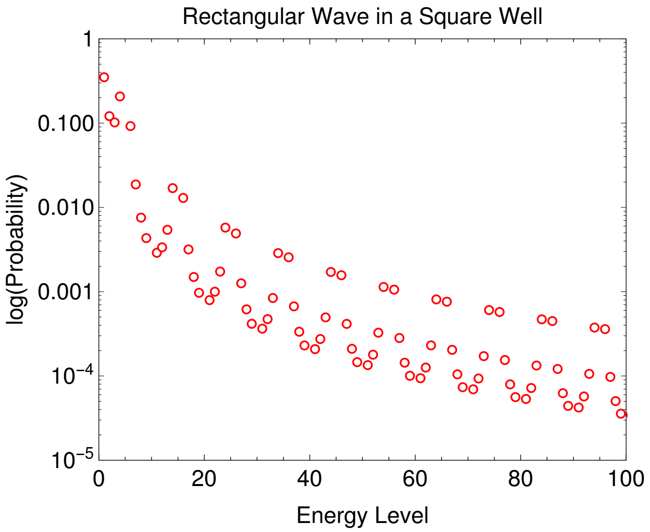


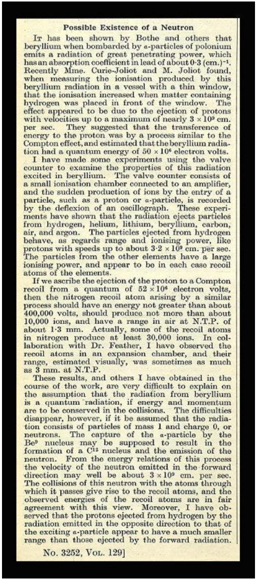
Probabilities of Different Final States - 3



Probabilities of Different Final States - 4



Why a page limit?



Possible Existence of a Neutron

It has been shown by Bothe and others that beryllium when bombarded by α-particles of polonium emits a radiation of great penetrating power, which has an absorption coefficient in lead of about 0·3 (cm.)$^{-1}$. Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionisation produced by this beryllium radiation in a vessel with a thin window, that the ionisation increased when matter containing hydrogen was placed in front of the window. The effect appeared to be due to the ejection of protons with velocities up to a maximum of nearly 3×10^9 cm. per sec. They suggested that the transference of energy to the proton was by a process similar to the Compton effect, and estimated that the beryllium radiation had a quantum energy of 50×10^6 electron volts.

I have made some experiments using the valve counter to examine the properties of this radiation excited in beryllium. The valve counter consists of a small ionisation chamber connected to an amplifier, and the sudden production of ions by the entry of a particle, such as a proton or α-particle, is recorded by the deflexion of an oscillograph. These experiments have shown that the radiation ejects particles from hydrogen, helium, lithium, beryllium, carbon, air, and argon. The particles ejected from hydrogen behave, as regards range and ionising power, like protons with speeds up to about $3{\cdot}2 \times 10^9$ cm. per sec. The particles from the other elements have a large ionising power, and appear to be in each case recoil atoms of the elements.

If we ascribe the ejection of the proton to a Compton recoil from a quantum of 52×10^6 electron volts, then the nitrogen recoil atom arising by a similar process should have an energy not greater than about 400,000 volts, should produce not more than about 10,000 ions, and have a range in air at N.T.P. of about 1·3 mm. Actually, some of the recoil atoms in nitrogen produce at least 30,000 ions. In collaboration with Dr. Feather, I have observed the recoil atoms in an expansion chamber, and their range, estimated visually, was sometimes as much as 3 mm. at N.T.P.

These results, and others I have obtained in the course of the work, are very difficult to explain on the assumption that the radiation from beryllium is a quantum radiation, if energy and momentum are to be conserved in the collisions. The difficulties disappear, however, if it be assumed that the radiation consists of particles of mass 1 and charge 0, or neutrons. The capture of the α-particle by the Be^9 nucleus may be supposed to result in the formation of a C^{12} nucleus and the emission of the neutron. From the energy relations of this process the velocity of the neutron emitted in the forward direction may well be about 3×10^9 cm. per sec. The collisions of this neutron with the atoms through which it passes give rise to the recoil atoms, and the observed energies of the recoil atoms are in fair agreement with this view. Moreover, I have observed that the protons ejected from hydrogen by the radiation emitted in the opposite direction to that of the exciting α-particle appear to have a much smaller range than those ejected by the forward radiation.

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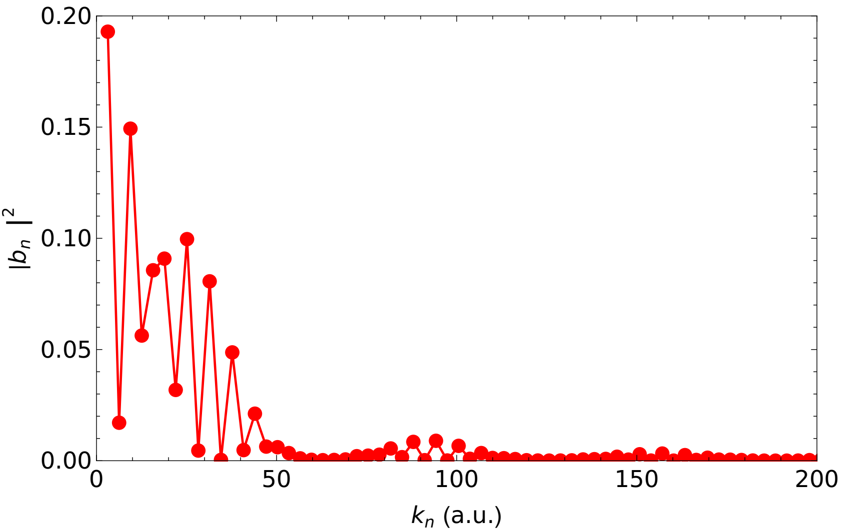
Width of a distribution?



The Quantum Program in One Dimension - So Far

1. Solve the Schroedinger equation to get eigenfunctions and eigenvalues.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\phi(x)}{\partial x^2} + V\phi(x) = E_n\phi(x)$$

2. For an initial wave packet $\psi(x)$ use the completeness of the eigenfunctions.

$$|\psi(x)\rangle = \sum_{n=1}^{\infty} b_n|\phi(x)\rangle$$

3. Apply the orthonormality $\langle\phi_m|\phi_n\rangle = \delta_{m,n}$.

$$\langle\phi_m|\psi\rangle = \langle\phi_m|\left(\sum_{n=1}^{\infty} b_n|\phi\rangle\right) = b_m = \int_{-\infty}^{\infty} \phi_m^* \left(\sum_{n=1}^{\infty} b_n|\phi\rangle\right) dx$$

4. Get the probability P_n for measuring E_n from $|\psi\rangle$.

$$P_n = |b_n|^2$$

5. Do the free particle solution.
6. Put in the time evolution.

Nuclear Fusion!

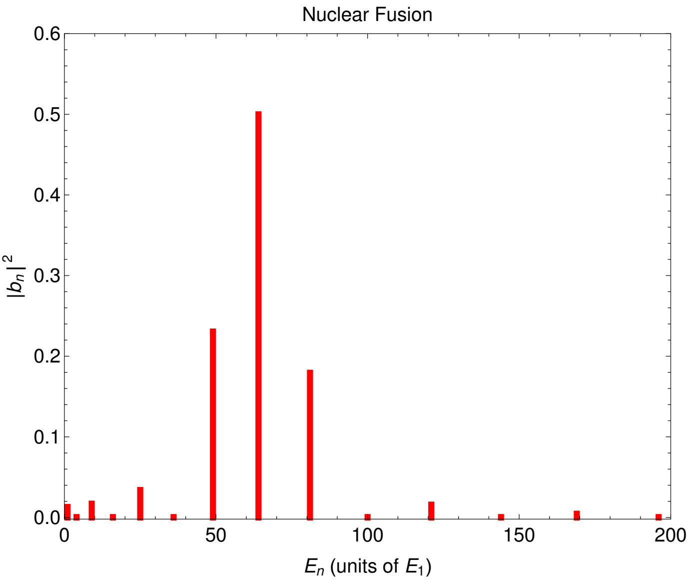
Consider a case of one dimensional nuclear 'fusion'. A neutron is in the potential well of a nucleus that we will approximate with an infinite square well with walls at $x = 0$ and $x = a$. The eigenfunctions and eigenvalues are

$$E_n = \frac{n^2\hbar^2\pi^2}{2ma^2} \qquad \begin{aligned} \phi_n &= \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \qquad & 0 \le x \le a \\ &= 0 & x < 0 \text{ and } x > a \end{aligned} \quad .$$

The neutron is in the $n = 4$ state when it fuses with another nucleus that is the same size, instantly putting the neutron in a new infinite square well with walls at $x = 0$ and $x = 2a$.

1. What are the new eigenfunctions and eigenvalues of the fused system?
2. What is the spectral distribution?
3. What is the average energy? Use the b_n's.

Spectral Distribution for One-Dimensional Nuclear Fusion



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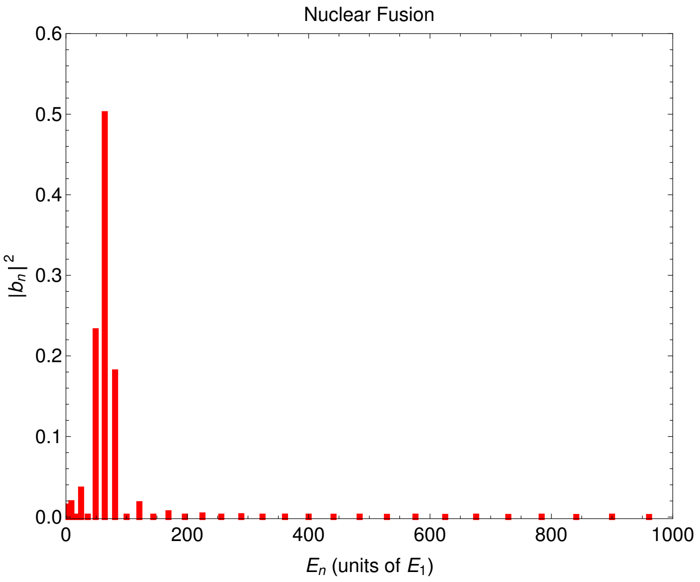

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