Solar Fusion

1. The solar constant is the average intensity of sunlight on the Earth's surface. Assuming the Sun's output is isotropic, what is the total power output (energy/time) of the Sun?

2. Consider the reaction $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$ which releases energy $\Delta H = 1.2 \times 10^6$ J/mole. What mass of hydrogen would be consumed if this reaction was used to power the Sun? What mass of oxygen would be consumed if this reaction was used to power the Sun? What is the total mass per second needed to power the Sun from this reaction?

3. Suppose the Sun had just the right composition to efficiently turn hydrogen and oxygen into water. Use the total mass rate from the previous question and the mass of the Sun $M_{Sun} = 2 \times 10^{30}$ kg to calculate the lifetime of the Sun.

4. Fusion reactions in the Sun are responsible for the production of solar energy. One of these reactions is the fusion of a proton with a deuteron (a proton and neutron bound together).

$$p + d \rightarrow {}^3He + \gamma$$

The deuteron has a radius of about $1.2fm$ and the proton has a radius of about $1fm$ where $1\ fm = 10^{-15}\ m$.

   (a) What is the energy released in the reaction shown above?

   (b) Calculate the inter-nuclear distance when the proton and the deuteron are just touching and call this distance r'. Estimate the Coulomb barrier, V_C, between the proton and the deuteron using

$$V_C = \frac{Z_1 Z_2 \hbar c}{137} \frac{1}{r'}$$

   where $\hbar c = 197\ MeV - fm$ and Z_i is the charge of the appropriate nucleus.

   (c) The proton is incident on the deuteron because of its thermal motion in the Sun's core. Estimate the incident energy of the proton by assuming that its energy is $\frac{3}{2}kT$ where k is Boltzmann's constant ($8.62 \times 10^{-11} MeV/K$) and T is the temperature of the solar core ($\approx 10^7 K$). Assume the deuteron is stationary. How does this compare with the Coulomb barrier?

   (d) For the colliding proton and deuteron in the previous part, what is the distance of closest approach?

5. The velocities of particles in a gas can be described classically with the Maxwellian velocity distribution

$$P(v)dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv$$

where $P(v)$ is the probability of the particle having a speed in the range $v \rightarrow v + dv$, v is the speed of a particle, and m is the mass of an individual particle.

   (a) What is the most probable speed of a particle in a gas in terms of T and any other constants?

(b) Consider a hydrogen molecule (H_2) in the Earth's upper atmosphere. Starting from basic physics principles derive the escape velocity for a particle near the Earth's surface. What is the temperature at which the most probable velocity will equal the escape velocity from the Earth? The temperature of the upper atmosphere is typically a few hundred degrees kelvin. should we worry about the atmosphere leaking away in the near future? Explain.

(c) Describe in words and with an equation or two how you would calculate the total probability for a particle in the Earth's upper atmosphere to escape. Neglect the effects of collisions and let the temperature of the upper atmosphere be a uniform value of T_A. Your answer should be a symbolic one; no numerical calculations are necessary.

6. We want to determine the probability of a proton in the stellar core having enough energy to overcome the Coulomb barrier between two protons. To find this we have to integrate the probability distribution (see Problem 5) of the protons from a lower limit derived from the Coulomb barrier out to infinity. This lower limit is the minimum velocity a proton can have incident on a stationary proton and still pass the Coulomb barrier. Write down the equations to perform this calculation including the integral itself and equations for any limits that are needed. If you're adventurous, give this integral a shot.

7. We have expressed the solution to the rectangular barrier problem in the form of a transfer matrix so

$$\begin{aligned} \zeta_1 &= \mathbf{t}\zeta_3 \\ &= \mathbf{d}_{12}\mathbf{p}_2\mathbf{d}_{21}\zeta_3 \end{aligned}$$

where ζ_1 and ζ_3 are vectors representing the amplitudes of the incoming and outgoing waves, $\mathtt{t}$ is the transfer matrix, and $\mathtt{d}_{12}$, $\mathtt{p}_2$, and $\mathtt{d}_{21}$ are the propagation and discontinuity matrices that compose $\mathtt{t}$. The propagation and discontinuity matrices for the rectangular barrier are

$$\mathbf{d_{12}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_2}{k_1} & 1-\frac{k_2}{k_1} \\ 1-\frac{k_2}{k_1} & 1+\frac{k_2}{k_1} \end{pmatrix} \qquad \mathbf{d_{21}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_1}{k_2} & 1-\frac{k_1}{k_2} \\ 1-\frac{k_1}{k_2} & 1+\frac{k_1}{k_2} \end{pmatrix}$$

$$\mathbf{p_2} = \begin{pmatrix} e^{-ik_2 2a} & 0 \\ 0 & e^{+ik_2 2a} \end{pmatrix}.$$

The transmission coefficient is the following.

$$T = \frac{1}{|t_{11}|^2}$$

You should be able to multiply the appropriate matrices together and show

$$t_{11} = \frac{1}{4}\left[\left(1+\frac{k_2}{k_1}\right)e^{-ik_2 2a}\left(1+\frac{k_1}{k_2}\right) + \left(1-\frac{k_2}{k_1}\right)e^{ik_2 2a}\left(1-\frac{k_1}{k_2}\right)\right]$$

For a barrier with a height $V_0 = 0.5\ MeV$ and half-width $a = 4.0\ fm$ what are the inputs to the transmission coefficient? For the velocity of the incident proton use the average velocity of a proton in the Sun (where $T = 10^7\ K$). If you're still adventurous, calculate the transmission coefficient. You should find a result far larger than the result in the previous problem.