Physics 309

Solving The Harmonic Oscillator Schroedinger Equation - 2

1. If one makes an astute choice of units then the first two harmonic oscillator wave functions can be written as:

$$\psi_0 = A_0 e^{-\xi^2/2} \qquad and \qquad \psi_1 = A_1 \xi e^{-\xi^2/2}$$

where

$$\xi = \beta x \qquad , \qquad \beta^2 = \frac{m\omega_0}{\hbar}.$$

Find the normalization constants, A_0 and A_1.

2. The energy eigenvalues of a molecule indicate the molecule is a one-dimensional harmonic oscillator. In going from the second excited state to the first excited state, it emits a photon of energy $h\nu = 0.1\ eV$. Assuming that the oscillating portion of the molecule is a proton, calculate the probability that a proton in the first excited state is at a distance from the origin that would be forbidden to it by classical mechanics. You may have difficulty performing the integration necessary for the final answer. In that case, express that answer in terms of the unsolved integral and propose some method for calculating it. A special prize awaits anyone who can solve the integral.

3. The first few, normalized, dimensionless harmonic oscillator eigenfunctions are

$$\psi_0 = A_0 e^{-\xi^2/2} \qquad \psi_1 = A_1 2\xi e^{-\xi^2/2} \qquad \psi_2 = A_3(4\xi^2 - 2)e^{-\xi^2/2}$$

where

$$A_n = (2^n n! \sqrt{\pi})^{-1/2} \qquad .$$

See page 201 in Liboff for a more complete list. Using the creation operator we defined in class

$$\hat{a}^\dagger = \sqrt{\frac{\alpha}{2}}\left(\hat{x} - \frac{i\hat{p}}{m\omega_0}\right)$$

where $\alpha = m\omega_0/\hbar = 2\pi m\nu/\hbar$ show that

$$\hat{a}^\dagger|1> = \sqrt{2}|2> \qquad .$$

4. Calculate the expectation value of the momentum operator $\hat{p}$ for the n=10 harmonic oscillator state.