Physics 309

Matter Waves - 1

1. Consider a particle of mass m that is freely allowed to move along the x axis in the region $0 \leq x \leq a$, but is strictly forbidden to be outside this region. The particle bounces between the impenetrable walls located at $x = 0$ and $x = a$. The wave functions for the ground and first excited states are

$$\Psi_0(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) e^{-iE_0 t/\hbar} \qquad \Psi_1(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right) e^{-iE_1 t/\hbar}$$

   for the region within the walls and are zero outside. What is the energy in each case in terms of the other constants in the problem? What is the expectation value of the position $\langle x \rangle$ (or average x) for each state of the system?

2. A hypernucleus is an atomic nucleus which contains hyperons, particles that are not nucleons (protons or neutrons) and contain a strange quark replacing one of the u or d quarks in a nucleon. They can move freely throughout the nuclear volume. Suppose a hyperon is confined in a nucleus of diameter a and has the following initial wave function

$$\begin{aligned} \psi(x,0) &= A \sin\frac{4\pi x}{a} & \frac{a}{4} \leq x \leq \frac{3a}{4} \\ &= 0 & \text{otherwise} \end{aligned}$$

   where A is a normalization constant. Treat the system as a one dimensional infinite rectangular well. The mass of the hyperon in energy units is $m_h c^2 = 1405\ MeV$.

   (a) What is the normalization constant A? Show the full calculation.

   (b) What are the eigenfunctions and eigenvalues of the system (just write down what we developed in class)?

   (c) These eigenfunctions are supposed to be orthogonal so $\langle \phi_m | \phi_n \rangle = \delta_{mn}$. Calculate the inner product ($\langle \phi_m | \phi_n \rangle$) and show this is true.

   (d) What are the coefficients of the Fourier series describing the initial wave function (the b_n's)?

   (e) If $a = 1.0\ fm$, what is the probability of the hyperon being in the ground state? What is the probability of the hyperon being in the first excited state?

   (f) What is the average energy of the particle $\langle E \rangle$? Give your answer in MeV. It is often convenient to use an alternate form of Planck's constant when you are dividing by a mass

$$\frac{\hbar^2}{m} = \frac{\hbar^2 c^2}{mc^2} = \frac{(\hbar c)^2}{mc^2}$$

   so use $\hbar c = 197\ MeV - fm$.