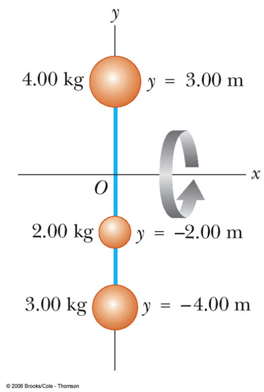
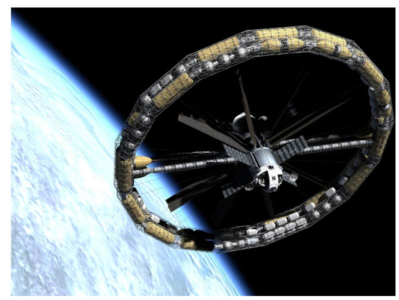
Physics 309 - Solving the Three Dimensional Schroedinger Equation - 2

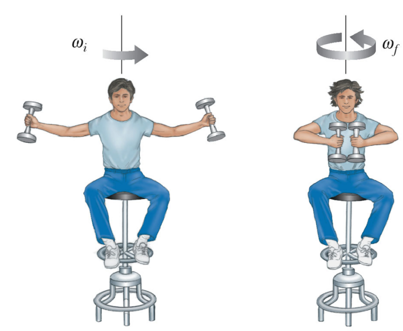
1. Rigid rods of negligible mass lying along the y axis connect three particles (see figure). The system rotates about the x axis with an angular speed of 2.00 rad/s. Find (a) the moment of inertia about the x axis and the total rotational kinetic energy evaluated using $\frac{1}{2}I\omega^2$ and (b) the tangential speed of each particle and the total kinetic energy evaluated using $\sum \frac{1}{2}m_i v_i^2$.



2. A von Braun wheel is a space station constructed in the shape of a hollow ring. Imagine the one in the figure has a mass $5.00 \times 10^4\ kg$. Members of the crew walk on a deck formed by the inner surface of the outer cylindrical wall of the ring, with radius 100 m. At rest when constructed, the ring is set rotating about its axis so that the people inside experience an effective free-fall acceleration equal to g. (the figure shows the ring together with some other parts that make a negligible contribution to the total moment of inertia.) The rotation is achieved by firing two small rockets attached tangentially to opposite points on the outside of the ring. What angular momentum does the space station acquire?



3. A student sits on a freely rotating stool holding two weights, each of mass $m = 3.00\ kg$ as shown in the figure. When his arms are extended horizontally, the weights are $\ell_1 = 1.00\ m$ from the axis of rotation and he rotates with an angular speed of $\omega_1 = 0.750\ rad/s$. The moment of inertia of the student plus stool is $I_s = 3.00 kg - m^2$ and is assumed to be constant. The student pulls the weights inward horizontally to a position $\ell_2 = 0.300\ m$ from the rotation axis. (a) Find the new angular speed of the student. (b) Find the kinetic energy of the rotating system before and after he pulls the weights inward.

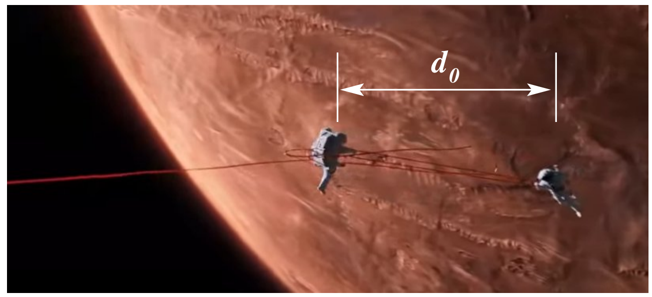


4. In the final rescue scene of the movie *The Martian* Commander Lewis and Mark Watney are spinning around each other on a massless tether of length d_0. They orbit the center-of-mass of the tether located at the midpoint. Commander Lewis pulls Watney toward her until she can grab him. Assume the masses of each astronaut are m (the commander is lighter, but she has a manned maneuvering unit attached to her spacesuit) and their initial tangential speed is $v_{\perp 0}$. Treat the astronauts as point particles as they orbit around one another. What is the astronauts' angular momentum in terms of d_0, m,

and $v_{\perp 0}$? Commander Lewis pulls Watney toward her until she can reach him at a distance d_1. What is the tangential velocity $v_{\perp 1}$ when she grabs him in terms of d_0, d_1, m, and $v_{\perp 0}$? What is the change in the kinetic energy ΔKE of the astronauts? Use the following values to obtain a numerical value for ΔKE ONLY. Compare your result with the energy of a car going 55 mph - $3 \times 10^5\ J$. Could she do it or is Hollywood violating the laws of physics?

$m = 320\ kg \quad d_0 = 7\ m$

$v_{\perp 0} = 2\ m/s \quad d_1 = 0.5\ m$



5. The z component of the angular momentum operator is

$$\hat{L}_z = -i\hbar\left(x\frac{d}{dy} - y\frac{d}{dx}\right) \quad .$$

We want to express this operator in terms of the spherical coordinates r, θ, ϕ. The transformation from Cartesian coordinates to spherical coordinates is

$$x = r\sin\theta\cos\phi \quad (1)$$

$$y = r\sin\theta\sin\phi \quad (2)$$

$$z = r\cos\theta \quad . \quad (3)$$

(a) First, we need the total derivatives with respect to x and y in spherical coordinates. The general form of the total derivative operator for x is the following.

$$\frac{d}{dx} = \frac{\partial}{\partial x} + \sum_{j=1}^{k} \frac{dy_j}{dx}\frac{\partial}{\partial y_j}$$

Write down the total derivative d/dx needed in L_z using this definition and the fact that any function f we encounter will use spherical coordinates r, θ, and ϕ.

(b) Now go after the pieces in the derivative of part 5a. Show

$$\frac{\partial r}{\partial x} = \frac{x}{r} = \sin\theta\cos\phi$$

recalling that $r = \sqrt{x^2 + y^2 + z^2}$.

(c) Now show

$$\frac{\partial\theta}{\partial x} = \frac{\cos\phi\cos\theta}{r} \quad .$$

(Hint: Use the fact that $\partial z/\partial x = 0$ where $z = r\cos\theta$ and solve for $\partial\theta/\partial x$.)

(d) Continuing on show

$$\frac{\partial \phi}{\partial x} = -\frac{\sin\phi}{r\sin\theta} \qquad .$$

(Hint: Calculate $\partial(\tan\phi)/\partial x$ where $\tan\phi = y/x$ and solve for $\partial\phi/\partial x$ after taking derivatives and converting everybody to spherical coordinates.)

(e) Combine the results of parts 5b-5d and show that

$$\frac{d}{dx} = \sin\theta\cos\phi\frac{\partial}{\partial r} + \frac{\cos\phi\cos\theta}{r}\frac{\partial}{\partial\theta} - \frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi} \qquad .$$

(f) Soldiering on we can now obtain an expression for d/dy by following essentially the same steps as above (parts 5b-5e) and substituting y for x in those steps. The results will be similar to, but still different from the results of parts 5b-5e. You should find that

$$\frac{d}{dy} = \sin\theta\sin\phi\frac{\partial}{\partial r} + \frac{\sin\phi\cos\theta}{r}\frac{\partial}{\partial\theta} + \frac{\cos\phi}{r\sin\theta}\frac{\partial}{\partial\phi} \qquad .$$

(g) Last leg!!!! Combine the results of parts 5b-5f to show that

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$$

where $\hat{L}_z$ is defined above.