Modern Physics
Activities for Physics 205

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August 23, 2021

Welcome to Physics 205!

The exercises in this manual have been developed to support an investigative physics course that emphasizes active learning. Your written work will consist primarily of documenting your class activities by filling in the entries in the spaces provided in the units. The entries consist of observations, derivations, calculations, and answers to questions. Although you may use the same data and graphs as your partner(s) and discuss concepts with your classmates, all entries should reflect your own understanding of the concepts and the meaning of the data and graphs you are presenting. Thus, each entry should be written in your own words. It is very important to your success in this course that your entries reflect a sound understanding of the phenomena you are observing and analyzing.

Contents

Lab 1   Galilean Relativity

Name: _____   Lab Partner(s): _____

Objective

To investigate the relationship between different inertial reference frames and develop the equations of Galilean relativity.

Overview

Before we begin our study of Einstein's special theory of relativity we will first investigate the effects of observing a phenomenon in two different inertial frames of reference. A spring-launched projectile will be "fired" from a moving platform and we will discover how the description of this phenomenon changes (or doesn't change) in a frame that moves along with the platform. You will use the video analysis software and mathematical modeling tools to find the equations that describe the horizontal motion (x *vs.* t) and the vertical motion (y *vs.* t) of the projectile in a stationary frame and then in an inertial frame moving at constant speed with the launcher. The set of relationships between the inertial reference frames forms Galilean relativity. These relationships provide an intuitive, "common sense" picture of the world that works well at low velocities, but fails in many surprising ways at high velocities. At high velocities we must resort to Einstein's special theory of relativity, which we will discuss later. To do the activities in this unit you will need:

- A video analysis system (*Tracker*).

- The video *Galilean_Transformation.mp4*

- Graphing and curve fitting software (*Excel*).

Activity 1: Observing Projectile Motion From a Moving Launcher

(a) Use *Tracker* to analyze the film *Galilean_Transformation.mp4* and determine the position of the projectile in each frame with a fixed origin. To do this task follow the instructions of **Appendix A: Video Analysis** for "Analyzing the Movie." When analyzing the movie, place the fixed origin at the dark spot that appears in front the cart the launcher rides. This makes the comparison with the activities below easier. Change the position of origin in the frame by performing the following steps.

1. Click on the axes icon on the menu bar. The axes will appear over the movie.

2. Click on the origin (where the axes cross each other) and drag it to the desired location.

3. Click on the *Create* icon located on the menu bar and then select "Point Mass."

(b) Collect the data for analysis by following the instructions in **Appendix A**. The data table should contain three columns with the values of time, x-position, and y-position. Copy this data into an Excel spreadsheet for analysis.

(c) Determine the position of the launcher at the first and last frames of the movie. Using these results, what is the horizontal and vertical speed of the launcher during the movie?

$x_0 =$ \qquad $x_1 =$ \qquad $\Delta t =$ \qquad $(v_{\text{launcher}})_x =$

$y_0 =$ \qquad $y_1 =$ \qquad\qquad $(v_{\text{launcher}})_y =$

(d) Plot and fit the position versus time data for the horizontal and vertical positions of the projectile. Use *Excel* to make the plot. Print each plot and attach it to this unit. Record the equation of each fit here. Be sure to properly label the units of each coefficient.

$x(t) =$

$y(t) =$

What is the horizontal speed of the projectile? How did you determine this?

Activity 2: Changing Reference Frames

(a) We now want to consider how the phenomenon we just observed would appear to an observer that was riding along on the launcher at a constant speed. Assume the moving observer places her origin at the same place you put your origin on the first frame of the movie. Predict how each graph will change for the moving observer.

Horizontal position versus time:

Vertical position versus time:

(b) Use *Tracker* to analyze the film *Galilean_Transformation.mp4* again and determine the position of the projectile in each frame. This time, though, use a moving origin that is placed at the same point on the cart on each frame. Use the same point on the launcher that you used to define the origin in the first frame during the previous activity. To change the origin from frame to frame follow these instructions.

1. Open the movie again in *Tracker* and click on the Axes icon to add your coordinate system (note that the origin will again appear over the movie). Next click the *Coordinate System* drop down menu and unselect the "Fixed Origin" option. This will allow you to move the position of the origin as you advance frames.

2. Click on the origin (where the axes cross each other) and move the origin to the dark spot that appears in front of the cart the launcher rides.

3. Click on the *Create* icon located on the menu bar and then select "Point Mass." To begin collecting data you will need to click on *mass A* in the *Track Control* box each time before marking the position of the projectile. Additionally, you will need to move the location of the origin to the dark spot that appears in front of the cart the launcher rides each time after the position of the projectile is marked. The film will advance as usual. Repeat this procedure to accumulate the x and y positions relative to the origin you've defined in each frame.

(c) Collect the data for analysis. The data table should contain four columns with the values of time, x-position, and y-position and the position of the origin. Print the data table and attach it to this unit.

(d) Plot and fit the position versus time data for the horizontal and vertical positions. See Appendix A for details using *Excel*. When you have found a good fit to the data, record your result below, print the graph, and attach a copy to this unit. Be sure to properly label the units of each coefficient in your equation.

$x(t) =$

$y(t) =$

What is the horizontal speed of the projectile? How did you determine this?

Activity 3: Relating Different Reference Frames

(a) Compare the two plots for the vertical position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected "prediction" here.

(b) Compare the two plots for the horizontal position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected "prediction" here.

(c) What is the difference between the horizontal velocities in the two reference frames? How does this difference compare with the horizontal velocity of the launcher? How are the horizontal velocities of the projectile in each inertial reference frame and the velocity of the launcher that you determined above related to one another? Does this relationship make sense? Why or why not?

(d) Consider a point $\vec{r} = x\,\widehat{i} + y\,\widehat{j}$ on the ball's trajectory in the stationary observer's reference frame. If the moving observer's time frame is moving at the speed $(v_{\text{launcher}})_x$ then what would the moving observer measure for x? Call this horizontal position of the moving observer x'.

(e) What would the moving observer measure for y? Call this vertical position of the moving observer y'.

(f) The relationships you found above are from Galilean relativity. You should have obtained the following results.

$$x' = x - (v_{\text{launcher}})_x t$$

$$y' = y$$

$$v_x = v'_x + (v_{\text{launcher}})_x$$

The primes refer to measurements made in the moving frame of reference in this case. If you did not get these expressions consult your instructor.

Activity 4: Testing Galilean Relativity

You can test your mathematical relationships with *Excel*. You will use your data for the stationary observer and the relationship you derived to calculate what the observer moving with the launcher would measure.

In your data table for the stationary observer, you have columns giving x, the horizontal position, as a function of time, t. Using an *Excel* formula, apply the Galilean transformation above to these numbers to determine x', the horizontal position as measured by the moving observer. Make a plot showing x' as a function of t.

Your "transformed" data for the stationary observer should closely resemble the results for the moving observer. Is this what you observe? If not, consult your instructor. Print and attach a copy of your plot to this unit.

Lab 2   Galilean Relativity and Spacetime Diagrams

Name: _____   Lab Partner(s): _____

Apparatus:

- *Mathematica*, with notebook file *minkowski_events.nb*

Activity 1: Drawing Spacetime Graphs

A spacetime graph, also called a *Minkowski diagram*, is just like a position vs. time graph, except that the axes are flipped.

(a) Suppose that you start at the origin and walk at a speed of 0.5 m/s to a vending machine located at $x = 10$ m). Once you are there, it takes you an additional 20 seconds to buy a candy bar. You then return to your original starting place at a speed of 0.25 m/sec. On the axes below, draw a spacetime graph of the following situation. This graph of your motion is called your "worldline."
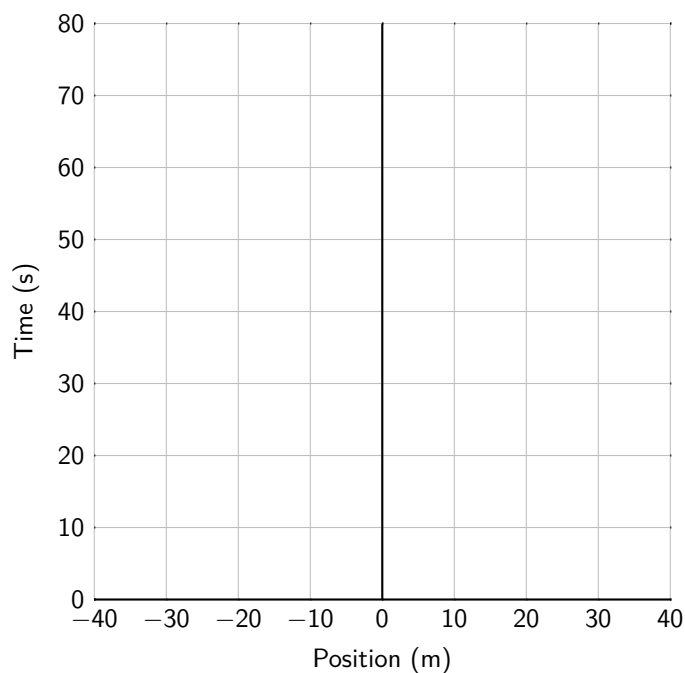

(b) There are four "events" that take place in the graph you just drew:

- You start walking at ($x = 0$ m, $t = 0$ s).
- Your arrival at the machine at ($x = 10$ m, $t = 20$ s).
- Your departure from the machine at ($x = 10$ m, $t = 40$ s).
- Your arrival back where you started ($x = 0$ m, $t = 80$ s).

Use the Galilean transforms that you developed in the previous lab to transform the x coordinates of each of those events into a reference frame that starts with you at the origin at time $t = 0$ and moves at a constant velocity of $v = +0.5$ m/s throughout your journey.

(*c*) On the axes below, draw your "worldline" for your round-trip journey as seen in the moving reference frame above.
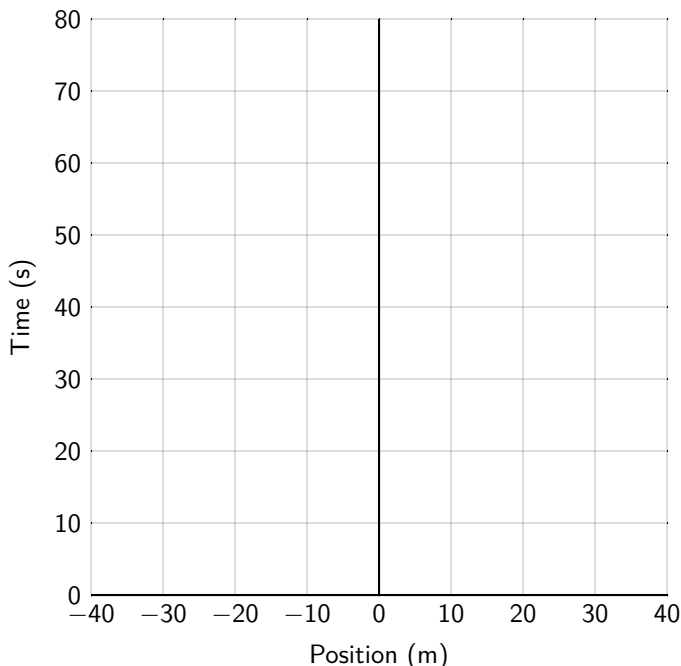
(*d*) Open the file *minkowski_events.nb*, which will open in Mathematica. Type *Ctrl-A* to select everything, and type *Shift-Enter* to execute all of the lines you have selected. Then scroll down to the first graph you see, titled "Galilean Transformation". It shows a spacetime graph for a regular grid of events, shown as blue dots. Two particular events are highlighted in green. As you move the slider left and right, you can see how the events transform into reference frames of different speeds. Go ahead, try it!

(*e*) You can see the line of code above the graph that generates the two points at (5,1) and (3,2). Edit these points, and add a third one, to show the last three points in your round-trip journey to the vending machine from part (a). Divide both x and t by 10 to fit on the graph without having to rescale the axes. When you are done editing, hit *Ctrl-Enter* to make the changes to the graph.

(*f*) Move the slider to view the events in a reference frame with speed $v = 0.5$. Do the results match with your graph above?

Activity 2: Crossing a River

Problem 1: Anna can swim 2 m/s in still water. She wants to swim across a river that flows east with a speed of 1 m/s.

(*a*) Suppose the river is 100 m wide. If she aims directly north, how far downstream from her intended landing point will she end up?

(*b*) What is her speed relative to the shore if she aims due north across the river, as in part (a)?

Answer: 2.24 m/s

(*c*) What is her speed relative to the shore if she swims at an angle so that she ends up on the shore directly across from where she started? Drawing a picture below will help.

Answer: 1.73 m/s

Problem 2: Anna and Bob can both swim at speed v_s in still water. They decide to have a race in a river of width L that flows east at speed v_r.

- Anna swims across the river and back. She swims at an angle to the current so she returns exactly to her starting point. (Anna swims north, then south.)
- Bob swims upstream in the river against the current for a distance L (relative to the shore), then swims back downstream with the current, also returning exactly to his starting point. (Bob swims west, then east).

(*a*) What is Anna's speed relative to the shore?

Answer: $\sqrt{v_s^2 - v_r^2}$

(*b*) What is Bob's speed relative to the shore when he swims upstream? What is his speed relative to the shore when he swims downstream?

(*c*) What is Anna's time for her round trip?

Answer: $\dfrac{2L}{\sqrt{v_s^2 - v_r^2}}$

(*d*) What is Bob's time for his round trip?

Answer: $\dfrac{2Lv_s}{(v_s^2 - v_r^2)}$

(*e*) Who wins the race? *Hint: Put your answers to (c) and (d) over a common denominator.*

Lab 3   The Twins Paradox

Name: _____   Lab Partner(s): _____

Objective

To investigate some of the unusual implications of Einstein's special theory of relativity.

Overview

Einstein's theory of special relativity leads to a variety of apparent paradoxes that depart radically from our everyday expectations. One of the most celebrated is the twins paradox, in which an identical twin makes a long interstellar journey while the other twin remains on the (roughly) stationary Earth. When the space-faring twin returns she finds her partner has aged considerably more than she has. In this unit you will explore the quantitative aspects of the paradox and some of the surprising consequences.

Activity 1: Setting Things Up

Problems in special relativity are often very counterintuitive, so it is instructive to consider the situation non-relativistically first. Investigate this problem without applying any of the new ideas you have learned about the theory of special relativity.

One member of a pair of identical twins has decided to embark on a long space voyage. The two twins have lived their lives in close proximity to one another and are very similar in appearance. The adventurous twin boards a fast spacecraft and leaves the Earth behind at a speed of 0.99c or 99% of the speed of light. The space-faring twin's itinerary is rather monotonous, and she simply travels at this constant speed for a time, turns around, and returns to the Earth at the same speed. She measures the time of her trip to be Δt_0. In the meantime the Earth-bound twin has seen twenty years pass by. We will refer to this time as Δt.

(a) In mathematical terms, what is the relationship between the times Δt_0 and Δt?

(b) Which time is associated with which twin?

(c) When the twins are reunited will their appearances differ?

Activity 2: Applying Special Relativity

(a) Now we want to apply the lessons of special relativity. Time dilation implies that moving clocks run more slowly when observed by someone in a different inertial frame. For the twins paradox what does this imply about the time interval the space-faring twin measures during her trip? Will it be less than, equal to, or greater than the interval measured by the Earth-bound twin? Will the space-faring twin age more, less, or the same amount as the Earth-bound twin?

(b) What is the mathematical relationship between Δt_0 and Δt according to the special theory of relativity?

(c) How much time has passed on the Earth-bound twin's clock?

(d) How much time has passed on the space-faring twin's clock?

(e) If this result is inconsistent with your prediction above how should you resolve the contradiction?

(f) How will the two twins' appearances differ, if at all? Is the difference only in the measurement of the time intervals or are there real physiological differences between the twins after the trip?

(g) If the average speed of the space-faring twin was more like the typical orbital speed of the space shuttle (about 7.4 km/s) what would the time difference between the twins' clocks be?

Activity 3: Graphical Analysis

(a) Find a mathematical relationship for the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. Show your work and record your result here.

(b) You will now use *Excel* to make a plot showing how this ratio behaves as a function of β (the speed expressed as a fraction of the speed of light). To do this, start up *Excel*, and create a column headed "beta." This column should contain the series of numbers 0,0.05,0.10,0.15,...,1. To create such a column of numbers, enter the first two rows and highlight them. Then grab the lower-right corner of the second cell with the mouse and drag down (in the same way as if you were dragging down a formula).

After you have created the β column, create a second column containing the ratio of time intervals (i.e., the relationship you found in part (a)). Use an *Excel* formula.

(c) Make a plot of the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. At what speed does the effect of time dilation become significant? Is there a limit to the ratio? Is there any reason to restrict the range of β to 0-1? Clearly state your reasoning. Print your plot and attach a copy to this unit.

(d) Consider the following scenario. As the space-faring twin's craft recedes from the Earth it is moving at a constant speed. Since no inertial frame can be considered "better" than any other there is nothing physically inconsistent with the view that the space-faring twin is observing the Earth recede from her at a constant velocity. Hence, the space-faring twin will observe clocks on the Earth to move slowly and the Earth-bound twin will age at a slower rate than the space-faring one. Is this reasoning flawed? How?
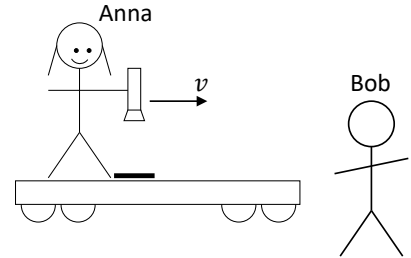
(e) If the scenario is not flawed how can it be that the space-faring twin was found to have aged less in the original problem?

Lab 4   Time Dilation and Length Contraction

Name: _____   Lab Partner(s): _____

Activity 1: Time Dilation

Anna rides a train towards the east at a speed v such that $\gamma = 1.5$. She turns on a flashlight, which shines onto a mirror directly below it.

(*a*) If Anna measures the light's round trip time to be 6 nanoseconds, what does Bob measure?

(*b*) Now Anna and Bob try a variation of the experiment. Anna rides on the train, as before, and the train moves east again, at the same speed v, so that $\gamma = 1.5$. But this time Bob holds the flashlight and the mirror. Both Anna and Bob time the round trip. What do Anna and Bob each measure for the round trip time Δt?

(*c*) In an experiment like what Anna and Bob are doing, the shortest measured time between two events (like the emission and detection of a photon) is called the "proper time." What determines who measures the shortest time? Is it who's on the train? Is it who holds the flashlight? Is it who holds the mirror?

(*d*) Write a definition of the *proper time,* Δt_0, between two events.

Activity 2: What about Length?

Anna rides the train again at the same speed, and Bob stands on the side, holding a long rod along the direction of the train tracks. Bob measures the rod to have length L_0. Bob sees Anna traveling past at speed v. Events "A" and "B" are Anna's nose being even with the left and right ends of the rod, respectively. Bob measures the time between event A and event B as $\Delta t = L_0/v$.
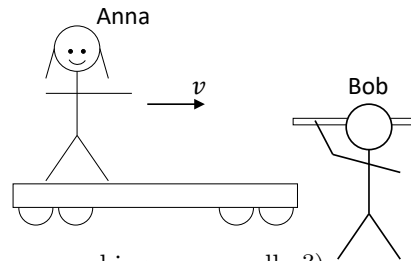
(*a*) What does Anna say is their relative speed v? (Same as what Bob measures, bigger, or smaller?)

(*b*) In Anna's frame, what is the time between events A and B, in terms of γ and Bob's Δt? (Hint: which of the two observers measures the proper time between the two events?)

(*c*) What does Anna infer is the length L of the rod? (Answer in terms of L_0 and γ.)

(*d*) Bob's measurement, L_0, is the "proper length" of the rod. Write a definition of "proper length" that highlights why Bob's measurement is different from that made in any other reference frame.

Lab 5 Muon Lifetimes

Name: _____ Lab Partner(s): _____

Background

Muons are rare subatomic particles which are not usually found in ordinary matter on Earth. Muons can be created when protons or atomic nuclei, traveling at high speeds from the sun or other stars, strike the Earth's upper atmosphere and collide with molecules in the air. As a result of these collisions, there are always muons raining down from the upper atmosphere towards the Earth's surface.

But muons don't last long. At rest in a laboratory, muons have an average lifetime of only 2.2 μs, spontaneously decaying into (usually) an electron and two neutrinos. Given an initial number of muons N_0 at time $t = 0$, the number of muons N at any later time t is given by

$$N = N_0 e^{-t/\tau}, \tag{1}$$

where $\tau = 2.2$ μs is the muons' average lifetime. Note that this equation can also be written as $N = N_0 e^{-\lambda t}$, where λ is called the "decay constant," and is simply $1/\tau$.

The average lifetime τ of a muon is slightly different from the "half life" of a muon. The half life would be the time it takes for the number of muons to fall to $1/2$ of the original number. The average lifetime τ used in this lab is the time it takes for the number of muons to fall to $1/e$ of the original number.

Activity: Working through an example of muon lifetimes

(a) Suppose that muons are created in the Earth's atmosphere at a height of 4 km above the surface. The muons stream towards the ground at a speed of $0.93c$. In the reference frame of the Earth, how long does it take these muons to reach the ground? (This is a straightforward question using physics you likely learned in high school.)

(b) What percent of the original muons should be remaining after this time, according to Equation (1)?

(c) The time Δt that you calculated in (a) is the time between two specific events: the muon being created and the muon hitting the Earth's surface. What reference frame measures the *proper time* Δt_0 between these two events? Is it the reference frame of a scientist on Earth, or the reference frame of the muon?

(*d*) What is the time of the journey from the upper atmosphere to the ground in the reference frame of the muon, traveling at speed $0.93c$?

(*e*) What percent of the original muons should be remaining after this time Δt_0, in the reference frame of the muons?

(*f*) Whatever causes a muon to decay takes place entirely within the muon itself. (That's why it's called "spontaneous" decay; it's not caused by anything external to the muon.) Is the time t that governs the average lifetime of a muon the time as measured in the Earth's reference frame, or the muon's reference frame?

(*g*) If you measure the fraction of muons that *actually* reach the ground, what fraction will you observe? (This observation was first done in the 1920s, and was an early experimental confirmation of Einstein's theory of relativity.)

(*h*) One more thing. In part (d), you probably calculated the muons' Δt_0 using the idea of time dilation. Now, try doing it again using the idea of length contraction. In the reference frame of the muon, what is the distance between the upper atmosphere and the ground? (*Hint: Who measure's the proper length L_0 of the atmosphere? Is it a scientist on Earth, or the muons?*)

(*i*) How long does it take the muon to travel that distance?

# Lab 6   Galilean Transformations and Lorentz Transformations

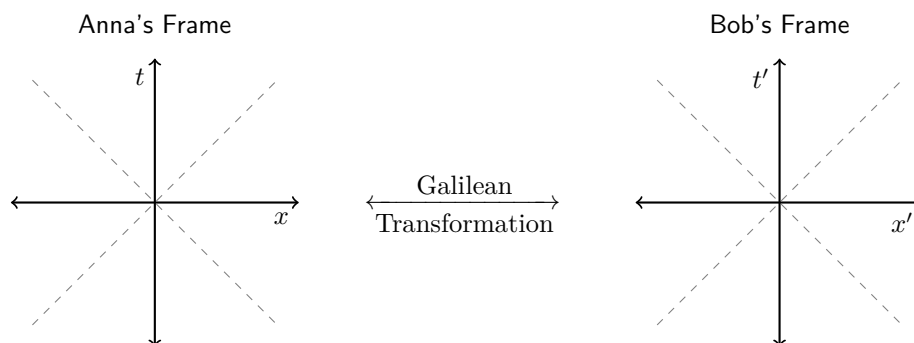Name: _____   Lab Partner(s): _____

Apparatus:

- *Mathematica*, with notebook file *minkowski_events.nb*

Activity 1: Transformations in Space and Time

You have previously discussed a thought experiment in which Anna rides a train at speed v. She aims a laser beam at a mirror on the floor of the train, and measures the time Δt_0 that it takes from the time the photon is emitted to when the photon returns to the laser. Bob observes the same experiment, standing near the tracks beside the train.
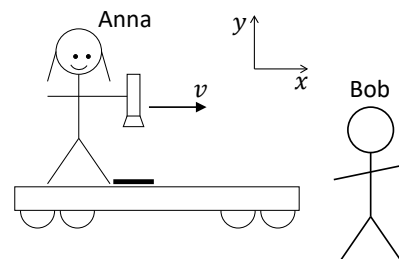
(*a*) On the axes below on the left, draw a spacetime graph (Minkowski diagram) of these two events as seen in Anna's reference frame, taking the emission of the photon to happen at $x = 0$ and $t = 0$. *Hint: think about your axes. In Anna's frame, the photon only moves in y, not x.*

<div align="center">
Anna's Frame   Bob's Frame
</div>

$$t \qquad\qquad\qquad\qquad t'$$

$$x \qquad\qquad\qquad\qquad x'$$

<div align="center">
Galilean
Transformation
</div>

(*b*) Now consider the same two events in the reference frame of Bob, who is standing beside the tracks so that the emission of the photon happens at $x' = 0$ and $t' = 0$ in his reference frame too. On the axes above on the right, draw a spacetime graph of these two events as seen in Bob's reference frame, according to the principle of Galilean relativity (as described the Galilean transformations).

(*c*) Once you have drawn your graph, you can check your answer by opening the file *minkowski_events.nb*, which will open in Mathematica. Type *Ctrl-A* to select everything, and type *Shift-Enter* to excute all of the lines you have selected. Then scroll down to the first graph you see, titled "Galilean Transformation". You can see the line of code above the graph that generates the two points at (5,1) and (3,2). Edit the locations of those points to the values for the two events in Anna's reference frame, and hit *Shift-Enter* to excute your changes. Use the slider to see the events in Bob's reference frame, and make any corrections needed on your graphs above.

(*d*) The graph you drew for Bob's reference frame is inconsistent with the principle of relativity, because it implies that the speed of light is not the same in all reference frames. In Anna's frame, light travels only the vertical distance down and back in some time t. In Bob's frame, the light travels a longer, diagonal path. According to the Galilean transformations, it does so in the same time t as in Anna's frame, which would mean that Bob would see the light going faster than Anna does. To correct this, would the time t of the light returning to the flashlight be *earlier* or *later* than in Anna's frame?

(*e*) On the axes below, redraw the two events as seen in Anna's reference frame on the axes to the left. On the axes on the right, draw the two events as you know they should be, consistent with the principle of relativity.

Anna's Frame   Bob's Frame

Lorentz
Transformation
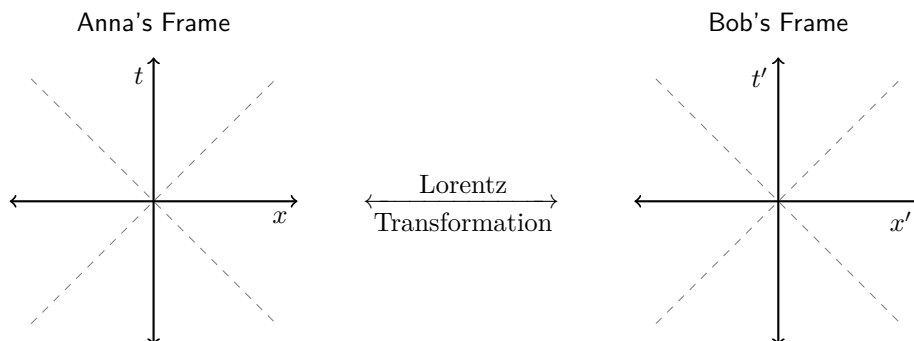
(*f*) Now that you have drawn your graph above, you can check your answers again using the same Mathematica simulation as before, but this time scroll down further to the second graph, titled "Lorentz Transformation". Again, edit the positions of the two green points, and move the slider to represent the speed of Bob's reference frame relative to Anna's. Was your prediction correct? Note any corrections on the graphs above, as needed.

Galilean transformations just don't account for relativistic effects like time dilation or length contraction at very high speeds, though they are an excellent approximation at low speeds, $v \ll c$. The second graph in the Mathematica file uses *Lorentz transformations* instead, which properly account for changes in space and time at high speeds.

Activity 2: Simultaneity and the Speed of Light

Anna rides on a train in the $+x$ direction. Facing the side of a train car, she holds out her arms (each one meter long) to the sides, with one lightbulb in each hand. In Anna's reference frame, the bulbs turn on simultaneously. Bob views this while standing beside the tracks as the train zooms by.
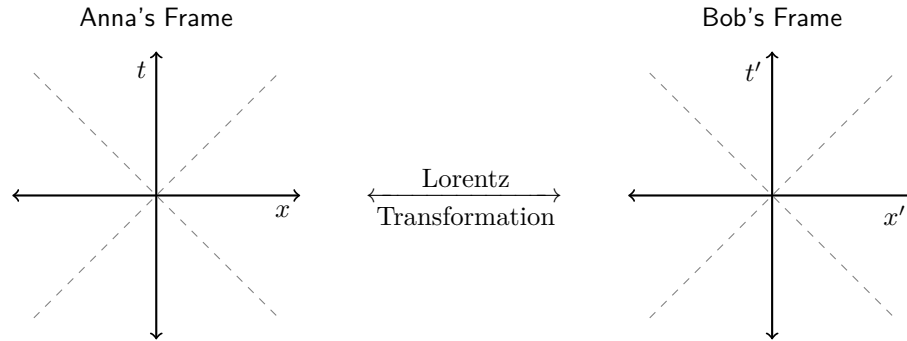
(*a*) On the axes below on the left, draw a spacetime diagram in the reference frame of Anna showing three events: each of the two lightbulbs turning on, and the photons from both bulbs arriving at the tip of Anna's nose. Place the photons hitting Anna's nose at $x = 0$ and $t = 0$, so that the bulbs turn on at some $t < 0$. On the axes to the right, draw the same three events as they would occur in Bob's reference frame, moving to the left with velocity $v = -0.5c$, *according to the Galilean transformations.*

Anna's Frame   Bob's Frame

Galilean
Transformation
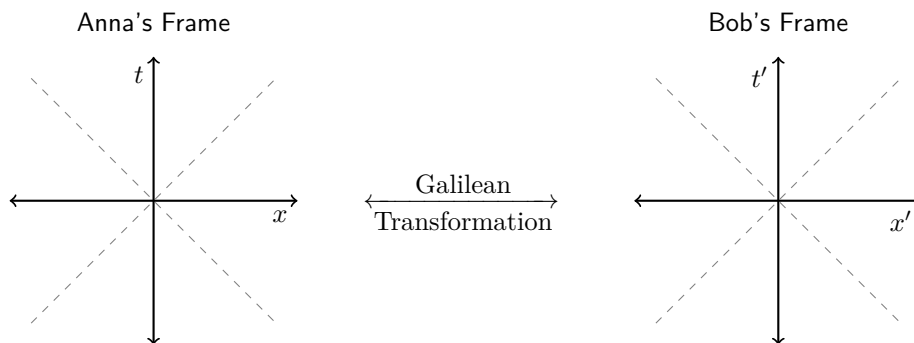
(*b*) As before, check your results using the *Galilean Transformation* simulation in Mathematica, noting any corrections on your axes above. Was your graph for Bob's frame correct?

(*c*) Are the two graphs above consistent with the postulate of relativity? Why or why not?

(*d*) On the axes below, redraw the same three events in both Anna's reference frame and Bob's reference frame, according to the Lorentz transformation. You can try it on your own first if you like, then use the Lorentz Transformation simulation in Mathematica to confirm or correct your understanding.

Anna's Frame $\qquad\qquad$ Bob's Frame

t $\qquad\qquad\qquad\qquad\qquad$ t'

$\xleftarrow{\text{Lorentz}}$
$\overrightarrow{\text{Transformation}}$

x $\qquad\qquad\qquad\qquad\qquad$ x'

(*e*) In the graphs above, is light traveling at speed c in both reference frames?

(*f*) Are the two events of the bulbs turning on simultaneous in both reference frames?

Lab 7   The Pole and Barn Paradox

Name: _____   Lab Partner(s): _____

Apparatus:

- *Mathematica*, with notebook file *pole_and_barn.nb*

An unrealistically speedy pole vaulter named Anna runs with her 10 meter long pole at $0.8c$ in the positive x direction. She runs into an 8 meter-long barn, which has doors on both sides.

Here's what happens, according to farmer Bob, standing inside the barn:

- The pole is length-contracted so that it's shorter than the barn.
- When the pole is centered inside the barn, Bob pushes a button so that both doors shut very quickly with the pole entirely inside the barn.
- Bob says, "Ah-ha! I've closed Anna's pole inside the barn."

But here's what happens according to Anna:

- Her pole is 10 meters long.
- The barn is length-contracted to less than 8 meters long.
- Anna says, "My pole couldn't possibly have been closed inside the barn!"

This seems like a paradox, since both observers have to agree on any tangible outcome of an experiment!

1. Open the file *pole_and_barn.nb* in Mathematica. If you get a pop-up error message, you may need to click *Enable Dynamic Content*. Type *Ctrl-A* to select all lines, and hit *Shift-Enter* to execute them. The graph you see represents a "spacetime diagram" (or "Minkowski diagram") of this set of events, with x on the horizontal axis and t on the vertical axis. If the units on the horizontal axis are in meters, and the diagonal dotted lines represent objects traveling from the origin at the speed of light, what are the units of the vertical (time) axis?

2. With the velocity slider set to $v = 0$ (the default), the diagram is in the reference frame of farmer Bob. The red lines represent the positions in time, or *worldlines*, of the two barn doors, and the purple lines represent the worldlines of the two ends of Anna's pole. Based on this diagram, what is the approximate length of Anna's pole according to Bob?

3. Verify your answer in part 2 by doing a calculation to determine the precise length contraction of the pole according to Bob.

4. Now, move the slider so that you view these events from Anna's reference frame. From the graph, how long is the barn according to Anna?

5. Verify your reading of the graph in part 4 by calculating the precise length of the barn in Anna's reference frame.

6. Now move the slider back to Bob's reference frame. According to Bob, at what time is the pole exactly centered inside the barn?

7. There are two red dots in the graph at random locations. Edit the Mathematica file in the line that looks like

```
ListPlot[{lorentz[-3,3,v],lorentz[-3,4.5,v]},PlotStyle ->{PointSize[Large],Red}]
```

to change the coordinates of those dots so that they mark the space and time coordinates of the closing of the two barn doors in Bob's reference frame. What are those exact coordinates?

8. Describe the sequence of events in Bob's reference frame (move upward in time). An event description would be something like "front end of pole through left barn door" (the first event), or "front end of pole crashes through right barn door."

9. Now move the slider back to Anna's reference frame. Describe the sequence of events according to Anna. Hint: The first event is still "front end of pole through left barn door".

10. So, was the pole ever closed inside the barn in either frame? Who has the "correct" view—Anna? Bob? Neither? Both? (The resolution lies in the idea that different observers can measure different times and events that are simultaneous in one frame may not be simultaneous in another.)

Lab 8   Relativistic Velocity Addition

Name: _____   Lab Partner(s): _____

Objective To investigate relativistic effects on an object that is moving away from an observer. This problem is an important step in understanding relativistic velocity addition.

Overview

Einstein's theory of special relativity leads to a variety of effects that are dramatically different from our everyday expectations. Special relativity predicts that no object can move faster than the speed of light. An important step in understanding this effect can be made by examining the time interval between light pulses emitted by a receding beacon.

Activity 1: Doing It The Easy Way First

Problems in special relativity are often very counterintuitive so it instructive to consider the situation non-relativistically first. Investigate this problem classically first.

An observer on the (roughly) stationary Earth watches a spaceship traveling away and towards the galactic core at a speed v_r. The craft emits a burst of light from a beacon that is later observed by the Earthbound observer. After a proper time interval Δt_{rocket} as measured by the captain of the spaceship, another burst of light is emitted. A schematic drawing of the two events is shown below. The ship travels a proper distance Δd between pulses as measured by the Earthbound observer.
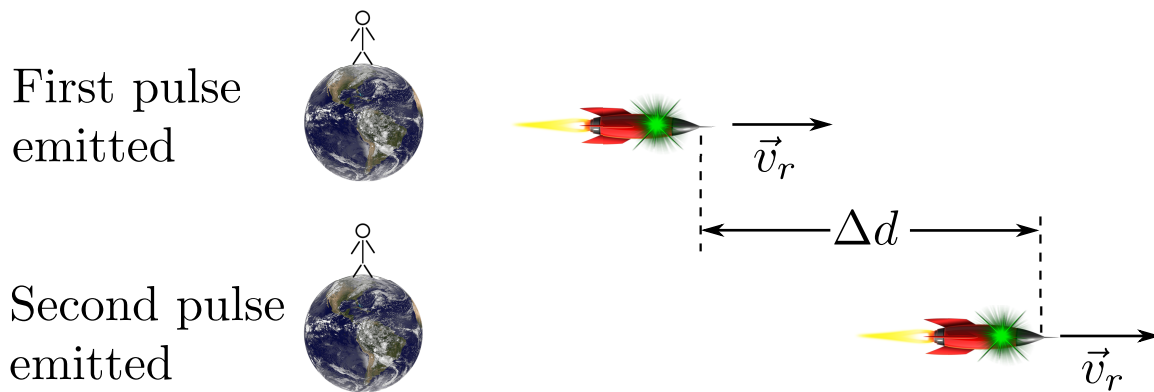
How far does the Earthbound observer see the receding spaceship travel between light pulses? Call this Δd. Express your answer in terms of the speed of the spaceship v_r and the time interval between pulses as measured by the captain of the spaceship.

How long is the time interval between light pulses as measured by the Earthbound observer?

Activity 2: Applying Special Relativity

You should have found in the previous section that the time interval between the pulses as measured on the Earth Δt_{Earth} is

$$\Delta t_{Earth} = \Delta t_{rocket} + \frac{v_r \Delta t_{rocket}}{c}$$

which includes the interval between the receding spacecraft emitting the light pulse and the effect of the craft moving away from the Earth between pulses. We will now analyze the same problem using the tools of special relativity. We expect time dilation to alter the measurements made by the Earthbound way. Will the time interval measured by the Earthbound observer be shorter, longer, or unchanged? Why?

The captain of the spaceship measures the time interval between the emission of the light pulses as Δt_{rocket}. What would the Earthbound observer measure for the time interval between the emission of the light pulses of the spaceship? Be careful here of an important distinction. The time interval between the emission of the light pulses on the spaceship is different from the time interval between the arrival of those pulses at the Earthbound observers location because the spaceship is receding from the earth.

What is the time interval between the arrival of the two pulses of light at the Earth?

Show that your result above can be written as

$$\Delta t_{Earth} = \sqrt{\frac{1 + \beta}{1 - \beta}} \, \Delta t_{rocket}$$

where $\beta = v_r / c$.

Activity 3: Adding Velocities

Now consider two rockets B and C receding from the Earth at A as shown in the figure. Rocket C is going faster than rocket B. Let v_B be the speed of rocket C RELATIVE to rocket B and Δt_C be the proper time between light pulses emitted by rocket C as measured on C. What is the time interval Δt_B between the pulses from C as measured on rocket B? (Hint: Consider using the result from Activity 2.)
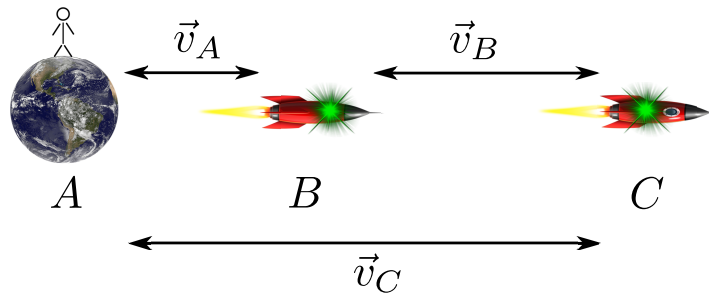
Each time the pulse from rocket C reaches rocket B someone on rocket B sends out a light pulse to Earth. Rocket B is moving at a velocity v_A relative to the Earth. What time interval Δt_A does the observer on Earth measure in terms of Δt_B?

You should have found in the previous step that

$$\Delta t_{Earth} = \Delta t_A = \sqrt{\frac{1 + \beta_A}{1 - \beta_A}} \ \Delta t_B$$

where $\beta_A = v_A/c$ and v_A is the speed of rocket B relative to rocket A. Now use your result above for Δt_B to get Δt_A in terms of Δt_C, β_A, and β_B.

Now use the result of Activity 2 to write down a different expression for Δt_A in terms of Δt_C (time interval between pulses as measured on rocket C) and v_C which is the speed of rocket C relative to rocket A.

You should now have two expressions for Δt_A.

$$\Delta t_A = \sqrt{\frac{1+\beta_A}{1-\beta_A}}\sqrt{\frac{1+\beta_B}{1-\beta_B}}\,\Delta t_C = \sqrt{\frac{1+\beta_C}{1-\beta_C}}\,\Delta t_C$$

where $\beta_B = v_B/c$ and v_B is the speed of rocket B relative to A (the Earth). Equate the factors under the square roots and generate an expression for β_C in terms of just β_A and β_B. Transform that result into velocities (v_A, v_B, v_C, and c). Compare this result with the expression in your text.

Lab 9   Einstein Solid

Name: ————————————————   Lab Partner(s): ————————————————————

Objective

To develop a quantum-mechanical model of an elemental solid (*e.g.* aluminum) and introduce the ideas of statistical mechanics.

Overview of the Model

One of the earliest successful applications of quantum mechanics was done by Albert Einstein in 1907 when he developed a model of an elemental solid (*i.e.* one that consists of a single element from the periodic table like aluminum, lead, *etc.*). We start by assuming that each atom in the solid is bound in a square lattice with each of six neighbors. Each bond is treated as a simple spring so the mechanical energy for a single atom is

$$E = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + \frac{1}{2}k(x^2 + y^2 + z^2) \tag{1}$$

where k is the spring constant of the bond, the coordinates x, y, and z are relative to the equilibrium position of the atom and v_x, v_y, and v_z are the components of the velocity. Einstein used an idea pioneered by Max Planck in 1901 and guessed the energy in the solid came in discrete pieces or quanta that were all the same size. Adding or removing these quanta heated or cooled the solid. Many years later the quantum mechanical energy E for a mass on a spring was found to be

$$E = (n_x + n_y + n_z + \frac{3}{2})\hbar\omega \tag{2}$$

where $\hbar$ is Planck's constant, ω is related to the spring constant k of the bond mentioned above, and n_x, n_y, and n_z represent the number of quanta associated with each degree of freedom of the spring. The degrees of freedom here correspond to the three possible directions each atom can vibrate. The size of each energy quantum is $\epsilon = \hbar\omega$. The total number of energy quanta in the solid is labeled q_A so the internal energy is $E_{int} = q_A\hbar\omega$. We then assume that all microstates of the solid have an equal probability of being populated. A microstate is a specific arrangement of the quanta on the atoms in the solid.

Activity 1: The Statistics of Matter

Before you embark on building the model of the Einstein solid consider some ideas from your previous study of gases. You will make some predictions here about the statistical nature of matter that you can refer back to later on in this unit.

(a) Consider a gas in a container. Would it violate Newton's Laws or any other physical law if all the particles in the gas collided in such a way that all of the gas particles ended up in the bottom half of the container leaving the top half empty?

(b) Is such a scenario likely? Explain.

(c) If you started out with all the gas in the bottom half of the container how likely is it to stay there?

The questions you answered above are addressing the notion of irreversibility. Many processes in nature appear to proceed in one 'direction' only. When you add milk to coffee it disperses throughout the coffee. After it is

dispersed, the milk never re-concentrates into a blob of milk in the middle of the coffee. These processes go from a more orderly configuration (a concentrated drop of milk) to a disordered state (milk spread throughout the liquid). The reverse never happens. We will return to this notion again in this laboratory.

Activity 2: Calculating the Multiplicity of Some 'Solids'

(a) You will first calculate the configurations of the quanta (the microstates) for a VERY simple solid consisting of a single atom! The number of atoms for solid A is $N_A = 1$ so there are three degrees of freedom $N_a = 3$ because there is one degree of freedom for each spatial direction. The atom's vibration can be decomposed into three components, one for each direction. Let the 'solid' contain two quanta of energy so $q_A = 2$. Make a table with the headings n_1, n_2, and n_3 and in each row enter one arrangement of the two quanta. Each row in the table is a microstate. Make a table with all of the possible microstates. The multiplicity Ω_A of the system is the number of all possible microstates. What is your multiplicity? Record it here.

(b) You can calculate the multiplicity Ω_A using the expression

$$\Omega(N_A, q_A) = \frac{(q_A + 3N_A - 1)!}{q_A!(3N_A - 1)!} \tag{3}$$

Make the calculation for $N_A = 1$ and $q_A = 2$. Does this agree with your result in part 2.a?

(c) Now do the same thing for a different 'solid'. This time for solid B, let $N_B = 2$ (two whole atoms!) and $q_B = 1$. How many degrees of freedom does solid B have? Make a table analogous to the one in part 2.a on the same sheet as before. What is the multiplicity of solid B? Record it here. Use the expression in Activity 2.b to check your calculation.

Activity 3: Putting the 'Solids' Together

When two solids are brought together heat/energy can flow between the two objects. For the model of the Einstein solid you are building this corresponds to energy quanta ($\hbar\omega$) moving from atom to atom and occupying different microstates of the combined system.

(a) Now bring the your solids A and B 'together' into a single system. What is N_{AB} the total number of atoms? What is the number of degrees of freedom of the combined system?

(b) What is the total number of energy quanta q_{AB} for the combined system?

(c) The system is in its initial macrostate. A macrostate is a configuration of the system defined here by the total number of atoms and quanta in each solid. In this case the macrostate is defined by $N_A = 1$, $q_A = 2$, $N_B = 2$, and $q_B = 1$. What is the total multiplicity Ω_{AB} for the combined system with $q_A = 2$ and $q_B = 1$ in its initial macrostate?

(d) Now take the energy quantum in solid B and put it in solid A, *i.e.*, let heat flow from solid B into solid A. This is now a macrostate where $q_A = 3$ and $q_B = 0$. What is the new multiplicity Ω_A for solid A and the multiplicity Ω_B for solid B?

(e) What is the multiplicity Ω_{AB} for the combined system (solids A and B)?

(f) Remember that a macrostate is defined by the combination of N_A, N_B, q_A, and q_B. Which macrostate had the greatest multiplicity, ($q_A = 2$, $q_B = 1$) or ($q_A = 3$, $q_B = 0$) (remember that N_A and N_B are the same in each configuration so we don't list those parameters here)?

(g) If the energy quanta can move from atom to atom which macrostate ($q_A = 2$, $q_B = 1$) or ($q_A = 3$, $q_B = 0$) is most probable? Why?

(h) If you started out in the ($q_A = 3$, $q_B = 0$) macrostate is it more likely that you will remain in that macrostate of evolve to the ($q_A = 2$, $q_B = 1$) macrostate? Why?

What you have discovered is a version of the irreversibility mentioned earlier, One macrostate ($q_A = 2$, $q_B = 1$) is preferred over the other because it has more microstates than the other. This result depends critically on your assumption that all states are equally populated.

Activity 4: Using *StatMech* **For More Complex Cases**

You should have found in the previous activity that the ($q_A = 2$, $q_B = 1$) macrostate was more likely to occur and the process proposed in part 3.d is relatively unlikely. In other words, it is more likely for energy to be spread evenly throughout the system. This is good news because it means the statistical picture we are painting is consistent with reality. Remember what happens to the blob of milk in the coffee.

(a) You should realize that making the sorts of calculations you did in Activity 3 above would become rather painful for say $N_A = 300$ atoms. In order to push the model further you will use a software packaged called *StatMech* to perform the same calculations. To run the program go to the `Physics Applications` menu and click on `StatMech`. You should see a window like the one below. The top of the window has several entry boxes
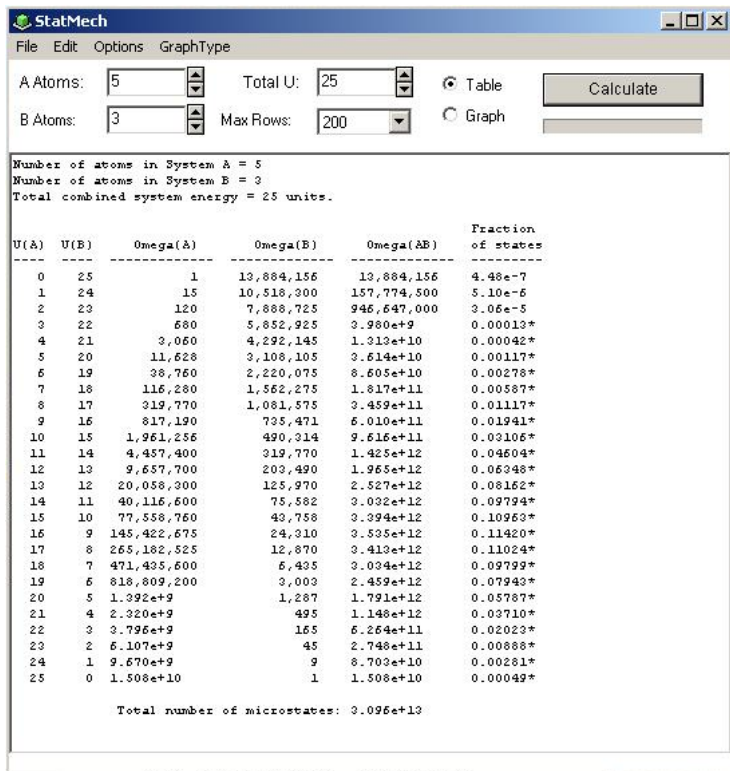

Figure 1: The `StatMech` window showing the table of multiplicities for each microstate. Each row corresponds to a different value of q_A.

where you can set the number of atoms (N_A and N_B) and the total number of energy quanta in the system U. The parameter U is the total internal energy E_{int} in the system in units of $\epsilon = \hbar\omega$. It is equivalent to the sum $q_{AB} = q_A + q_B$. You can also set the number of rows of microstates to print out or choose to view a graph instead of the table. To test the operation of *StatMech* redo the calculations of the microstates that you did in Activity 3. Make sure your results in Activity 3 agree with the output of *StatMech*. You will also see there are other macrostates that were ignored in Activity 3 for simplicity.

(b) Now run *StatMech* for the case where $N_A = 10$, $N_B = 20$, $U = 500$. What is the value of q_A for the most probable macrostate? Record it here. Click on the button at the top of the *StatMech* window and choose graph. You will see a graph of the table and it should look something like Figure 2. The vertical axis is the probability of a particular macrostate divided by the maximum probability of any macrostate. The horizontal axis is U_A/U_{TOT} where U_A is the energy of solid A in units of $\hbar\omega$ (equivalent to q_A) and U_{TOT} is the total internal energy of the solid in units of $\hbar\omega$ (equivalent to the total number of quanta q_{AB}). What is the value of U_A/U_{TOT} for the most probable state? How is this value related to the value of q_A for the most probable macrostate? Also, explain in words what this plot is showing you.
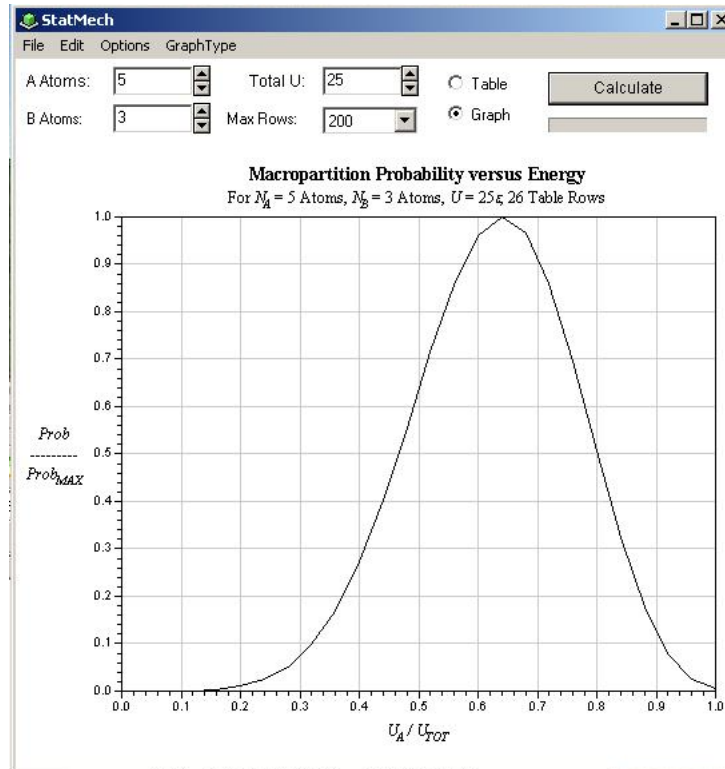
Figure 2: The `StatMech` window showing a graph of the multiplicities as a function of E_A/E_{int} where $E_A = q_A\hbar\omega$ and $E_{int} = q_{AB}\hbar\omega = (q_A + q_B)\hbar\omega$.

(c) How wide is the distribution of microstates? Measure this number by estimating the full-width-half-maximum (FWHM) from your graph. Do this by finding the largest value on the vertical axis, divide that value by two, and find the two points on either side of the peak where the distribution is equal to that half-maximum. Take the difference between these two points along the horizontal and this is the FWHM. Record your result here.

(d) Now repeat steps 4.b-c with $U = 100,000$ and N_A and N_B at their last values. What is the most probable value of U_A/U_{TOT} and the FWHM? How have things changed?

(e) Keeping $U = 100,000$ now repeat steps 4.b-c, but this time double the values of N_A and N_B. Record the most probable macrostate and the FWHM. Repeat this doubling of the number of atoms in each solid while keeping U fixed at least 3-4 times. Record the most probable macrostate and the FWHM each time along with N_A and N_B.

(f) How does the value of q_A for the most probable macrostate change as the number of atoms increases?

(g) How does the FWHM change as the number of atoms increases?

Activity 5: Irreversibility

You will now use the results from the previous Activity to delve into some of the implications of the statistical mechanics of the Einstein solid.

(a) As the number of atoms increases, what happens to the probability for finding the system in a macrostate different from the most probable one? Use the results of your calculations to explain your answer.

(b) When the system is in a macrostate far from the most probable one, what is the most likely thing to happen as energy or heat flows around the system?

(c) For the last calculation in Activity 4 what is the probability of the state with the minimum value of q_A? In other words what is the probability that all of the quanta would end up all in solid B? What is the probability of the most probable macrostate?

(d) Go back to the questions in Activity 1 and look at your answers. Do they still appear to be correct? A situation where all of the gas particles end up in one part of the container is a macrostate of the system analogous to the situation in Activity 5.c where all of the quanta end up in one of the solids and not the other. Answer those questions in Activity 1 again in terms of microstates, macrostates, and probability.

The behavior you are seeing here is for an Einstein solid, but is actually typical for most macroscopic systems. These systems have a large number of atoms or molecules with a variety of different energy states available.

They evolve to the most probable macrostate and there is essentially no chance to occupy a state far from the most probable one. When two materials are first put in thermal contact they may be far from the most probable macrostate, but they equilibrate at that most probable one (where the temperatures are equal) and never go back. This is irreversibility.

Lab 10   Entropy and Temperature

Name: ——————————————   Lab Partner(s): ——————————————

Objective

To explore the connection between the fundamental definition of entropy and temperature in an Eintein solid.

Overview

Consider the microscopic definition of the entropy of a system

$$S = k_B \ln \Omega \tag{1}$$

where k_B is Boltzmann's constant and Ω is the multiplicity or number of microstates. A microstate is defined by a particular arrangement of energy quanta among the atoms. A macrostate is defined by the total number of energy quanta q and the number of atoms N. We are building a model of an elemental solid (*e.g.*, like aluminum) where the total internal energy in the solid E_{int} is described by

$$E_{int} = q\hbar\omega = q\epsilon \tag{2}$$

where $\hbar$ is Planck's constant divided by 2π, ω is a constant that characterizes the strength of the bonds between the atoms, and ϵ is shorthand for the combination. The parameter q is the total integer number of quanta in the system and is a constant. These quanta are statistically distributed over the N atoms of the solid so all possible states of the system are equally likely and the multiplicity Ω is

$$\Omega = \frac{(q + 3N - 1)!}{q!(3N - 1)!} \qquad . \tag{3}$$

This model of an elemental solid is called an Einstein solid.[1]

We want to find a connection between the entropy defined in Equation 1 and the temperature. Recall how temperature is usually defined relative to some properties of matter like the freezing and boiling points of water. In gases temperature is connected to the average kinetic energy of the gas molecules. You are developing the microscopic picture of entropy, but it won't be successful until you can connect it to the observed, macroscopic behavior of bulk matter and our familiar notions of temperature.

Activity 1: The Entropy of Einstein Solids in Thermal Equilibrium

(a) To start connecting the entropy to the temperature you have to study the behavior of the entropy as the energy changes. To do this we will study two Einstein solids (A and B) in thermal equilibrium with each other. Their total internal energy will be

$$E_{int} = q_{AB}\epsilon = (q_A + q_B)\epsilon \tag{4}$$

where q_A and q_B are the numbers of energy quanta in each solid and q_{AB} is their sum.

Use the program *StatMech* (see the `Physics Applications` menu) for the configuration where you choose $N_A > 100$, $N_B > 80$ and $U > 400$. The label U in the *StatMech* window refers to the total number of energy quanta in the system in units of $\hbar\omega$ and is equivalent to q_{AB} in Eq 4. An example of the output of *StatMech* is shown in Figure 1. The first two columns in the lower panel of Figure 1 represent $U(A)$ and $U(B)$, the energies in each individual solid (again in units of $\epsilon = \hbar\omega$) and are equivalent to q_A and q_B. After you perform the calculation with *StatMech* scan quickly down the column labeled 'Omega(AB)'. If any of the exponents you see exceed the value 307, then run the calculation again with smaller inputs until no exponent exceeds 307. This limitation is a restriction on MicroSoft *Excel* that you will use later to make plots. Record your values of N_A, N_B, and U.

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[1] "A different approach to introducing statistical mechanics", T.A.Moore and D.V. Schroeder, AJP 65 (1), 26-36, (1997), `http://physics.weber.edu/schroeder/statmech.pdf`
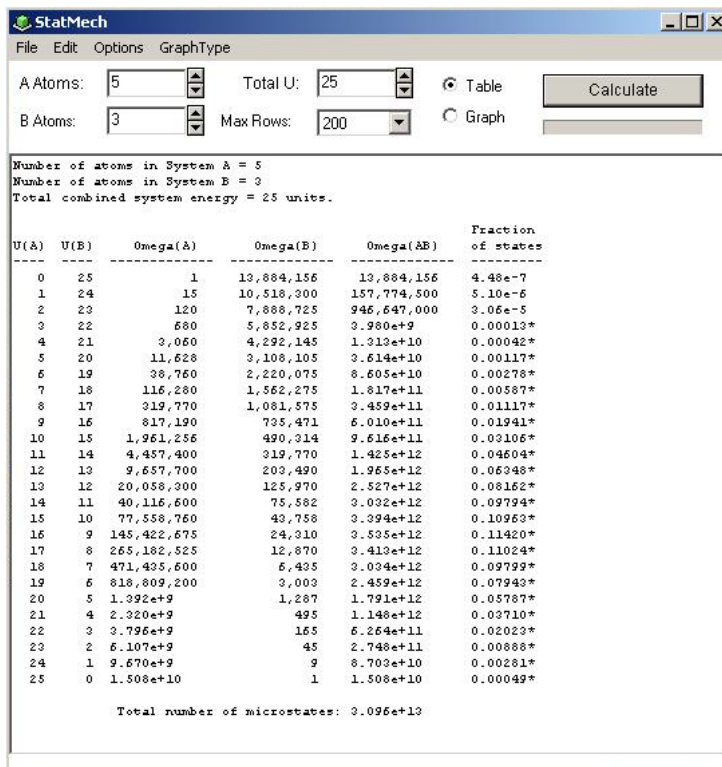
Figure 1: The `StatMech` window showing the table of multiplicities for each microstate. Each row corresponds to a different value of q_A.

(b) Now generate plots of $S_{AB} = S_A + S_B$, S_A, and S_B from the *StatMech* table. You can do this with *Excel*, but there are some intermediate steps necessary. Start Microsoft *Word* first. Next, go to the *StatMech* window, highlight the table, copy it (see the `Edit` menu on the *StatMech* window), and paste it into the *Word* document. In *Word* edit out all the commas (',') and asterisks ('*') in the file (use the `Replace` option under the `Edit` menu). Save the Word file, but save it as a plain text ('.txt') file. You can now open the file in *Excel*. When you open the file, *Excel* pops up a `Text Import Wizard` that will guide you through the format of the input file. The defaults usually seem to work. Use *Excel* to calculate and plot on one graph S_{AB}, S_A, and S_B as a function of E_A. You should get something resembling the figure below. Print out your plot and attach it to this unit.

(c) What is E_A for the most probable macrostate? What mathematical condition can you impose on the total entropy S_{AB} to determine the most probable macrostate? How do you think the temperatures of solids A and B are related at the most probable microstate?

(d) How are the slopes of S_A and S_B related to one another at the most probable macrostate?

(e) How is E_A, the energy in solid A related to q_A? How is E_B, the energy in solid B related to q_A? Remember that q_{AB} is a constant and $q_{AB} = q_A + q_B$. Calculate the differentials dE_A and dE_B and rewrite the answer in part 1.d in terms of dS_A/dE_A and dS_B/dE_B.

(f) If the energy E_A and q_A of solid A increases what should the temperature of solid A do? If the energy E_A of solid A increases what happens to dS_A/dq_A in your plot? Do the temperature and dS_A/dq_A change in the same way or in a different way as E_A increases?

You should have found that

$$\frac{dS_A}{dE_A} = \frac{dS_B}{dE_B} \qquad \text{and} \qquad T_A = T_B \tag{5}$$

at the most probable microstate. If not, consult your instructor.

Activity 2: Relating Entropy and Temperature

(a) Using the spreadsheet you generated in Activity 1, calculate dS_A/dE_A as a function of E_A (recall $E_A = q_A \epsilon$) and plot it. You can do this to an adequate approximation by taking the difference between S_A at adjacent values/rows of q_A. The difference in q_A between adjacent rows is just one. We can treat this difference as the energy difference in units of ϵ. To do this suppose your spreadsheet has the values of S_A in column H. The *Excel* syntax for estimating the derivative for the first value of q_A (the first row) is '`=(H2-H1)/1.0`' where `H2` is
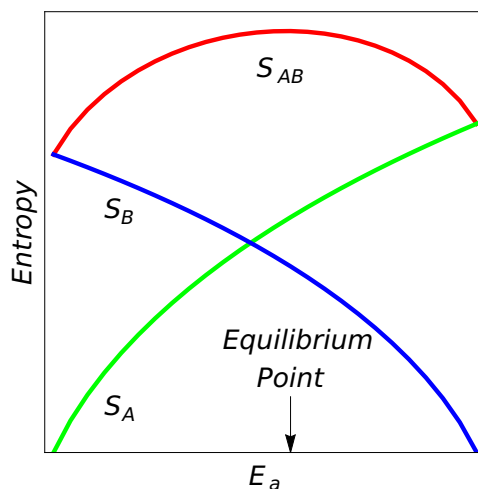

Figure 2: Plot of the entropies as a function of q_A.

the value in the second row and `H1` is the value in the first row. The numerator of one is redundant, but it shows you are approximating the derivative using the data from points that differ by one quantum in q_A, the number of quanta in solid A (or by ϵ in E_A). Recall that $E_A = q_A \epsilon$ where q_A is an integer. The syntax for dS_A/dq_A for the second value of q_A is '`=(H3-H2)/1.0`' and so on. Do the same for dS_B/dE_A and dS_{AB}/dE_A. Does the slope of S_{AB} pass through zero at the correct spot (recall part 1.c)? How are dS_A/dE_A and dS_B/dE_A related at the most probable macrostate. Does your plot agree with that result?

(b) Recall how we expect the energy E_A and number of quanta q_A to behave with temperature. If the energy E_A and q_A of solid A increases what does the temperature of solid A do? If the energy E_A of solid A increases what happens to dS_A/dE_A? Do the temperature and dS_A/dE_A change in the same way as E_A increases?

(c) We want to come up with a relationship between temperature and the entropy. From the results above (parts 1.a-e) you should have found

$$\frac{dS_A}{dE_A} = \frac{dS_B}{dE_B} \tag{6}$$

and

$$T_A = T_B \tag{7}$$

for the most probable macrostate. This means there is some function of the temperature $f(T)$ such that

$$f(T) = \frac{dS}{dE} \tag{8}$$

for each solid that will be equal at equilibrium. We want $f(T)$ to behave like the temperatures we are accustomed to using. In other words, as the energy in the solid increases T should increase. Recall part 2.b and the behavior of dS/dE as T increases. Try to guess a mathematical form of $f(T)$ that acts like 'normal' temperatures and one that doesn't. Explain your reasoning.

(d) How would you choose which of the forms you guessed in part 2.c is the correct one?

Activity 3: Determining $f(T)$ and the Heat Capacity

In the previous Activity you should have found that the mathematical form of $f(T)$ has to be something like $1/T^n$ where n is some positive number. This is necessary because your graphs should show that as the energy E_A (and the number of quanta q_A) of the solid increases $f(T) = dS/dE$ goes down. To make sure the temperature T behaves reasonably (and goes up with E_A and q_A) $f(T)$ must be some inverse of function of T. To decide exactly which function is right requires comparing Equation 8 or some result from it to some data.

Recall the table of heat capacities we generated in the laboratory entitled *Calorimetry*. Use those results to fill in Table 1 making sure that you are using molar heat capacities. The heat capacities are constant with respect

Solid	dE/dT per mole	Solid	dE/dT per mole

Table 1: Molar specific heats ($(1/n)dE/dT$) for several elemental solids.

to temperature and are similar in value to one another. These are the data that will help us determine $f(T)$. To calculate dE/dT we must find a relationship between E and T for the Einstein solid.

(a) Start with Equations 1 and 8 and the chain rule and show the following.

$$\frac{dS}{dE} = k_B \frac{1}{\Omega} \frac{d\Omega}{dE} \tag{9}$$

(b) Use Equation 2 to show

$$dE = \hbar\omega dq \qquad \text{and} \qquad \frac{d\Omega}{dE} = \frac{1}{\hbar\omega} \frac{d\Omega}{dq} \qquad . \tag{10}$$

(c) Starting with Equations 1 and 3 one can show

$$\frac{d\Omega}{dE} = \frac{3N}{E} \Omega \qquad . \tag{11}$$

Combine this equation (number 11) and the results from 3.a-b to get a relationship for dS/dE for the Einstein solid in terms of N and E. Set that expression equal to $f(T)$, and solve for E the internal energy. It is the derivative of this last equation $((1/n)dE/dT)$ that will give you the molar specific heat. What function of $f(T)$ will give a result that is independent of temperature when you take the derivative with respect to T of your expression for the internal energy E?

(d) What is the final form of Equation 8 and $f(T)$?

(e) Determine the mean and standard deviation of the heat capacity of the elemental solids in Table 1. Calculate the molar specific heat ($(1/n)dE/dT$) for the Einstein solid using your results from parts 3.c-d. Is the molar specific heat for the model of the Einstein solid consistent with the measured ones?

Activity 4: The Second Law of Thermodynamics

(a) Go back to your plots of the entropy as a function of q_A from part 1.b. Consider two Einstein solids that are brought together at a value of q_A that is higher than the equilibrium one at the most probable macrostate. Choose a value of q_A that is halfway between the most probable value and the maximum. Once the two Einstein solids are in contact, how will the system evolve? What happens to S_A and S_B? Do they go up, down, or stay the same? What happens to the total entropy S_{AB}? In fact, based on your plot from part 2.b, is there any circumstance where S_{AB} will not increase?

(c) To vividly see what happens when Einstein solids come in thermal contact, run the program *equilib.exe* available in the `Physics Applications` menu. This program starts with two Einstein solids with all the energy quanta in solid B. It simulates the evolution of the two solids as they march toward thermal equilibrium. Click `Evolve` to see the simulation run.

What you should have discovered in the previous part is that the entropy of the combined systems always increases regardless of what configuration the Einstein solids are in when they come in contact. The system always evolves to the most probable, most disordered macrostate where the temperatures will be equal and the entropy is a maximum. The energy quanta are most spread out. This result is stated in several different ways, but the most succinct is simply $\Delta S > 0$ for an isolated system. The entropy of an isolated system always increases. This is called the Second Law of Thermodynamics.

Lab 11   The Optical Spectrum of Hydrogen

Name: _____   Lab Partner(s): _____

Objective

- To determine the wavelengths of the visible lines in the hydrogen spectrum using a spectrometer and a diffraction grating.

- To determine the value of Rydberg's constant.

- To compare the predicted energy levels with the measured ones.

Introduction

The spectral lines of the hydrogen spectrum that fall in the visible region are designated as the H_α, H_β, H_γ, and H_δ lines. All (there happen to be four of them) belong to the Balmer series. In general, the spectrum of hydrogen can be represented by Rydberg's formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \tag{1}$$

where n_f can be any positive integer and n_i takes on the values of $n_f + 1$, $n_f + 2$, $nf + 3$, and so on and R_H is the Rydberg constant for hydrogen and equals $1.097 \times 10^7 m^{-1}$.

Apparatus

- spectrometer

- diffraction grating

- hydrogen gas spectral tube

- power supply for hydrogen tube

Activity 1: Measuring Spectral Lines

Diffraction grating spacing d = _____ Å

Using the spectrometer, measure the angular position of the direct beam, θ_o, and record it here: θ_o = _____ degrees. For each spectral line, measure the angular position of the first order maximum (once on each side) and record them in columns 2 and 3 below. Then determine the diffraction angle on each side (columns 4 and 5) and an average of the two (column 6). Calculate the wavelength of each line using the relation:

$$\lambda = d \sin \theta \tag{2}$$

where d is the diffraction grating spacing.

Line	θ_{left} (degrees)	θ_{right} (degrees)	$\theta_o - \theta_{left}$ (degrees)	$\theta_{right} - \theta_o$ (degrees)	$\theta_{average}$ (degrees)	Wavelength (Å)	Color
H_α							red
H_β							blue
H_γ							violet

Activity 2: Calculating the Rydberg Constant

The visible spectral lines for hydrogen correspond to transitions where $n_f = 2$. Using values for n_i of 3, 4, and 5 for the three lines you have measured, and the wavelengths you have determined, calculate a value of R_H for each line (from equation 1) and compare the average of these with the accepted value. Use your results to predict the wavelength of the next line in the series λ_δ (using $n_i = 6$). It's measured value is 4101Å. How does your prediction compare?

$R_\alpha \;\; =$

$R_\beta \;\; =$

$R_\gamma \;\; =$

$R_{average} \;\; =$ %%%%%%%% % difference =

$\lambda_\delta \;\; =$ %%%%%%%% % difference =

Collect values of $R_{average}$ from the other groups in the class and calculate an average and standard deviation. Record them below. How does this result compare with the accepted value? How does it compare with your individual measurement (i.e. does your measurement fall within the range indicated by the average and standard deviation)?

Activity 3: Hydrogen Transition Energies

(a) Experimental: The relationship between the wavelength of the emitted light and its energy is $E = hc/\lambda$ where c is the speed of light and h is Planck's constant. Calculate the transition energies (experimental) from $E = hc/\lambda$, based on your measured wavelengths.

(b) Theoretical: The energy levels of hydrogen can be described by the equation

$$E_n = -\frac{13.6 \ eV}{n^2} \tag{3}$$

where n is called the principal quantum number. Make an energy level diagram showing the transitions you believe you have measured. Calculate the theoretical values of the transition energies predicted by equation 3, noting that $E = E_i - E_f$.

How do your measured transition energies compare with the predicted ones? Be quantitative in your answer.

Lab 12   Orbits and the Effective Potential

Name: _____   Lab Partner(s): _____

Objective

To study classical orbits of systems like the Sun and Earth or the hydrogen atom to understand the physics of these orbits and the impact of angular momentum on the shape and other properties of the orbit.

Apparatus

- *Excel* for plotting curves

- *PhET Gravity and Orbits* simulation software.[1]

Overview

Classical physics successfully describes many fundamental properties of the world around us including the motion of the Sun, the Earth, and the denizens of our Solar System. It also provides a starting point to study the hydrogen atom where the mathematical form of the equations that describe the interaction of a proton and an electron (the Coulomb force) is the same. In this laboratory we will study the links between mechanical energy, potential energy, and kinetic energy including the role of angular momentum in influencing orbits. We will perform a simulation of the Earth-Sun system which serves as a model of the hydrogen atom.

Activity 1: Energy in Three Dimensions

Consider the form of the total mechanical energy of a light particle in three dimensions orbiting a much heavier one

$$E = \frac{1}{2}m\vec{v} \cdot \vec{v} + V(r) = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + V(r) = \frac{1}{2}mv^2 + V(r) \tag{1}$$

where m is the particle's mass, $\vec{v}$ is its velocity, $r = \sqrt{x^2 + y^2 + z^2}$ is the distance between Earth-Sun or electron-proton, and $V(r)$ is the potential energy of the particle. This potential depends only on the light particle's distance from the heavy one. The potential energy between the electron and proton in a hydrogen atom is the Coulomb potential so

$$V_C(r) = k_e \frac{q_1 q_2}{r} = -k_e \frac{e^2}{r} \tag{2}$$

where we have used the known proton and electron charges. The potential energy between the Earth and the Sun is the gravitational potential so

$$V_G(r) = G\frac{m_1 m_2}{r} = -G\frac{m_E m_S}{r} \tag{3}$$

where G is the gravitational constant and m_E and m_S are the mass of the Earth and Sun respectively. Note the similarity between Equations 2 and 3.

If we have an electron in the vicinity of a proton (*i.e.* a hydrogen atom), then it is convenient to rewrite the kinetic energy part of Equation 1 in polar coordinates. The proton, electron and the electron's velocity vector form a plane where we can describe the velocity in terms of a radial component and an angular one

$$\vec{v} = v_r\hat{r} + v_t\hat{\theta} \tag{4}$$

where $\hat{r}$ points radially along a line from the origin at the proton center to the electron's position and $\hat{\theta}$ is perpendicular to $\hat{r}$ and points so counter-clockwise rotations are positive.

(a) Get an expression for v^2 in terms of v_r and v_t and substitute your result in the total energy equation (Equation 1). Note the differences for the square of the velocity in Equation 1.

(b) We now define the momentum associated with the radial motion as $\vec{p}_r = m\vec{v}_r$ and the momentum associated with the angular motion $\vec{L} = mr\vec{v}_t$. The angular momentum $\vec{L}$ is a constant of the motion so it is unchanging. Rewrite your energy equation in terms of p_r, L, r, and any other constants. Include the explicit version of the Coulomb potential from Equation 2.

(c) You should have found in part 1.b that the three-dimensional mechanical energy of a particle in a Coulomb field can be written as

$$E = \frac{p_r^2}{2m} + \frac{L^2}{2mr^2} - k_e \frac{e^2}{r} \tag{5}$$

where L is a constant so the energy only depends on r and p_r. Make a sketch of the potential energy as a function of r. What are the limiting values of the potential?

(d) The angular momentum L is a constant of the motion so the last two terms in Equation 5 can be treated as a single 'effective' potential V_{eff} that governs the radial motion of particles. Make a sketch of those last two terms added together as a function of r for constant L. How is your figure different from the one in 1.c?

Activity 2: Plotting the Energy

(a) To visualize the motion of the Earth around the Sun or a classical electron around a proton we will plot combinations of the components of the total energy in Equation 5 using *Excel*. To start, generate a column of values of the radial distance between the two particles. In the top cell of Column A enter 'r' to label the column. In the second cell of the column type in `=ROW()/10` and hit enter. You should see a number appear in the cell. This is the value of 'r'. Click on the small square in the lower, right corner of the cell and drag down the column until you have eighty values in the column.

(b) In the top cell of the adjacent column (Column B) enter '`7.6/r^2`' to label the column and then calculate $7.6/r^2$ using the values of r from Column A you just created. You might enter `=7.6/(A2*A2)`. Click and drag

down that new cell to match all the entries in Column A. The constants here are chosen for the hydrogen atom so the radius r is in angstroms (10^{-10} m) and the energies are in electron-Volts (eV).

(c) Go to the top cell of the next column to the right (Column C), enter '`-14.4/r`' to label the column and calculate $-14.4/r$ for all the values of r in Column A.

(d) Go to the top cell of the next column (you're now on the fourth column, Column D), enter '`Veff`' to label it, and calculate the sum of the calculations in the previous two columns.

(e) Now make charts in *Excel* of the three functions of r you generated above. Put all of the curves on a single plot for comparison. To do this highlight all the data you have generated in Columns A-D including the labels at the top of each column and go to `Insert` $\rightarrow$ `Chart` $\rightarrow$ `Scatter(X,Y) Chart`. You should immediately get a plot with r on the horizontal axis and the three curves you just calculated along with a legend defining each curve. To make the plot easier to interpret zoom in on the central region of the plot by right-clicking on the vertical axis labels and select `Format Axis`. Under `Axis Options` change the minimum and maximum `Bounds` to -8 and $+2$, respectively. Print the plot and attach it to this unit.

(f) We can now investigate what these curves mean. Use a ruler to draw a straight, horizontal line at an energy value of -3.4 eV on your plot. We are considering bound states (the hydrogen atom or the Earth orbiting the Sun) so $E < 0$. You can think of this straight line as representing the value of the total mechanical energy. See Equation 5.

(g) Does your total energy 'curve' intersect the effective potential curve V_{eff} anywhere? If not, consult your instructor. For a classical particle like a ball rolling on a hill or a satellite orbiting the Earth, what happens at this intersection? What is the radial part of the kinetic energy at this point (the first term on the right-hand-side of Equation 5)? Describe the motion of a classical particle (the Earth orbiting the Sun or an electron orbiting a proton) in this potential with this value of the total mechanical energy.

(h) Draw a dashed, straight, horizontal line at a value of -7.5 eV on your plot. This means the total energy is less than the efffective potential energy. What does that imply about the radial part of the kinetic energy? See Equation 5. Does this make sense? What restrictions are there on the energy E of a classical particle?

Activity 3: Simulating Orbits

We will now use a simulation of the Earth-Sun system as a surrogate for the hydrogen atom. We can do this because the form of the Coulomb force is mathematically the same as the gravitational force.

(a) Obtain the *Gravity and Orbits* simulation at the address below

`https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits_en.html`

and start it up. Select the `Models` option and then configure the software using the checkboxes on the right-hand-side of the tabletop. See the figure below for the options to pick.

(b) The green vector attached to the Earth shows the velocity and moves with the Earth. The initial configuration of the Earth is motion in a circle. Is there any component of the Earth's velocity in the radial direction? Can you draw any conclusions about the value of the first term in Equation 5. Recall your figure of the different
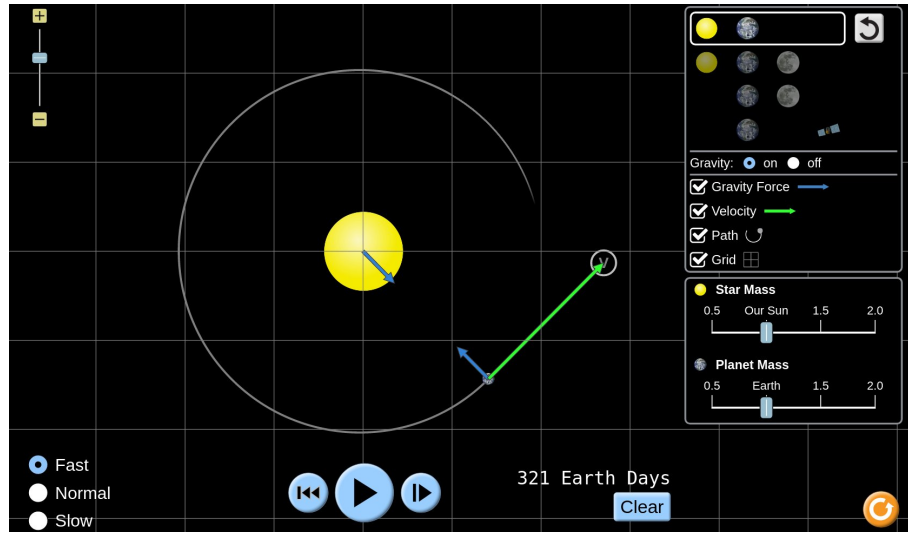
Figure 1: Tabletop of *Gravity and Orbits* simulation from PhET.

components of the mechanical energy in Activity 2. If the Earth is moving in a circle where would you draw the horizontal line representing the total mechanical energy on your plot? Make that drawing.

(c) Without changing anything in the simulation what do you predict would happen to the Earth's trajectory if you increased its velocity? Would the radius of the circular orbit increase? Explain your choice.

(d) Stop the simulation. Increase the length of the green velocity vector without changing its direction by clicking on the arrowhead and dragging it to increase its length. Restart the simulation. You can change the overall scale of the tabletop by moving the slide in the upper, left-hand corner. Is the orbit still a circle? Is the first term in Equation 5 (the radial part of the kinetic energy) constant? What happens to the length of the year, *i.e* the time for one full orbit?

(e) Stop the simulation. Rotate the Earth's green velocity vector about 30° inward. How does the orbit change? Where is the Earth's speed largest? Where is the Earth's speed smallest? Explain your observations using the

conservation of mechanical energy. What happens to the length of the year, *i.e* the time for one full orbit?

References

1. PhET Interactive Simulations, University of Colorado Boulder, `http://phet.colorado.edu`, last accessed April 16, 2021.

Lab 13   A Theory for the Hydrogen Atom

Name: _____   Lab Partner(s): _____

Objective

To use a numerical technique to solve the Schroedinger equation for the hydrogen atom, compare calculations with spectroscopic measurements from hydrogen, and explore the implications of the quantum theory especially the quantization of energy in bound systems.

Apparatus

- *Schrödinger Shooter* software

- Results from previous measurements of hydrogen energy transitions.

Overview

Quantum mechanics is the theoretical structure used to make sense of the atomic and sub-atomic world. It is based on a small set of assumptions and a lot of associated mathematics.

The Postulates of Quantum Mechanics

1. The quantum state of a particle is characterized by a wave function $\Psi(\vec{r}, t)$, which contains all the information about the system an observer can possibly obtain. The square of the magnitude of the wave function $|\Psi(\vec{r}, t)|^2$ is interpreted as a probability or probability density for the particle's presence.

2. The things we measure (*e.g.* energy, momentum) are called observables. Each observable has a corresponding mathematical object called an operator that does 'something' to the wave function $\Psi(\vec{r}, t)$. The radial dependence of the wave function $\Psi(\vec{r}, t)$ is governed by the energy operator which generates a famous expression called the Schrödinger equation.

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + \frac{L^2}{2mr^2}\right)\Psi(r) + U\Psi(r) = E\Psi(r) \tag{1}$$

   Ultimately, the success of this approach will depend on how the theoretical results we generate from solving the Schrödinger equation compare with our measurements.

We will focus on these postulates in this laboratory and in particular on understanding and solving Equation 1, the Schrödinger equation.

Activity 1: Energy in Three Dimensions

Consider a particular form of the total mechanical energy of a particle in three dimensions

$$E = \frac{1}{2}m\vec{v}\cdot\vec{v} + V(\vec{r}) = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + V(\vec{r}) = \frac{1}{2}mv^2 + V(\vec{r}) \tag{2}$$

where m is the particle's mass, $\vec{v}$ is its velocity, and $V(\vec{r})$ is the potential energy of the particle and depends only on the particle's position. We have used the definition of the momentum $\vec{p} = m\vec{v}$. The potential energy between the electron and proton in a hydrogen atom is the Coulomb potential so

$$V(\vec{r}) = k_e\frac{q_1 q_2}{r} = -k_e\frac{e^2}{r} \tag{3}$$

where we have used the known proton and electron charges.

If we have an electron in the vicinity of a proton (*i.e.* a hydrogen atom), then it is convenient to rewrite the kinetic energy part of Equation 2 in polar coordinates. The proton, electron and the electron's velocity vector form a plane where we can describe the velocity in terms of a radial component and an angular one

$$\vec{v} = v_r \hat{r} + v_t \hat{\theta} \tag{4}$$

where $\hat{r}$ points radially along a line from the origin to the electron's position and $\hat{\theta}$ is perpendicular to $\hat{r}$ and points so counter-clockwise rotations are positive.

(a) Get an expression for v^2 in terms of v_r and v_t and substitute your result in the energy equation (Equation 2).

(b) We can now define the momentum associated with the radial motion as $\vec{p}_r = m\vec{v}_r$ and the momentum associated with the angular motion $L = mrv_t$. The angular momentum L is a constant of the motion so it is unchanging. Rewrite your energy equation in terms of p_r, L, r, and any other constants. Include the explicit version of the potential in Equation 3.

(c) You should have found in part 1.b that the three-dimensional energy of a particle in a Coulomb field can be written as

$$E = \frac{p_r^2}{2m} + \frac{L^2}{2mr^2} - k_e \frac{e^2}{r} \tag{5}$$

where L is a constant so the energy only depends on r and p_r. Note the similarities with the forms of the Schrödinger equation in Equation 1. Make a sketch of the potential energy as a function of r. What are the limiting values of the potential?

(d) The angular momentum L is a constant of the motion so the last two terms in Equation 5 can be treated as a single 'effective' potential that governs the radial motion of particles. Make a sketch of those last two terms as a function of r for constant L. How is your figure different from the one in 1.c?

(e) Since the energy is constant you can draw it on your previous sketch as a flat, straight line. We are considering bound states (the hydrogen atom) so $E < 0$. Draw an energy line for a bound state on your previous sketch. Does your energy 'curve' intersect the effective potential curve anywhere? For a classical particle like a ball rolling on a hill or a satellite orbiting the Earth, what happens at this intersection? Describe the motion of a classical particle in this potential. What restrictions are there on the energy E of a classical particle?

Activity 2: Quantum Predictions

(a) What do you expect the wave function of the electron in a hydrogen atom to look like? Recall Postulate 1 which says we interpret the square of the magnitude of the wave function $|\Psi(\vec{r}, t)|^2$ as the probability distribution for measuring the position of the particle. Copy your drawing from part 1.d and add a sketch of your expectation for the hydrogen atom wave function on the same graph. Consider what happens as the effective potential changes with r.

(b) What happens to your wave function as the energy increases? Make another copy here of the graph of the effective potential energy that you made in 2.a. Draw an energy curve for a higher energy state than your previous one. Now sketch the new, higher-energy wave function. How is it different from the curve you made in 2.a?

Activity 3: Solving the Schrödinger Equation

(a) We are now ready to start solving the Schrödinger Equation. Go first to the `All Programs` menu, then to `Physics Applications` and click on `Schrödinger Shooter`. You will see the `Schrödinger Shooter` window like the one in the next figure. If you don't see this window, consult your instructor.

(b) The first step is to select a potential energy function. Click on `File` and go to `New Potential Energy Function`. Select the potential energy function most appropriate for the hydrogen atom. Next select the charge of the atomic nucleus by entering that value in the box labeled `Z:` in units of the electronic charge e. Set the quantum number of the angular momentum to zero in the box labeled `L:`. Use the second menu button from the left-hand side at the bottom of the `Schrödinger Shooter` window to choose `Normalized` wave functions (it should be labeled `Not Normalized` when you start the program). This last choice makes it easier to compare wave functions for different quantum numbers. Last, set the initial value of the energy to -1.5 in the box labeled
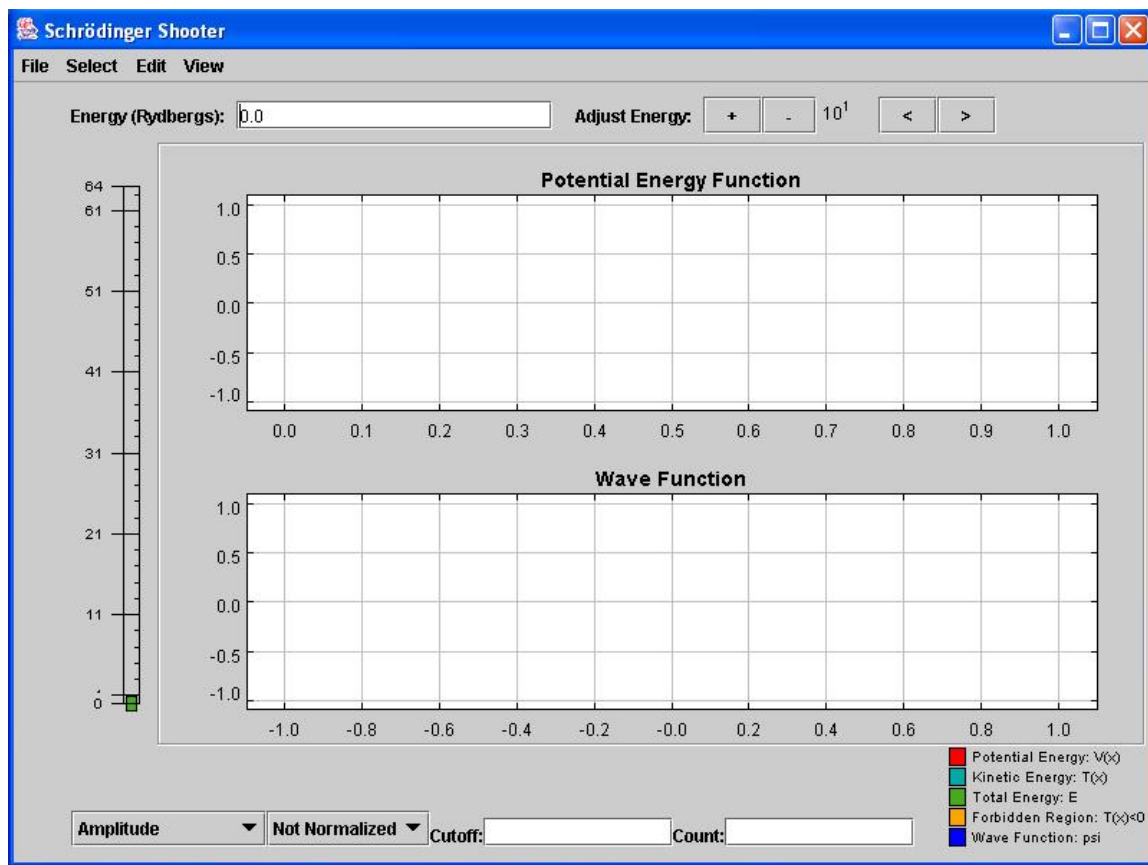
Figure 1: Initial panel for *Schrödinger Shooter*

`Energy (Rydbergs):` and hit return. The conversion between eV and rydbergs is 13.6056923(12) eV/rydberg. The number in parentheses is the uncertainty on the last two digits in the experimental value. At this point you should see curves in both of the panels of the `Schrödinger Shooter` window. If you don't, consult your instructor.

(c) You should see several curves in the upper panel. What quantities do the blue, red, yellow, and green curves represent? Use the legend in the lower right portion of the `Schrödinger Shooter` window. Record them here.

(d) You can now adjust the energy of the electron in the field of the proton to find the energy levels of hydrogen. There are three ways to change this energy. There is a slide on the left-hand side of the `Schrödinger Shooter` window. Click and drag the slide to change the energy and read the value in the box labeled `Energy(Rydbergs)` located near the top of the `Schrödinger Shooter` window. You can also enter the energy in the same box labeled `Energy(Rydbergs):` as you did in part 3.b. Last, to the right of this last box there are buttons labeled `Adjust Energy:` that will increase or decrease the energy and change the step size.

(e) Tune the energy to find lowest five energy levels in your hydrogen atom by obtaining physically acceptable wave functions. You may need to increase the horizontal range to view the full wave function. You can do this by increasing the value in the box labeled `Cutoff:` at the bottom of the `Schrödinger Shooter` window. If you

increase the horizontal scale, then increase the next box labeled `Count:` (next to the `Cutoff` box) by the same ratio. In other words, if you double the cutoff, then double the count. This last parameter controls the stepsize used in the numerical integration of the Schrödinger equation. What does the wave function look like when you have found the correct energy level? What postulate did you exploit to find the correct solutions? How does the radial wave function change as the energy increases? Record your values for each of the energy levels below.

Activity 4: Analysis and Comparison with Previous Data

(a) Enter the values you found for the energies in an *Excel* spreadsheet and convert the values to eV. Use the scale below to make an energy level diagram for hydrogen.
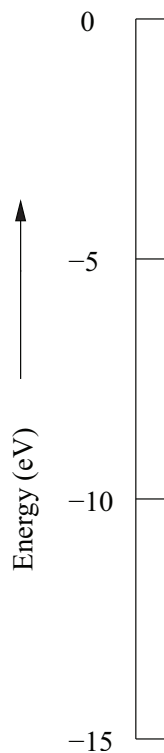
(b) In the lab entitled *Optical Spectrum of Hydrogen* you measured and identified transitions in hydrogen between different principle quantum levels. Using the results in part 4.a calculate the energies of the same transitions and the wavelength of the emitted photon. Record your results below.

(c) Compare your measured results from the *Optical Spectrum of Hydrogen* with the predictions of your model in part 4.b. Calculate the percentage difference for each transition. Does your model agree with the data? Be quantitative in your answer.

Activity 5: The Meaning of Quantum Theory

(a) For one of your hydrogen energy states, change the angular momentum of the quantum state by setting $L = 1$ in the box labeled `L:`. How does the radial wave function change? How much does the energy change? Do the same thing with the next higher energy state. Can you jump to any conclusions about the effect of angular momentum on the energy of a quantum state in hydrogen?

(b) Return to the questions in Activity 2 and examine your predictions. How did you do? Correct any statements that you now find are wrong.

(c) What requirement or postulate forces us to choose particular energy states (*i.e.* what causes energy quantization)?

(d) Reconsider a variation of one of the questions in part 1.e. In that question you were asked to describe the motion of a classical particle like a satellite orbiting the Earth. Describe the motion of a quantum particle in the hydrogen atom potential.

Appendix A: Video Analysis Using Tracker

**Making a Movie with "Camera"**

To make a movie, perform the following steps:

1. Make sure the camera is connected to a USB port on your computer. Close all windows, applications, programs, and browsers.

2. Click the **Start** button in the lower-left corner of your screen and type "camera' in the Search programs and files box. Once the search results appear, click on **Camera** at top of window.

3. Position the camera 2-3 meters from the object you will be viewing. Adjust the camera height and orientation so that the field of view is centered on the expected region where the object will move.

4. Place a meter stick or an object of known size in the field of view where it won't interfere with the experiment. The meter stick should be the same distance away from the camera as the motion you are analyzing so the horizontal and vertical scales will be accurately determined. It should also be parallel to one of the sides of the movie frame. Make sure that the meter stick is not far away from the central region of field of view, and that it is perpendicular to the line of sight of camera.

5. To start recording click the video camera icon on the right of the window. When you are done, click the red square.

   The video will save to camera roll located in **This PC → Pictures → Camera Roll**

Analyzing the Movie

To determine the position of an object at different times during the motion, perform the following steps:

1. Start up Tracker by going to **Start → Programs → Physics Applications → Tracker**. When **Tracker** starts it appears as shown below. The menu icons and buttons that we will use are identified by arrows.

2. Click the **Open Video** button on the toolbar to import your video. After your video is imported, Tracker will warn you that the video frames don't have the same time duration. This is okay since **Camera** uses a variable frame rate. Click **OK** on the warning window to ignore Tracker's recommendation.

3. Click the **Clip Settings** button to identify the frames you wish to analyze. A clip settings dialog box appears. Here, you only need to identify and set the start and end frames. Leave everything else in the dialog box unchanged. To find and set the start frame, drag the player's left slider to scan forward through the video, and stop when you get to the first frame of interest. Now, the start frame is set and the corresponding frame number should be displayed in the dialog box. If not, then click on the **Start Frame** in the dialog box, enter the number of the frame (printed in the lower left part of the Tracker window), and click outside the box. Then, click the **Play video** button to go to the last frame in the video. Next, drag the player's right slider to scan backward through the video to find the last frame of interest. Stop when you get to the frame of interest. Now, the end frame is also set and the corresponding frame number should be displayed in the dialog box. If not, then click on the **End Frame** in the dialog box, enter the number of the frame, and click outside the box. Finally, click the **OK** button to close the dialog box, and then click the player's **Return** button to return to the start frame.

4. Click the **Calibration** button and select the **calibration stick**. A blue calibration line appears on the video frame. Drag the ends of this blue line to the ends of your calibration meter stick. Then click the readout box (the number indicating length) on the calibration line to select it. Enter the length of the meter stick in this box (without units). For example, if your calibration meter stick is 1.00 meter long, enter 1.00 in the box (if not already there) and then click outside the box to accept the value. At this point, you can right-click the video frame to zoom in for more accurate adjustment of the ends of the calibration stick. Right-click the video again to zoom out.
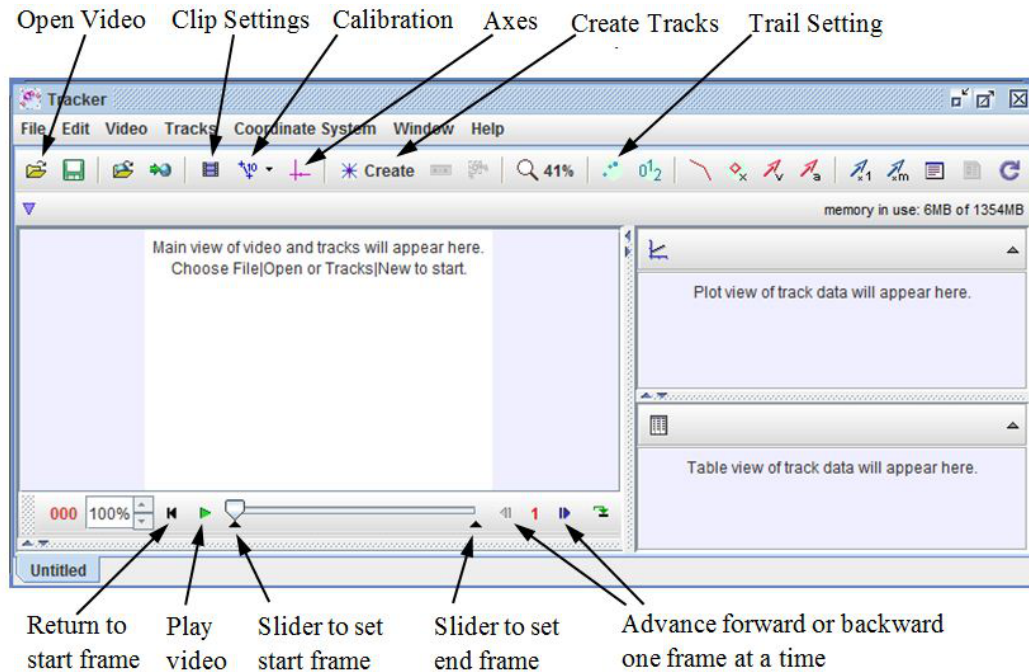
Figure 1: Initial **Tracker** window for video analysis.

5. Click the **Axes** button to set the origin and orientation of the x-y coordinate axes. Drag the origin of the axes to the desired position. Sometimes this will be the initial position of the object of interest. For rotating objects it will typically be the axis of rotation. Click the video outside the origin to fix the position of the origin. To change the orientation (angle) of the axes, drag the x axis. Click the video to fix the new orientation.

6. Click the **Create** button to track the object of interest in the video. From the menu of choices select **Point Mass** for the track type. Make sure the video is at the start frame, which shows the initial position of the object of interest. Mark this position by holding down the **shift key** and clicking the mouse (crosshair cursor) on the object. As the position is marked, the video automatically advances to the next frame. Similarly, mark the position of the object on this and subsequent frames by holding down the **shift key** and clicking the mouse. Do not skip any frames.

   After marking the position on the end frame, you can adjust any one of the marked positions. Advance the video to the frame where you would like to make a fine adjustment. Right-click the video frame to zoom in and drag the marked position with the mouse.

   If you would like to track additional objects, repeat the procedure outlined here for each object.

7. **Plotting and Analyzing the Tracks:** The track data (position versus time) are listed in the Table View and plotted in Plot View sections of the Tracker screen. Click the vertical axis label of the plot to change the variable plotted along that axis. To plot multiple graphs, click the **Plot** button, located above the plot, and select the desired number. Right-click on a plot to access display and analysis options in a pop-up menu. To fit your data to a line, parabola, or other functions, select the **Analyze** option. On the Data-Tool window that opens up, click the **Analyze** button and check the **Curve Fits** box from the drop-down menu. Then select the fit type from the **Fit Name** drop-down menu (lower left). Make sure the **Auto fit** box is checked. Note that the curve fitter fits the selected function to the data in the two leftmost columns of the displayed data table. The leftmost column, identified by a yellow header cell, defines the independent variable, and the second leftmost column, identified by a green header cell, defines the dependent variable. So, to fit the data in other columns, their corresponding headers must be dragged

to the two leftmost columns. To print out the displayed plot and data table on the Data Tool screen, select **Print** from the **File** menu on the Data Tool screen.

8. **Printing and Exporting Data:** Track data can also be easily exported to *Excel* for further analysis by copying the data from the data table to the clipboard and pasting into *Excel*. Select the desired data (on Data Tool screen), go to **Edit → copy → selected data** (on Data Tool screen). Open *Excel* (in Microsoft Office folder), click on "HOME", and click the Paste clipboard. Data should appear in *Excel*. Graph can now be plotted in *Excel* (see **Appendix B: Excel**) and printed.

Appendix B: Introduction to Excel

Microsoft Excel is the spreadsheet program we will use for much of our data analysis and graphing. It is a powerful and easy-to-use application for graphing, fitting, and manipulating data. In this appendix, we will briefly describe how to use Excel to do some useful tasks. The current version is Excel 2013.

B.1   Data and Formulas

Figure 1 below shows a sample Excel spreadsheet containing data from a made-up experiment. The experimenter was trying to measure the density of a certain material by taking a set of cubes made of the material and measuring their masses and the lengths of the sides of the cubes. The first two columns contain her measured results. **Note that the top of each column contains both a description of the quantity contained in that column and its units.** You should make sure that all of the columns of your data tables do as well. You should also make sure that the whole spreadsheet has a descriptive title and your names at the top, as indicated in the sample spreadsheet below.

In the third column, the experimenter has figured out the volume of each of the cubes, by taking the cube of the length of a side. To avoid repetitious calculations, she had Excel do this automatically. She entered the formula =B5^3 into cell C5. Note the equals sign, which indicates to Excel that a formula is coming. The ^ sign stands for raising to a power. After entering a formula into a cell, you can grab the square in the lower right corner of the cell with the mouse and drag it down the column, or you can just double-click on that square. (Either way, note that thing you're clicking on is the tiny square in the corner; clicking somewhere else in the cell won't work.) This will copy the cell, making the appropriate changes, into the rest of the column. For instance, in this case, cell C6 contains the formula =B6^3, and so forth.

Column D was similarly produced with a formula that divides the mass in column A by the volume in column C.

At the bottom of the spreadsheet we find the mean and standard deviation of the calculated densities (that is, of the numbers in cells D5 through D8). Those are computed using the formulas =average(D5:D8) and =stdev(D5:D8).

B.2   Graphs

Here's how to make graphs in Excel. First, use the mouse to select the columns of numbers you want to graph. (If the two columns aren't next to each other, rearrange them so that they are next to each other. The variable you want on the horizontal axis needs to be to the left of the variable you want on the vertical axis). Then click

Figure 1: Sample Excel spreadsheet

on the *Insert* tab at the top of the window. In the menu that shows up, there is a section called *Charts*. Almost all of the graphs we make will be scatter plots (meaning plots with one point for each row of data), so click on the *Scatter* icon, which is the lower one in the group that looks like this. Several possible scatter plots appear. Select the first one with the same icon as before, and your graph will appear.

Next, you'll need to customize the graph in various ways, such as labeling the axes correctly.

This is done in the *Chart Elements* menu which is accessed by clicking on the plus sign (+) in a box, at the upper right corner of your graph. Click on *Axis Titles*, and places appear for axis titles. Edit the text inside of the two axis titles so that it indicates what's on the two axes of your graph, *including the appropriate units for each.*

Next, give your graph an overall title. Click on *Chart Title* and a box appears around it. Delete "Chart Title" and enter a proper title such as "Position vs Time" or whatever.

Usually you want your graph to contain a best-fit line passing through your data points. To do this, *right* click on one of the data points and select *Add Trendline.* A number of options appear such as "linear", "polynomial", "power", and so on. Select the one you want, and also check the *Display Equation on chart* option near the bottom. You can then drag the equation to someplace else on the chart so you can read it better. Remember that Excel won't include the correct units on the numbers in this equation, but you should. Also, Excel will always call the two variables x and y, even though they might be something else entirely. Bear these points in mind when transcribing the equation into your lab notebook.

Sometimes, you may want to make a graph in Excel where the x column is to the right of the y column in your worksheet. In these cases, Excel will make the graph with the x and y axes reversed. Here's how to fix this problem: Before you make your graph, make a copy of the y column in the worksheet and paste it so that it's to the right of the x column. Then follow the above procedure and everything will be fine.

B.3   Making Histograms

A histogram is a useful graphing tool when you want to analyze groups of data, based on the frequency at given intervals. In other words, you graph groups of numbers according to how often they appear. You start by choosing a set of 'bins', *i.e.*, creating a table of numbers that mark the edges of the intervals. You then go through your data, sorting the numbers into each bin or interval, and tabulating the number of data points that fall into each bin (this is the frequency). At the end, you have a visualization of the distribution of your data.

Start by entering your raw data in a column like the one shown in the left-hand panel of Figure 2. Look over your numbers to see what is the range of the data. If you have lots of values to sift through you might consider sorting your data is ascending or descending order. (You don't need to do this if you can easily see the range of data.) To do this task, choose the column containing your data by clicking on the letter at the top of the column, go to *Data* in the menubar, select *Sort*, and pick ascending or descending. The data will be rearranged in the order you've chosen and it will be easier to see the range of the data. For an example, see the middle column of data in the left-hand panel of Figure 2.

Now to create your bins pick a new column on your spreadsheet and enter the values of the bin edges. Bins should be of equal size with the bin edges given by simple numbers. Make sure the bins you choose cover the range of the data. See the left-hand panel of Figure 2 again for an example.

You now have the ingredients for making the histogram. Go to *Data* in the menubar, select *Data Analysis* (on the right) and choose *Histogram*. Click *OK*. You should see a dialog box like the one in the right-hand panel of Figure 2. Click in the box labeled *Input Range* and then highlight the column on the spreadsheet containing your data. Next, click in the box labeled *Bin Range* and highlight the column on the spreadsheet containing the bins. Under *Output Options*, select *Chart Output* and click *OK*. The histogram should come up. If it doesn't, go to *Insert* in the menubar and select the histogram icon in the *Charts* section (the first icon in that section). Change the horizontal axis label to whatever you are plotting (including units). The vertical axis should already be labeled "Frequency". Change the chart title to an appropriate title for whatever you are plotting. The result should look like the right hand panel of Figure 3.
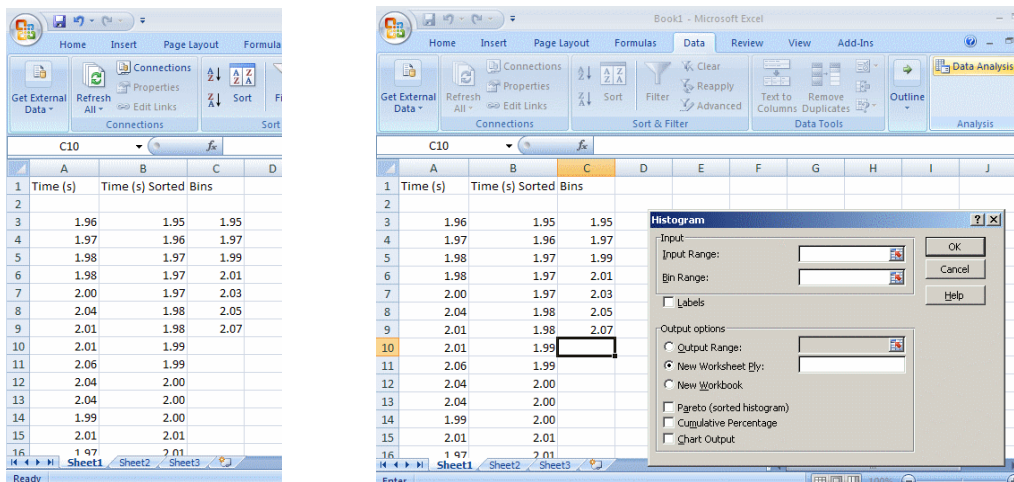

Figure 2: Column data and bins (left-hand panel) and dialog box (right-hand panel) for making a histogram in Excel.
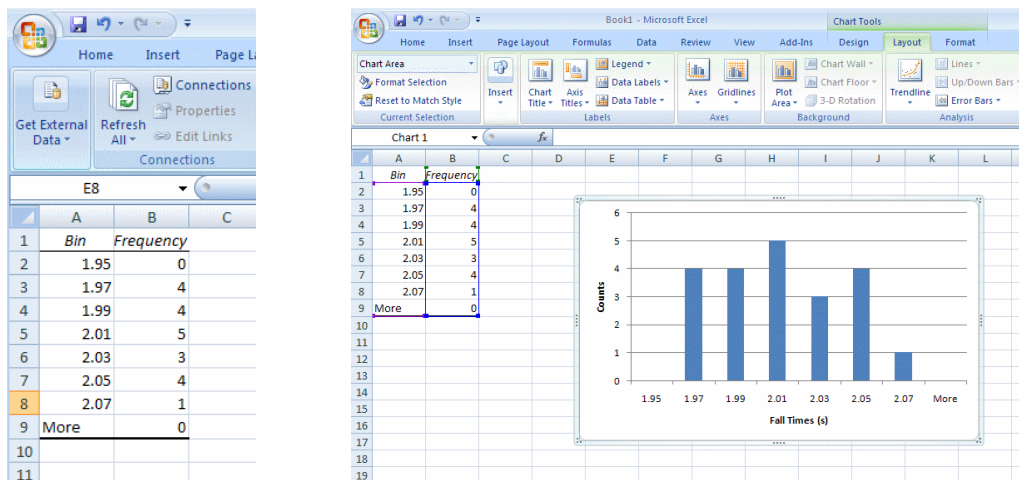

Figure 3: Newly-created worksheet (left-hand panel) and final plot (right-hand panel) for histogram worksheet in Excel.

B.4   LINEST

LINEST is a function in Excel which gives a LINear ESTimate of the slope and the uncertainty in the slope for any linear data set. It can also give the y-intercept and the uncertainty in the y-intercept. The uncertainties are based on the scatter of the data points about a perfect straight line. To use LINEST in Excel, perform the following steps:

1. Select a 2 x 2 group of cells (not including any cells with data)

2. In the function line type =LINEST(

3. Select range of y values

4. Type comma (,)

5. Select range of x values

6. Type comma (,)

7. To estimate both the slope and the y-intercept: Type 1,1)

   OR, to estimate only the slope, forcing a y-intercept of zero: Type 0,1)

8. Hold down Control AND Shift buttons, and press Enter

9. The result should look like this (if you typed "1,1"):

slope	intercept
δ slope	δ intercept

where δ slope represents the uncertainty in the slope value. You will still need to round off both numbers, depending on the relative magnitudes of the two numbers. It is also common practice to express both numbers to the same power of 10.