Homework 5   Quanta

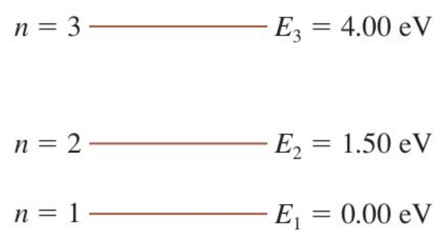
1. What is the wavelength, in nm, of a photon with energy (a) 0.30 eV, (b) 3.0 eV, and (c) 30 eV? For each, is this wave- length visible, ultraviolet, or infrared light?

2. For what wavelength of light does a 100 mW laser deliver 2.50×10^{17} photons per second?

3. Through what potential difference must an electron be accelerated from rest to have a de Broglie wavelength of 500 nm?

4. What is the de Broglie wavelength of a neutron that has fallen 1.0 m in a vacuum chamber, starting from rest?

5. The figure is an energy-level diagram for a simple atom. What wavelengths appear in the atoms (a) emission spectrum and (b) absorption spectrum?



6. The allowed energies of a simple atom are 0.00 eV, 4.00 eV, and 6.00 eV.
   (a) Draw the atoms energy-level diagram. Label each level with the energy and the quantum number.
   (b) What wavelengths appear in the atoms emission spectrum?
   (c) What wavelengths appear in the atoms absorption spectrum?

7. How much energy does it take to ionize a hydrogen atom that is in its first excited state?

8. What is the third-longest wavelength in the absorption spectrum of hydrogen?

9. Two hydrogen atoms collide head-on. The collision brings both atoms to a halt. Immediately after the collision, both atoms emit a 121.6 nm photon. What was the speed of each atom just before the collision?

10. A beam of electrons is incident upon a gas of hydrogen atoms.
    (a) What minimum speed must the electrons have to cause the emission of 656 nm light from the $3 \to 2$ transition of hydrogen?
    (b) Through what potential difference must the electrons be accelerated to have this speed?

11. The wave function for a particle is
$$\psi(x) = \sqrt{\frac{a}{\pi(x^2 + a^2)}}$$
for $a > 0$ and $-\infty < x < +\infty$. Determine the probability that the particle is located somewhere between $x = -a$ and $x = +a$.

12. An electron is contained in a one-dimensional box of length 0.100 nm. (a) Draw an energy level diagram for the electron for levels up to $n = 4$. (b) Find the wavelengths of all photons that can be emitted by the electron in making downward transitions that could eventually carry it from the $n = 4$ state to the $n = 1$ state.

13. The nuclear potential energy that binds protons and neutrons in a nucleus is often approximated by a square well. Imagine a proton confined in an infinitely high square well of length 10.0 fm, a typical nuclear diameter. Calculate the wavelength and energy associated with the photon emitted when the proton moves from the $n = 2$ state to the ground state. In what region of the electromagnetic spectrum does this wavelength belong?

14. A photon with wavelength λ is absorbed by an electron confined to a box. As a result, the electron moves from state $n = 1$ to $n = 4$. (a) Find the length of the box. (b) What is the wavelength of the photon emitted in the transition of that electron from the state $n = 4$ to the state $n = 2$?

15. The wave function of a particle is given by

$$\psi(x) = A\cos(kx) + B\sin(kx)$$

where A, B, and k are constants. Show that ψ is a solution of the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

assuming the particle is free ($U = 0$), and find the corresponding energy E of the particle.

16. The wave function of an electron is

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$$

Calculate the probability of finding the electron between $x = 0$ and $x = L/4$.

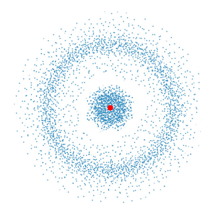
17. A particle of mass $m = 2.0 \times 10^{-28}\ kg$ is confined to a one-dimensional box of length $\ell = 1.0 \times 10^{10}\ m$. For $n = 1$, what are (a) the particle's wavelength, (b) its momentum, and (c) its ground-state energy?

18. Imagine that a particle has a wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{a}}e^{-x/a} & \text{for} > 0 \\ 0 & \text{for} < 0 \end{cases}$$

19. An atom (not a hydrogen atom) absorbs a photon whose frequency is $f = 6.2 \times 10^{14}\ Hz$. How much does the energy of the atom increase?

20. What is the energy of the hydrogen atom electron whose probability density is represented by the dot plot shown below? What minimum energy is needed to remove this electron from the atom?



21. What are the energy and wavelength of a photon emitted when a hydrogen atom undergoes a transition from the $n = 3$ state to the $n = 1$ state?

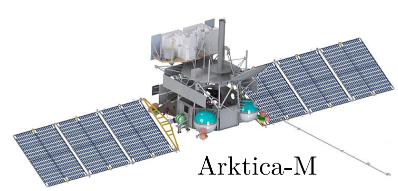
22. What is the wavelength of the least energetic photon emitted in the Lyman series (where $n_f = 1$) of the hydrogen atom spectrum lines?

23. What is the series limit for the Lyman series (where $n_f = 1$)?

24. A Russian Arktica satellite that monitors polar weather follows an elliptical orbit around the Earth at an altitude of $h = 300\ km$ above the surface (radius $r_s = 6.67 \times 10^6\ m$) at a velocity

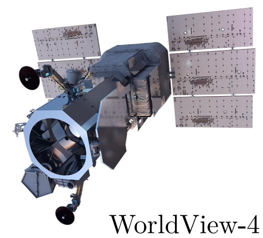
$$\vec{v} = 4.1 \times 10^3\ m/s\ \hat{r} + 7.5 \times 10^3\ m/s\ \hat{\theta} \qquad .$$

What is the angular momentum? What is the total energy? What is the distance of closest approach to the Earth? The satellite mass is $m_s = 600\ kg$.



Arktica-M

25. The WorldView-4 satellite is in a circular orbit around the Earth at an altitude of $h = 681\ km$ above the surface at a speed $v_0 = 7.52 \times 10^3\ m/s$. It quickly burns it rockets pointing in a direction away from the Earth and parallel to the direction of the gravitational force. It reaches a new mechanical energy $ME_1 = -1.14 \times 10^{10}\ J$. What will be its new angular momentum and new distance of closest approach to the Earth? Is this rocket burn a good idea? The satellite mass is $m_s = 726\ kg$.



WorldView-4

26. A satellite of mass $m_s = 726\ kg$ is orbiting the Earth and has a perigee (closest point to the center of the Earth) $r_p = 1.31 \times 10^7\ m$ with a velocity $v_p = 7.5 \times 10^3\ m/s$ at that point. How far away is apogee r_a (farthest point from the center of the Earth)?

27. A satellite of mass $m_s = 800\ kg$ is orbiting the Earth with a velocity $\vec{v}_i = 5.3 \times 10^3\ m/s\ \hat{r} + 6.8 \times 10^3\ m/s\ \hat{\theta}$ at a distance $r_s = 10^7\ m$ from the Earth. It has to burn its rocket so the apogee r_a (farthest point from the center of the Earth) is $r_a = 1.5 \times 10^8\ m$. How much does its speed have to increase?

28. A monochromatic point source of light radiates in all directions equally with power $P = 25$ W at a wavelength $\lambda = 5000$. A plate of metal is placed 100 cm from the source. Atoms in the metal have a radius of 1 Å. Assume the atom can continually absorb light. The work function of the metal is $W = 4$ eV. How long is it before an electron is emitted from the metal?

29. The work function of zinc is 3.6 eV. What is the energy of the most energetic electron emitted by ultraviolet light of wavelength 1900 Å?

30. Photoelectrons are observed when a metal is illuminated by light with a wavelength less than 388 nm. What is the metal's work function?

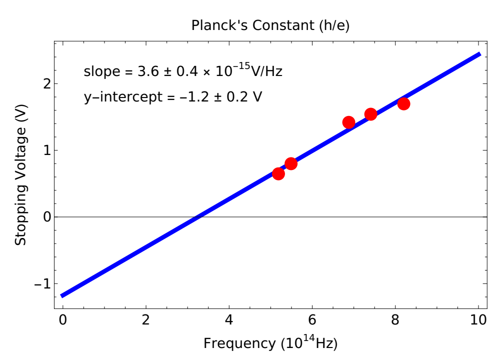
31. Electrons in a photoelectric-effect experiment emerge from an aluminum surface with a maximum kinetic energy of 1.30 eV. What is the wavelength of the light? The table to the right has a list of work functions of different metals.

32. A photoelectric-effect experiment finds a stopping potential of 1.56 V when light of 200 nm is used to illuminate the cathode. (a) From what metal is the cathode made? (b) What is the stopping potential if the intensity of the light is doubled? The table to the right has a list of work functions of different metals.

Element	$E_0(eV)$
Potassium	2.30
Sodium	2.75
Aluminum	4.28
Tungsten	4.55
Copper	4.65
Iron	4.70
Gold	5.10

33. A helium laser pointer has a power $P = 1\ mW$ with a beam spot of radius $w = 1\ cm$. You shine the laser on a piece of tungsten which has atoms of radius $r = 1.41$ Å and work function $E_{ej} = 4.55\ eV$. Use the 'swimming pool' model of the photoelectric effect. Assume all of the light striking a single atom gets absorbed and funneled into the kinetic energy of a single electron. What is the rate of electrons ejected from the surface? How long does it take for the first electron to get ejected?

34. The plot shows some results from Intermediate Lab on measuring Planck's constant and the work function of a metal. What is the relationship between the stopping voltage and the frequency f of the light striking the metal? What is the value of Planck's constant from the fit to the data? What is the work function of the metal? What is the metal?



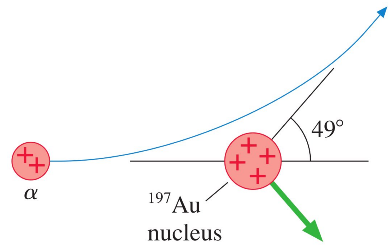
35. Through what potential difference would you need to accelerate an alpha particle (a ^{4}He nucleus), starting from rest, so that it will just reach the surface of a 15-fm-diameter ^{238}U nucleus?

36. The oxygen nucleus ^{16}O has a radius of 3.0 fm. (a) With what speed must a proton be fired toward an oxygen nucleus to have a turning point 1.0 fm from the surface? Assume the nucleus remains at rest. (b) What is the protons kinetic energy in MeV?

37. In a head-on collision, the closest approach of a 6.24 MeV ^{4}He nucleus to the center of a nucleus is 6.0 fm. The nucleus is an atom of what element? Assume the nucleus remains at rest.

38. A 100-eV hydrogen atom ^{1}H collides head-on in a perfectly elastic collision with a ^{28}Si atom at rest. What is the kinetic energy of each atom after the collision?

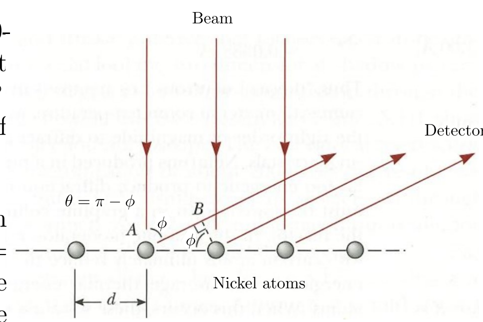
39. An alpha particle (a ^{4}He nucleus) approaches a ^{197}Au nucleus with a speed of $1.50 \times 10^7\ m/s$ as shown in the figure. The alpha particle is scattered at a 49° angle at the slower speed of $1.49 \times 10^7\ m/s$. In what direction does the ^{197}Au nucleus recoil, and with what speed?



40. (a) An electron has a kinetic energy of 3.00 eV. Find its wavelength. (b) Suppose a photon has energy 3.00 eV. Find its wavelength.

41. The nucleus of an atom is on the order of $10^{-14}\ m$ in diameter. For an electron to be confined to a nucleus, its de Broglie wavelength would have to be on this order of magnitude or smaller. (a) What would be the kinetic energy of an electron confined to this region? (b) Make an order-of-magnitude estimate of the electric potential energy of a system of an electron inside an atomic nucleus. (c) Would you expect to fi nd an electron in a nucleus? Explain.

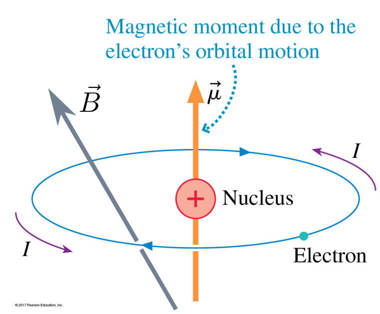
42. In the DavissonGermer experiment shown in the figure, 54.0-eV electrons were diffracted from a nickel lattice. If the first maximum in the diffraction pattern was observed at $\phi = 50°$ what was the lattice spacing a between the vertical columns of atoms in the figure?



43. A photoelectric experiment is performed with an aluminum cathode. An electron inside the cathode has a speed $v_i = 1.5 \times 10^6\ m/s$. If the potential difference between the anode and the cathode is $\Delta V = -2.0\ V$, what is the highest possible speed v_f the electron can have when it reaches the anode?

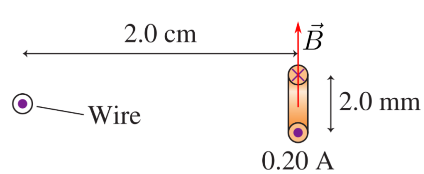
44. Consider the proton as a solid sphere of radius $r = 10^{-15} m$. (a) What is the mass density of a proton? (b) Treat the electron as a uniform solid sphere with the same density as the proton. What would be its radius? (c) Suppose further that this electron has spin angular momentum $S = I\omega = \hbar/2$ because of a classical rotation about the z axis. What is the speed of a point on the equator of this electron? How does this result compare with the speed of light?

45. Consider a classical electron orbiting a proton in a hydrogen atom at a radius $r_e = 0.53$ Å with a speed $v_e = 0.0073c$ as shown in the figure. The hydrogen atom is in a uniform $\vec{B}$ field that makes an angle $\theta = 30°$ with the plane of the electron's orbit and has strength $|\vec{B}| = 1.0\ T$. What is the magnetic moment $\vec{\mu}$ of the orbiting electron (ignore it's intrinsic magnetic moment) and the torque $\vec{\tau}$ on the current loop?



46. A square current loop 5.0 cm on each side carries a 500 mA current. The loop is in a 1.2 T uniform magnetic field. The axis of the loop, perpendicular to the plane of the loop, is 30° away from the field direction. What is the magnitude of the torque on the current loop?

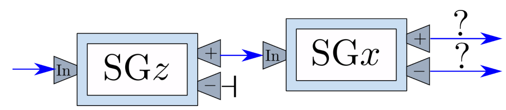
47. Consider the current loop near a current-carrying wire in the figure. The magnitude of the $\vec{B}$ field from the wire is $2 \times 10^{-5}\ T$. What is the magnitude of the torque on the current loop? What is the loop's equilibrium orientation?



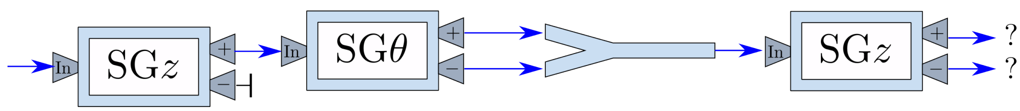
48. Consider the configuation of Stern-Gerlach devices in the figure. Suppose the input to the SGz device is

$$|\psi_i\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle$$

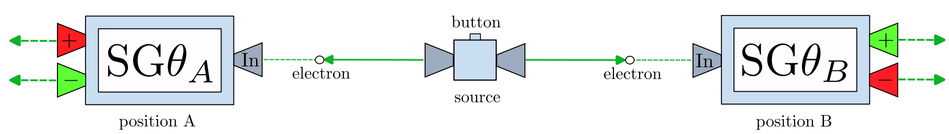
where a_1 and a_2 are constants. What is the probability of observing $+x$ in the final state? What is the probability of observing $-x$ in the final state? What other requirements are there on a_1 and a_2 (Hint: $\langle\psi_i|\psi_i\rangle = 1$)?



49. Suppose that N electrons enter the SGθ device shown in the figure below. Let θ be such that $\cos(\theta/2) = \sqrt{3/4}$ and $\sin(\theta/2) = 1/2$. (a) How many electrons will come out of the $+z$ channel of the second SGz device? (b) If the $+\theta$ channel of SGθ device were blocked before the electrons reached the funnel, how many electrons would come out of the $+z$ channel of the second SGz device?



50. Consider the EPR apparatus shown below where $\theta_A = 0°$, $\theta_B = -120°$, and A flashes green. (a) What is $|\psi_A\rangle$? (b) What is $|\psi_B\rangle$? (c) Which channel must the electron be in? (d) What is the probability, P_{g2}, for this to occur?



51. Just for fun consider the EPR apparatus shown above where $\theta_A = 0°$, $\theta_B = 120°$, and A flashes red. (a) What is $|\psi_A\rangle$? (b) What is $|\psi_B\rangle$? (c) Which channel must the electron be in? (d) What is the probability, P_{r3}, for this to occur?

52. Consider the EPR apparatus shown above where $\theta_A = 120°$, $\theta_B = -120°$, and A flashes red. (a) What is $|\psi_A\rangle$? (b) What is $|\psi_B\rangle$? (c) Which channel must the electron be in? (d) What is the probability, P_{r3}, for this to occur?

53. Consider the EPR apparatus shown above where $\theta_A = 120°$, $\theta_B = 120°$, and A flashes red. (a) What is $|\psi_A\rangle$? (b) What is $|\psi_B\rangle$? (c) Which channel must the electron be in? (d) What is the probability, P_{r4}, for this to occur?

54. Consider the results of a series of runs with a EPR device like the one above. Is the list of events below consistent with quantum mechanics? Remember that the uncertainty in any count N of random numbers is $\pm\sqrt{N}$.

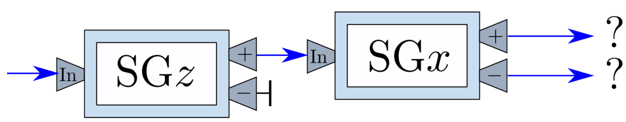
23GR	31RG	32RR	31GG	21GG	13GR	12RG	23GR
21GR	11GG	31RG	23RG	12RR	13RR	12RR	32GG
23GR	23GG	11GG	21GR	23RR	23RR	12GG	12RR
31GG	31GG	23GG	13RG	12RR	32GG	13RR	33RR
13GG	21GG	13RR	31RR	12RR	23GG	32GG	12RR
32RR	33GG	31GR	23GG	12GG	31RG	22GG	11GG
13GR	13RG	22RR	31RR	23GR	32RR	22GG	12RG
32RR	13RG	13RR	12GG	31GR	21RG	11RR	12RR

55. Consider a classical model for the electron that includes hidden variables consistent with locality and reality. This 'observable' will identify the color for for different orientations or values of θ for the SG devices. Call it the electron 'gizmo'. The colors assigned to the different orientations are, for example: $0° \to$ red, $120° \to$ green, and $-120° \to$ red. (a) What are all the different combinations for the gizmos? (b) What are all the different combinations for orientations of the SG devices?

56. Consider the hidden variable theory we developed in class where electrons carry additional information (hidden variables that tell which colors to flash for the different orientations). In the table below the columns represent all the combinations of orientations. The rows represent gizmo. The 'Pr' is the probability of getting the same color. Let 'S' mean the orientation is the same and 'D' means they are different. Fill in the table.

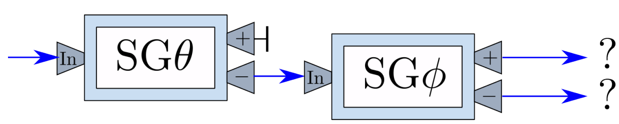
	11	12	13	21	22	23	31	32	33	Pr
RRR										
RRG										
RGR										
RGG										
GRR										
GRG										
GGR										
GGG										

57. Consider the Stern-Gerlach measurement shown below.



Electrons in the $|+z\rangle$ state emerge from the positive SGz channel and enter the SGx device. What does the $|+z\rangle$ state look like to an SGx device? In other words, write the $|+z\rangle$ state vector in terms of the eigenvectors of SGx. What are the numerical amplitudes of each SGx eigenvector? Use this result to show how the final SGx eigenvector is 'plucked' out of the incoming wave function. Is your result consistent with our previous results?

58. Consider the Stern-Gerlach measurement shown below.



Electrons in the $|-\theta\rangle$ state emerge from the negative SGθ channel and enter the SGϕ device. The eigenvectors of the SGϕ device are the same functions as the SGθ device with the angle ϕ replacing θ. What does the $|-\theta\rangle$ state look like to an SGϕ device? In other words, write the $|-\theta\rangle$ state vector in terms of the eigenvectors of SGϕ. What are the numerical amplitudes of each SGϕ eigenvector? Use this result to show how the final SGϕ eigenvector is 'plucked' out of the incoming wave function.

59. Define a q-vector for the spins state $|+z\rangle$ to be the following.

$$|+z\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad .$$

We know empirically that if we send electrons through an SGz device, select only those emerging from the + channel, and send them into an SGx device, the probability that a given electron will emerge from each output channel of the SGx device is 1/2. Show that this information and the assumption the components of $|+x\rangle$ and $|-x\rangle$ are real implies that

$$|+x\rangle \quad \text{and} \quad |x\rangle \quad = \begin{bmatrix} \pm\sqrt{1/2} \\ \pm\sqrt{1/2} \end{bmatrix} \quad .$$

However, $|+x\rangle$ and $|-x\rangle$ must be orthogonal since they are eigenvectors of S_x. Show that, up to an overall sign, $|+x\rangle$ and $|-x\rangle$ must be

$$|+x\rangle = \begin{bmatrix} \sqrt{1/2} \\ \sqrt{1/2} \end{bmatrix} \qquad |-x\rangle = \begin{bmatrix} \sqrt{1/2} \\ -\sqrt{1/2} \end{bmatrix}$$

or

$$|+x\rangle = \begin{bmatrix} \sqrt{1/2} \\ -\sqrt{1/2} \end{bmatrix} \qquad |-x\rangle = \begin{bmatrix} \sqrt{1/2} \\ \sqrt{1/2} \end{bmatrix}$$

60. The kinetic energy of a relativistic particle in SI units is

$$K = E - mc^2$$

where E is the particle's total relativistic energy and m is its mass. Recall the results of the metric equation for the inner product of a particle's 4-momentum with itself.

$$(mc^2)^2 = E^2 - |\vec{p}|^2 c^2$$

and

$$|\vec{p}|^2 c^2 = E^2 - (mc^2)^2 = (E - mc^2)(E + mc^2) \qquad .$$

Combine these equations with the deBroglie equation to prove that a formula for the deBroglie wavelength of relativistic and non-relativistic particles is

$$\lambda = \frac{hc}{\sqrt{K(K + 2mc^2)}}$$

Show this result reduces to the non-relativistic expression

$$\lambda = \frac{h}{\sqrt{2Km}} \qquad .$$

61. The 1990 Nobel Prize for physics was awarded to H.W.Kendall, J.I.Friedman, and R.E.Taylor for their discovery of quarks using 20-GeV electrons scattering from light nuclei.

    (a) Using the results from the previous problem, what is the deBroglie wavelength of a beam of 20-GeV electrons?
    (b) How does this compare to the size of typical nuclei of 10^{-15} m?
    (c) Does it matter much if the energy of 20 GeV refers to the total energy or the relativistic kinetic energy of the electrons?

62. In the 1940 book *Mr. Tompkins in Wonderland* physicist George Gamow imagined a trip into a "quantum jungle" where the value of Planck's constant was $h = 1.0\ J-s$ instead of its accepted value of $6.63 \times 10^{-34}\ J-s$. Imagine that while exploring this jungle you disturb a colony of bats in barn. Imagine a beam of identical bats comes flying out through two barn doors that are a distance $a = 3\ m$ apart at $v_b = 6\ m/s$. Suppose you are a distance $L = 30\ m$ from the doors. Where should stand in the barnyard to avid being struck by the bats? The bats all have mass $m = 0.5\ kg$.