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DEPARTMENT OF PHYSICS
PHY321F

MILLIKAN OIL-DROP EXPERIMENT

This experiment, first carried out by R.A. Millikan and H. Fletcher in 1909 is remarkable, in that it allows one of the fundamental constants of nature, the elementary unit of charge, *e*, to be determined with extremely simple apparatus and straightforward measurements, and also allows a change in charge by this extraordinarily small amount to reveal itself macroscopically as an easily observable change in velocity. (In fact, it is just because of this that this method has been used more recently in attempts to detect the possible existence of smaller sub-units of charge, as postulated for quarks - so far without success.)

The essential idea is that, by comparing the magnitude of the electrostatic force, qE, exerted by an electric field $\boldsymbol{E}$ on a charge q with that of the gravitational force, mg, exerted by gravity on the mass bearing the charge, one may determine q, the quantities E, m, and g, all being known or easily measurable. When this is done, it turns out that the value of q (at least for very small oil-drops) always turns out to be a fairly small multiple of a certain value, which is thus assumed to be the basic unit in which charges come.

The uniform electric field $\boldsymbol{E}$ is generated by a voltage V applied across two parallel horizontal plates a distance d apart. Minute oil drops are sprayed into this space by means of an atomiser. When the electric field is switched off they fall under gravity and reach a constant terminal velocity in a fraction of a second. This occurs when the weight of the droplet is balanced by the viscous resistance to its motion due to the air. According to Stokes's law this resistance (for small spheres of radius a) is given by $6\pi\eta a v_0$ where η is the viscosity of air and v_0 the velocity of fall under gravity. For the terminal velocity we have

$$6\pi\eta a v_0 = m_{eff}\, g \tag{1}$$

where m_{eff} is the “effective mass” of the droplet, equal to $\frac{4}{3}\pi a^3(\rho-\sigma)$, where a is its radius, ρ its density and σ the density of air (to allow for the Archimedean upthrust).

If the drop happens to carry an electric charge, q, and an electric field is applied vertically upwards, it will move with a different drift velocity v_d such that

$$6\pi\eta a v_d = m_{eff}\, g - qE\,. \tag{2}$$

If v_d is negative, the electrostatic force is large enough to overcome gravity, and the drop moves upwards.

Subtracting equation (2) from (1), we have

$$qE = 6\pi\eta a(v_0 - v_d)$$

and substituting for $6\pi\eta a$ from (1)

$$qE = m_{eff}\, g\, \frac{v_0 - v_d}{v_0} = \frac{4}{3}\pi a^3 g(\rho - \sigma)\frac{v_0 - v_d}{v_0}. \tag{3}$$

The velocities v_0 and v_d are measured by finding the times t_0 and t_d to cover a small distance *s*, say, determined by cross wires in the eyepiece of the viewing microscope. Thus

$$\frac{v_0 - v_d}{v_0} = \frac{t_d - t_0}{t_d} \quad .$$

The electric field *E* is given by *V/d*, where *V* is the voltage applied to the plates and *d* their separation. Substituting these expressions,

$$q = \frac{4}{3}\frac{\pi a^3 g(\rho - \sigma)d}{V} \cdot \frac{t_d - t_0}{t_d}. \tag{4}$$

In general it is advisable to measure only times of rise or fall greater than 10 seconds in order to obtain reliable results.

Waiting for a drop to change its charge accidentally is a tedious process. A change in charge may be assisted by exposing the drop to a beam of X-rays or to the γ-rays from a radioactive source.

Hold the radioactive source near the Millikan cell for a second or two. The field should be switched off during this procedure, as otherwise the ions produced are rapidly swept to the plates, and there is less chance of the drop picking up any.

All the other quantities being known or measurable it only remains to estimate *a*, the radius of the drop, in order to calculate the charge on it. This is done by assuming Stokes's law and substituting for m_{eff} in equation (1):

$$6\pi\eta a v_0 = \frac{4}{3}\pi a^3 (\rho - \sigma) g$$

$$a_{approx} = \left(\frac{9\eta v_0}{2(\rho - \sigma)g}\right)^{\frac{1}{2}} = \left(\frac{9\eta s}{2(\rho - \sigma)g}\right)^{\frac{1}{2}} \cdot \left(\frac{1}{t_0}\right)^{\frac{1}{2}}. \tag{5}$$

Using this value of *a* in equation (4) we obtain what we may call q_{approx}.

Correction to Stokes's Law

Millikan showed that Stokes's law needs a correction when the spheres involved are extremely small, as is usually the case in this experiment. Such spheres may be considered to accelerate freely in the empty spaces between the air molecules, thus acquiring a greater terminal velocity than they would if the air were a *continuous* medium of viscosity η.

Applying a first order correction in a power series he suggested that the effectively reduced viscosity could be represented by $\eta/(1+b/pa)$, where p is the ambient air pressure and b a suitable constant. From his measurements he concluded that b had the value 0.000617 cm × cm.Hg, if p was given in cm.Hg. and a in cm. (A simple calculation shows that this becomes 0.00822 if a is in metres and p in Nm^{-2} or pascals.) We accept this value for b, as it has proved in the past to bring the results obtained from drops of different size into agreement.

With this correction we obtain the corrected value as follows:

$$a_{corr} = \frac{a_{approx}}{\left(1+\frac{b}{pa_{corr}}\right)^{\frac{1}{2}}}. \tag{6}$$

Solving this quadratic equation for a_{corr},

$$a_{corr} = \sqrt{a_{approx}^2 + \frac{b^2}{4p^2}} - \frac{b}{2p} = \sqrt{\frac{9\eta s}{2(\rho-\sigma)gt_0} + \frac{b^2}{4p^2}} - \frac{b}{2p}. \tag{7}$$

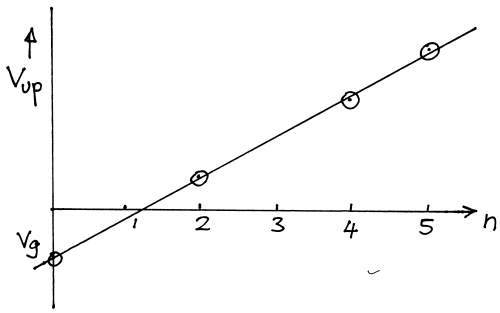
Next calculate the correction factor, f, say,

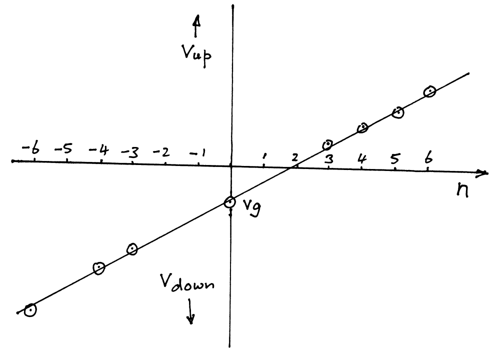
$$f = \frac{1}{\left(1+\frac{b}{pa_{corr}}\right)^{\frac{1}{2}}}.$$

Then $a_{corr} = fa_{approx}$, and replacing a_{approx} in equation (4) by a_{corr}, we see that

$$q_{corr} = f^3 q_{approx}. \tag{8}$$

If observations are continued for a considerable time it will be found that occasionally t_d changes suddenly by a discrete amount. When the values of q_{corr} obtained from the different timings are compared it will appear that they are relatively small integral multiples, ne, of a unit of charge, e. This is the elementary unit of charge, or "electronic charge" (the electron having charge $-e$ and the proton charge $+e$.) Equation (3) shows that, for a given drop, v_d/E is a linear function of $q = ne$. So, for constant potential difference, if the various values of v_d are compared, it will be found that they can be arranged on a straight line when plotted against appropriate values of n (found by inspection.). If the applied voltage V has been changed between different values of n one plots v_d/V against n in order to obtain a straight line.





Simple method

A simple method sometimes adopted is to vary the applied voltage V until the drop hangs stationary, neither rising or falling. Call this balancing voltage V_{bal}. Then $qE = m_{eff}\, g$, i.e.

$$\frac{qV_{bal}}{d} = \frac{4}{3}\pi a^3(\rho - \sigma)g \qquad (9)$$

$$q = \frac{4}{3}\frac{\pi a^3(\rho - \sigma)gd}{V_{bal}}$$

a is found as before from t_g, using equation (7).

With very small drops, on account of their Brownian motion (see later), it is difficult to be sure exactly when V_{bal} has been reached, and the accuracy of the measurements is more difficult to assess. The former method is thus preferable.

Experimental details

The Millikan experiment consists of a small cell with insulating walls, illuminated by a small solid state laser. The drops are observed with a microscope with a graticule in the eyepiece. It should be possible to spray the drops so that they fall through a small hole in the top plate. This can become blocked with oil, and it may be easier to remove the plate to spray a mist into the cell. A pin stuck through the hole through which the drops will fall aids in focusing the microscope and adjusting the illumination for maximum brilliance. ***Do not switch on the supply voltage while the pin is in position between the plates***.

To prevent the drop from drifting sideways, or moving longitudinally out of focus, it is important that the plates be accurately levelled.

Very small drops exhibit Brownian motion clearly. On account of this there will be a spread in the values of t obtained, and a large number of readings should be taken in order to obtain an accurate mean. (This spread actually provides a means of obtaining Avogadro's number - see later.)

It is more rewarding to take a number of readings with different charges on the same drop, rather than on a number of drops with only one charge on each. This, however, can be difficult to achieve and it may be necessary to measure different drops.

Data required

The density of the silicone oil provided was carefully measured some years ago, and was then found to be given by $\rho = (0.9904 - 0.0009\,t)$ g cm^{-3} where t is the temperature in degrees Celsius. It is hoped that it has not changed appreciably since it was measured. It is difficult to measure the temperature ***in*** an oil drop exactly, but t may be taken as the ambient air temperature read on the thermometer provided, as it is involved in what is really only a small correction term.

The barometric pressure may be read on the barometers in the second year laboratory. g at UCT is 979.6 cm s^{-2}.

The density of air may be taken as $\sigma = 0.0012$ g cm^{-3}.

The distance in the object space corresponding to traversal of 4 out of the 6 small squares in the eyepiece graticule is $s = 0.1712 \pm 0.0001$ cm.

The distance between the condenser plates is given by the makers as $d = 0.50 \pm 0.01$ cm. This distance is difficult to check directly and is probably the greatest source of uncertainty in the experiment, which cannot be expected to give a result more accurate than to about 2%.

In a more refined version of the experiment, with standardised meters and an accurately measured value of d, the greatest remaining source of uncertainty is the value of the viscosity of air. It was a redetermination of this that led Millikan to revise his original value of e, 4.77×10^{-10} e.s.u. upwards to 4.80×10^{-10} e.s.u. which agrees better with later determinations by other (and more precise) methods.

The weighted mean of a number of fairly reliable determinations gives η at 23 ^{o}C as $(1.8300 \pm 0.0025) \times 10^{-5}$ kg m^{-1} s^{-1} , with a temperature coefficient of $+\ 0.00483 \times 10^{-5}$ kg m^{-1} s^{-1} C^{-1} .

Note: the cgs unit of viscosity is the **poise** (1 g cm^{-1} s^{-1}) which is still widely used. 1 kg m^{-1} s^{-1} = 10 poise.

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Determination of Avogadro's Number from the Brownian motion

By considering the equipartition of energy between the molecules of a gas (or liquid) and small suspended particles Einstein and (independently) von Smoluchowski deduced the equation

$$\overline{(\Delta x)^2} = \frac{2RT}{NK}\tau \qquad (10)$$

giving the mean of the squared displacements in a large number of equal time intervals τ. Here *R* is the gas constant per mole, *T* the Kelvin temperature, *N* is Avogadro's number (number of molecules per mole), and *K* the constant in Stokes's law giving the frictional resistance to motion through a viscous fluid. (In its simple form, the law states that the resistance to a sphere of radius *a* moving with velocity *v* is $6\pi\eta av$, so *K* here is $6\pi\eta a$).

It is not very convenient to measure the Δ*x*'s in fixed time intervals τ. It is easier to measure the variations Δ*t* in the times to fall a fixed distance under gravity; and in carrying out the oil-drop determination of *e* one naturally accumulates a large number of these data. (This was shown to be equivalent by Schrödinger.)

Let $\overline{t_0}$ be the mean time to fall a distance *s* under gravity.

Suppose that during this time the particle has been displaced a distance Δ*x* (upwards, say) as a result of its Brownian motion. This will result in the observed t_0 being greater by an amount Δt and for ***small*** displacements we can put $\Delta x = \overline{v_0} \cdot \Delta t = \left(s/\overline{t_0}\right)\Delta t$.

Therefore $\overline{\Delta x^2} = \frac{s^2}{\overline{t_0}^2}\overline{\Delta t^2}$ (11)

The equal intervals τ envisaged in equation (10) are replaced by the approximately equal times t_0, $\tau \cong \overline{t_0}$.

If the fluctuations obey the normal distribution, $\overline{(\Delta t)^2} = \frac{\pi}{2}\left(\overline{|\Delta t|}\right)^2$, we can use either $\overline{(\Delta t)^2}$ or $\left(\overline{|\Delta t|}\right)^2$ (with the appropriate factor) giving two, in general, slightly different estimates of *N* (as we are dealing with a finite sample which naturally departs somewhat from an ideal normal distribution.)

Substituting (11) into (10)

$$\overline{(\Delta t)^2} = \frac{\overline{t_0}^2}{s^2}\frac{2RT}{NK}\tau \cong \frac{\overline{t_0}^3}{s^2}\frac{2RT}{NK} .$$

Therefore $N = \frac{2RT\overline{t_0}^3}{s^2 K\overline{(\Delta t)^2}}$ or $N = \frac{4RT\overline{t_0}^3}{\pi s^2 K\left(\overline{|\Delta t|}\right)^2}$, (12a,b)

according to the uncorrected form of Stokes's law $K = 6\pi\eta a$ for small droplets falling in air of viscosity η. For very small oil drops, such as those showing appreciable Brownian motion, however, Stokes's law needs correction, so it is best to eliminate the dubious quantities K and the mass, radius and density of the drop, as follows:

$m_{eff}\, g = Kv_0$ for free fall
$m_{eff}\, g - neE = Kv_d$ where v_d is the drift velocity when carrying charge ne

Combining: $neE = K(v_0 - v_d)$.

Therefore $K = \frac{neE}{(v_0 - v_d)} = \frac{neEt_0 t_d}{s(t_d - t_0)}$ using $v_0 - v_d = \frac{s}{t_0} - \frac{s}{t_d} = \frac{s(t_d - t_0)}{t_0 t_d}$.

Remember that if the drop drifts upwards, t_d is conventionally negative!

Substituting in equation (12)

$N = \frac{2RT\overline{t_0}^2\left(\overline{t_d} - \overline{t_0}\right)}{neEst_d\overline{(\Delta t_0)^2}}$ or $N = \frac{4RT\overline{t_0}^2\left(\overline{t_d} - \overline{t_0}\right)}{\pi neEst_d\left(\overline{|\Delta t_0|}\right)^2}$ (13a,b)

(E is of course the field, = V/d)

For a given drop the values of N obtained from measurements for different charges are not, of course, independent, as they are all based on the same variance $\overline{(\Delta t_0)^2}$, with slight differences due to experimental errors in the measurements of t_d.

Alternatively one may use Millikan's corrected form for $K, 6\pi\eta a/(1 + b/pa)$, in equations 12a or 12b, obtaining

$N = \frac{RT}{3\pi\eta a s^2}\frac{t_0^3}{\overline{(\Delta t_0)^2}}\left(1 + \frac{b}{pa}\right)$ or $N = \frac{2RT}{3\pi^2\eta a s^2}\frac{t_0^3}{\left(\overline{|\Delta t_0|}\right)^2}\left(1 + \frac{b}{pa}\right)$ (14a,b)

These values of N are obtained solely from the measurements of t_0, and do not depend on measurement of t_d and estimation of n.

Finally, N may also be estimated from the variance of t_d, if enough readings are available. It is easily shown that the appropriate formula equivalent to (14a) is

$$N = \frac{RT}{3\pi\eta a s^2} \frac{\overline{t_d^3}}{\overline{(\Delta t_d)^2}} \left(1 + \frac{b}{pa}\right), \tag{15a}$$

using only the magnitude of the drift time.

Eliminating the factor $K = 6\pi\eta a / (1 + b/pa)$ by using $mg = Kv_g$, we find

$$N = \frac{3RT}{2\pi(\rho - \sigma)gs} \frac{\overline{t_d^3}}{t_0 \overline{(\Delta t_d)^2}}. \tag{16a}$$

The most reliable estimates of N result from drops with fairly long t_0 or t_d, which show considerable Brownian motion. With larger drops the true variations in time are partly obscured by random errors in timing, which tend to increase the apparent Δt and lead to too small a value of N.