Homework 5 Electricity

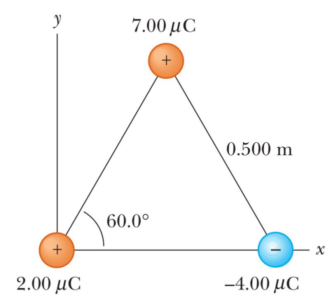
1. (a) Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 g/mol. (b) Electrons are added to the pin until the net negative charge is 1.00 mC. How many electrons are added for every 10^9 electrons already present?

2. Nobel laureate Richard Feynman once said that if two persons stood at arm's length from each other and each person had 1% more electrons than protons, the force of repulsion between them would be enough to lift a "weight" equal to that of the entire Earth. Carry out an order-of-magnitude calculation to substantiate this assertion.

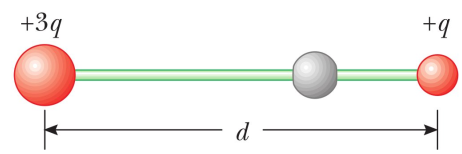
3. Two protons in an atomic nucleus are typically separated by a distance of $2 \times 10^{-15} m$. The electric repulsion force between the protons is huge, but the attractive nuclear force is even stronger and keeps the nucleus from bursting apart. What is the magnitude of the electric force between two protons separated by $2.00 \times 10^{-15} m$?

4. A charged particle A exerts a force of $2.62 \mu N$ to the right on charged particle B when the particles are 13.7 mm apart. Particle B moves straight away from A to make the distance between them 17.7 mm. What vector force does it then exert on A?

5. Three point charges are located at the corners of an equilateral triangle as shown in in the figure. What is the electric force on the 7.00 μC charge.



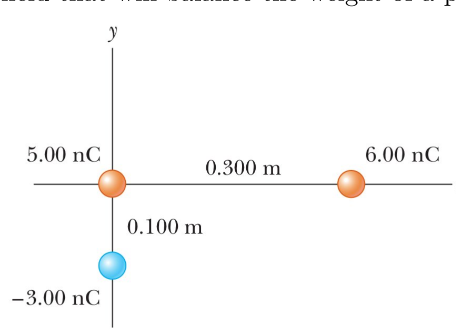
6. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in the figure, a third small, charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?



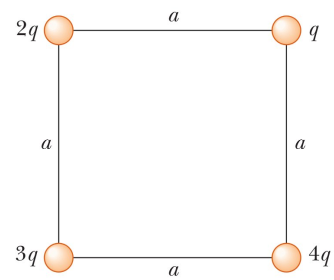
7. In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is $0.529 \times 10^{-10} m$. (a) What is the magnitude of the electric force each exerts on the other. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

8. What are the magnitude and direction of the electric field that will balance the weight of a proton?

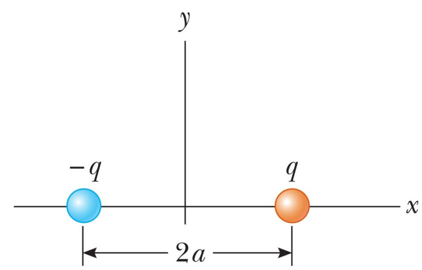
9. Three point charges are arranged as shown in the figure. (a) What is the vector electric field the 6.00-nC and -3.00-nC charges together create at the origin. (b) What is the vector force on the 5.00-nC charge.



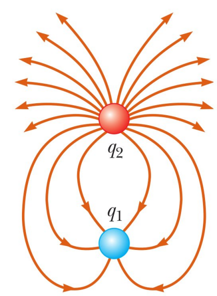
10. Four point charges are at the corners of a square of side a as shown in the figure. (a) Determine the magnitude and direction of the electric field at the location of charge q. (b) What is the resultant force on q?



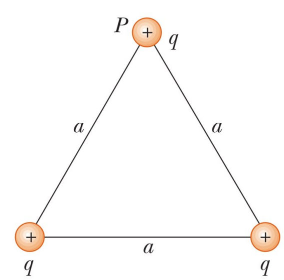
11. Consider the electric dipole shown in the figure. Show that the electric field at a distant point on the x axis is $E_x \approx 4k_e qa/x^3$.



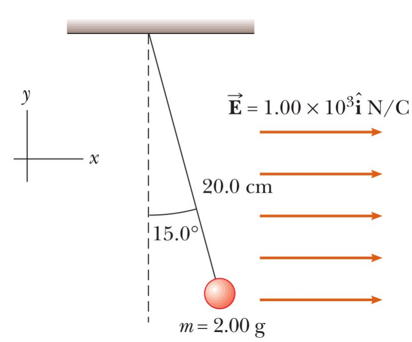
12. The figure shows the electric field lines for two point charges separated by a small distance. (a) What is the ratio q_1/q_2. (b) What are the signs of q_1 and q_2?



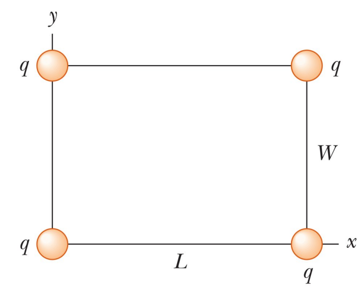
13. Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in the figure. (a) Assume that the three charges together create an electric field. Sketch the field lines in the plane of the charges. Find the location of a point (other than ∞) where the electric field is zero. (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?



14. A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown in the figure. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?



15. Four identical point charges ($q = 10.0\ \mu C$) are located on the corners of a rectangle as shown in the figure. The dimensions of the rectangle are $L = 60.0\ cm$ and $W = 15.0\ cm$. Calculate the magnitude and direction of the resultant electric force exerted on the charge at the lower left corner by the other three charges.

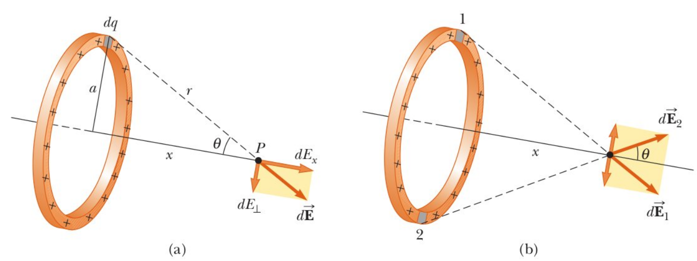


16. A rod 14.0 cm long is uniformly charged and has a total charge of $-22.0\mu C$. Determine the magnitude and direction of the electric field along the axis of the rod at a point 36.0 cm from its center.

17. A continuous line of charge lies along the x axis, extending from $x = x_0$ to positive infinity. The line carries charge with a uniform linear charge density λ_0 . What are the magnitude and direction of the electric field at the origin?

18. A uniformly charged ring of radius 10.0 cm has a total charge of 75.0 μC. Find the electric field on the axis of the ring at (a) 1.00 cm, (b) 5.00 cm, (c) 30.0 cm, and (d) 100 cm from the center of the ring.

19. Show that the maximum magnitude E_{max} of the electric field along the axis of a uniformly charged ring occurs at $x = a/\sqrt{2}$ (see figure) and has the value $Q/(6\sqrt{3}\pi\epsilon_0 a^2)$.



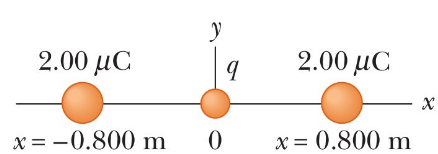
20. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

21. How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)

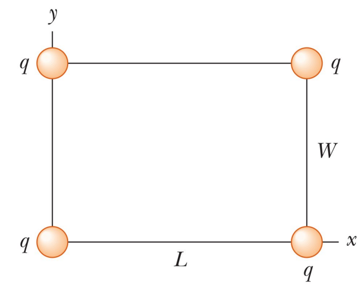
22. The difference in potential between the accelerating plates in the electron gun of a TV picture tube is about 25,000 V. If the distance between these plates is 1.50 cm, what is the magnitude of the uniform electric field in this region?

23. (a) Find the potential at a distance of 1.00 cm from a proton. (b) What is the potential difference between two points that are 1.00 cm and 2.00 cm from a proton? (c) Repeat parts (a) and (b) for an electron. Note: Unless stated otherwise, assume that the reference level of potential is $V = 0$ at $r = \infty$.

24. Given two $2.00 - \mu$C charges as shown in the figure and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two $2.00 - \mu$C charges on the test charge q? (b) What is the electric field at the origin due to the two $2.00 - \mu$ C charges? (c) What is the electrical potential at the origin due to the two $2.00 - \mu$C charges?

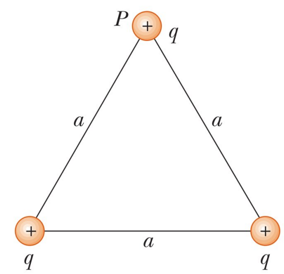


25. Four identical point charges ($q = 10.0\ \mu$C) are located on the corners of a rectangle as shown in the figure. The dimensions of the rectangle are $L = 60.0\ cm$ and $W = 15.0\ cm$. Calculate the change in electric potential energy of the system as the charge at the lower left corner is brought to this position from infinitely far away. Assume that the other three charges remain fixed in position.

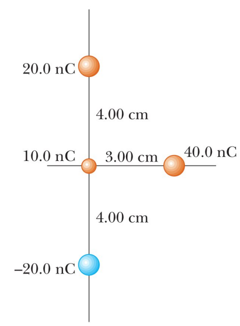


26. Show that the amount of work required to assemble four identical point charges of magnitude Q at the corners of a square of side s is $5.41 k_e Q^2/s$.

27. Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in the figure. (a) At what point, if any, in the plane of the charges is the electric potential zero? (b) What is the electric potential at the point P due to the two charges at the base of the triangle?



28. Two particles, with charges of 20.0 nC and -20.0 nC, are placed at the points with coordinates (0, 4.00 cm) and (0, -4.00 cm) as shown in the figure. A particle with charge 10.0 nC is located at the origin. (a) Find the potential energy of the configuration of the three fixed charges. (b) A fourth particle, with a mass of $2.00 \times 10^{-13}\ kg$ and a charge of 40.0 nC, is released from rest at the point (3.00 cm, 0). Find its speed after it has moved freely to a very large distance away.



29. In 1911, Ernest Rutherford and his assistants Hans Geiger and Ernest Marsden conducted an experiment in which they scattered alpha particles from thin sheets of gold. An alpha particle, having charge +2e and mass $6.64 \times 10^{-27}\ kg$, is a product of certain radioactive decays. The results of the experiment led Rutherford to the idea that most of the mass of an atom is in a very small nucleus, with electrons in orbit around it, in his planetary model of the atom. Assume that an alpha particle, initially very far from a gold nucleus, is fired with a velocity of $2.00 \times 10^7\ m/s$ directly toward the nucleus (charge +79e). How close does the alpha particle get to the nucleus before turning around? Assume that the gold nucleus remains stationary.

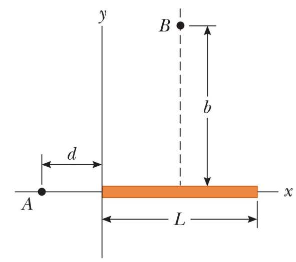
30. The potential in a region between $x = 0$ and $x = 6.00\ m$ is $V = a + bx$, where $a = 10.0\ V$ and $b = -7.00\ V/m$. What is (a) the potential at $x = 0, 3.00\ m$, and $6.00\ m$; and (b) the magnitude and direction of the electric field at $x = 0$, $3.00\ m$, and $6.00\ m$.

31. The electric potential inside a charged spherical conductor of radius R is given by $V = k_e Q/R$ and outside the potential is given by $V = k_e Q/r$. Using $E_r = -dV/dr$, derive the electric field (a) inside and (b) outside this charge distribution.

32. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. Starting from the electric field along the axis of the ring, what is the potential difference between the point at the center of the ring and a point on its axis a distance $2R$ from the center? The axis of the ring passes through the ring's center and is perpendicular to the plane of the ring.

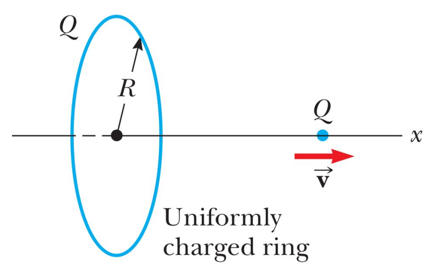
33. The liquid-drop model of the atomic nucleus suggests that high-energy oscillations of certain nuclei can split the nucleus into two unequal fragments plus a few neutrons. The fission products acquire kinetic energy from their mutual Coulomb repulsion. Calculate the electric potential energy (in electron volts) of two spherical fragments from a uranium nucleus having the following charges and radii: 38e and 5.50×10^{-15} m; 54e and 6.20×10^{-15} m. Assume that the charge is distributed uniformly throughout the volume of each spherical fragment and that just before separating each fragment is at rest and their surfaces are in contact. The electrons surrounding the nucleus can be ignored.

34. A rod of length L (see figure) lies along the x axis with its left end at the origin. It has a nonuniform charge density $\lambda = \alpha x$, where α is a positive constant. (a) What are the units of α? (b) What is the electric potential at A. (c) What is the electric potential at B? Get your answer in the form of an integral.



35. The x axis is the symmetry axis of a stationary, uniformly charged ring of radius R and charge Q (see figure). A particle with charge Q and mass M is located at the center of the ring. When it is displaced slightly, the point charge accelerates along the x axis to infinity. Show that the ultimate speed of the point charge is

$$v = \sqrt{\frac{2k_e Q^2}{MR}}$$



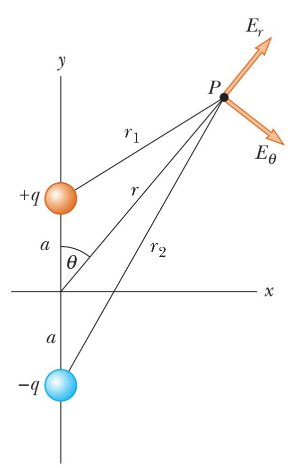
36. An electric dipole is located along the y axis as shown in the figure. The magnitude of its electric dipole moment is defined as $p = 2qa$. (a) At a point P, which is far from the dipole ($r >> a$), show that the electric potential is

$$V = \frac{k_e p \cos\theta}{r^2}$$

(b) Calculate the radial component E_r and the perpendicular component E_θ of the associated electric field. Note that $E_\theta = -(1/r)(\partial V/\partial\theta)$. Do these results seem reasonable for $\theta = 90°$ and $0°$? (c) For the dipole arrangement shown, express V in terms of Cartesian coordinates using $r = (x^2 + y^2)^{1/2}$ and

$$\cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Using these results and again taking $r >> a$, calculate the field components E_x and E_y.



37. In a particular cathode-ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?

38. A small sphere that carries a charge q is whirled in a circle at the end of an insulating string. The angular frequency of revolution is ω. What average current does this revolving charge represent?

39. Suppose the current in a conductor decreases exponentially with time according to the equation $I(t) = I_0 e^{-t/\tau}$, where I_0 is the initial current (at $t = 0$) and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) How much charge passes this point between $t = 0$ and $t = \infty$?

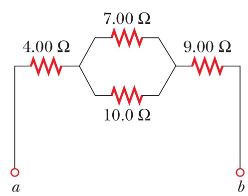
40. The quantity of charge q (in coulombs) that has passed through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds. What is the instantaneous current across the surface at $t = 1.00$ s?

41. A light bulb has a resistance of 240 Ω when operating with a potential difference of 120 V across it. What is the current in the light bulb?

42. A toaster is rated at 600 W when connected to a $120 - V$ source. What current does the toaster carry, and what is its resistance?

43. Suppose a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W light bulb increase? Assume that its resistance does not change.

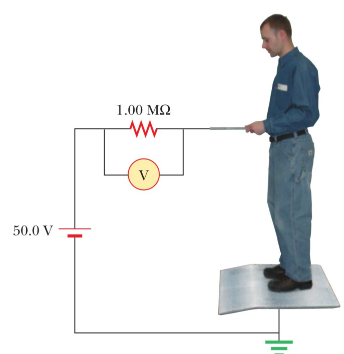
44. (a) Find the equivalent resistance between points a and b in the figure. (b) A potential difference of 34.0 V is applied between points a and b. Calculate the current in each resistor.



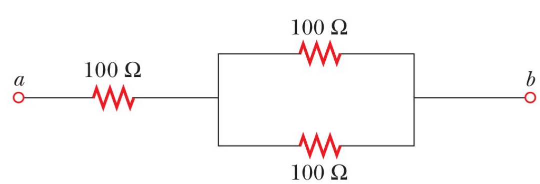
45. For the purpose of measuring the electric resistance of shoes through the body of the wearer to a metal ground plate, the American National Standards Institute (ANSI) specifies the circuit shown in the figure. The potential difference ΔV across the $1.00 - M\Omega$ resistor is measured with a high-resistance voltmeter. The resistance of the person's body is negligible by comparison. (a) Show that the resistance of the footwear is given by

$$R_{shoes} = 1.00 M\Omega \left(\frac{50.0V - \Delta V}{\Delta V} \right)$$

(b) In a medical test, a current through the human body should not exceed 150 μA. Can the current delivered by the ANSI-specified circuit exceed 150 μA? To decide, consider a person standing barefoot on the ground plate.



46. Three $100 - \Omega$ resistors are connected as shown in the figure. The maximum power that can safely be delivered to any one resistor is 25.0 W. (a) What is the maximum voltage that can be applied to the terminals a and b? (b) For the voltage determined in part (a), what is the power delivered to each resistor? What is the total power delivered?

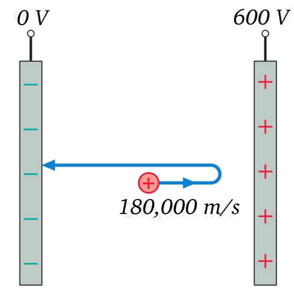


47. Currents of protons in the form of particle beams from accelerators show promise as a precise tool for destroying cancerous tumors. Consider the design of such a devise. Voltages of $V = 2.0 \times 10^6$ V can be

cheaply produced. Protons emitted from a source start from rest and are accelerated through this electric potential V. What would be the 'drift' velocity of protons that were emitted by such a machine? If the machine produces a current of $I = 10^{-5}\ A$ with a cross-sectional area of $A = 10^{-5}\ m^2$, then what is the density of protons in the beam? If a typical treatment session requires the tumor to receive a dose of $N = 10^{17}$ protons of this energy, then how long would the patient be the 'target' of this proton beam?

48. You are a summer research student at a medical-school lab that uses proton beams to treat cancer patients. The protons exit the machine with a speed $v_0 = 4.0 \times 10^5\ m/s$ and you've been asked to design a device to stop the protons safely. You can simply have the protons strike a metal plate to stop them, but protons traveling faster than $v_l = 2.0 \times 10^5\ m/s$ emit dangerous x-rays when they hit. You decide the best thing to do is to slow down the protons to below v_l and then let them hit the metal plate. You take two, flat, parallel, metal plates a distance $d = 0.02\ m$ apart and drill a small hole through the center of one plate to let the beam through. The second plate will stop the protons. What should be voltages on the two plates used to slow down the protons.

49. In the figure a proton is fired with a speed $v_i = 180,000\ m/s$ from the midpoint of the capacitor toward the positive plate. Does the proton reach the positive plate? What is the proton's speed as it collides with either the negative or positive plate?



50. Two 10-cm-diameter charged rings face each other, 20 cm apart. The left ring is charged to -20 nC and the right ring is charged to +20 nC. (a) What is the electric field E, both magnitude and direction, at the midpoint between the two rings? (b) What is the force F on a -1.0 nC charge placed at the midpoint?

51. Two 10-cm-diameter charged rings face each other, 20 cm apart. Both rings are charged to +20 nC. What is the electric field strength at (a) the midpoint between the two rings and (b) the center of the left ring?

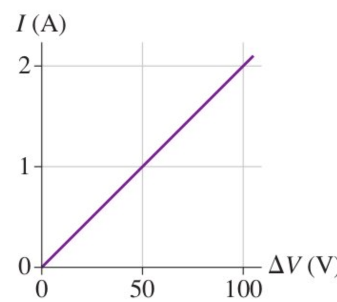
52. Two 10-cm-diameter charged disks face each other, 20 cm apart. Both disks are charged to +50 nC. What is the electric field strength at (a) the midpoint between the two disks and (b) a point on the axis 5.0 cm from one disk?

53. A 20 cm $\times$ 20 cm horizontal metal electrode is uniformly charged to +80 nC. What is the electric field strength 2.0 mm above the center of the electrode?

54. An aluminum wire having a cross-sectional area of $4.00 \times 10^{-6}\ m^2$ carries a current of $I = 5.0\ A$. Find the drift speed v_d of the electrons in the wire. The density of aluminum is $\rho = 2.70\ g/cm^3$. Assume that one conduction electron is supplied by each atom.

55. Four 1.50-V AA batteries in series are used to power a transistor radio. If the batteries can move a charge of $Q = 240\ C$, how long will they last if the radio has a resistance of $R = 200\ \Omega$?

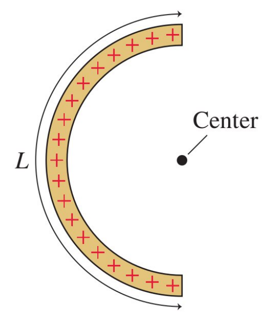
56. The figure shows a current-versus-potential-difference graph for a material. What is the material's resistance?



57. A rod of length $L = 14.0\ cm$ is uniformly charged and has a total charge $Q = -22.0\ \mu C$. Consider a point at a distance $y_0 = 36.0\ cm$ from its center and perpendicular to the axis of the rod. Starting from the field

of a point charge obtain the magnitude and direction of the electric field at this point in terms of L, Q, x_0, x and any other necessary constants. Once you have that expression calculate the numerical value of the electric field vector at x_0.

58. A charge Q is uniformly distributed over a thin, flexible rod that is bent into a semicircle as shown in the figure. (a) What is the expression for the electric field at the center of the semicircle in terms of L, Q, and any other constants? If $L = 0.08\ m$ and $Q = 4 \times 10^{-8}\ C$, calculate the field. (b) Starting from the electric potential for a point charge, what is the expression for the potential in terms of L, Q, and any other constants? If $L = 0.08\ m$ and $Q = 4 \times 10^{-8}\ C$, calculate the potential. Hint: A short, curved chunk of the arc ds is related to the angle covered by that short chunk by $d\theta = rds$ where r is the radius of the semicircle.



59. Consider the work done to move a charge q_{test} in the electric field of a point charge q

$$W = \int \vec{F}_{pt} \cdot d\vec{r} = \int_{r_A}^{r_B} q_{test}\vec{E}_{pt} \cdot d\vec{r} = \int_{r_A}^{r_B} \frac{k_e q q_{test}}{r^2}\ \hat{r} \cdot d\vec{r}$$

where $\hat{r}$ is a unit vector pointing radially outward. See the figure. In polar coordinates we can write the differential vector $d\vec{r}$ as

$$d\vec{r} = dr_{\|}\hat{r} + dr_{\perp}\hat{\theta}$$

where $\hat{r}$ again points radially outward and $\hat{\theta}$ is perpendicular to $\hat{r}$. The differential components $dr_{\|}$ and $dr_{\perp}$ are parallel and perpendicular to $\hat{r}$, respectively. Show the dot product in the equation for the work W is

$$\hat{r} \cdot d\vec{r} = dr_{\|}$$

using the properties of the dot product $\vec{A} \cdot \vec{B} = |\vec{A}|\ |\vec{b}| \cos\alpha$ where α is the angle between the vectors $\vec{A}$ and $\vec{B}$.