# Physics 131-01 Final Exam

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

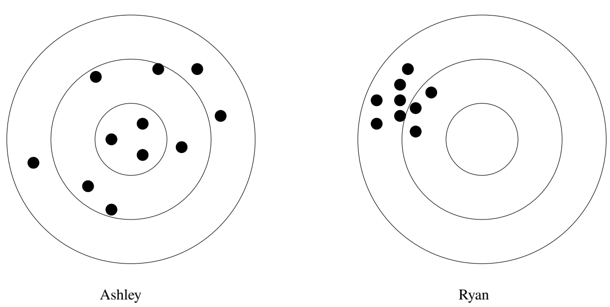
Signature ___________________________________

Questions (4 pts. apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. You are riding on a flat surface in a cart at a velocity $\vec{v}_{launcher}\hat{i}$ which is much less than the speed of light. A stationary observer is nearby. At $t = 0$ your coordinate systems coincide. At $t > 0$ you fire a toy cannon at an angle of about 45° from the moving cart. Consider a point $\vec{r} = x\hat{i} + y\hat{j}$ on the ball's trajectory in the stationary observer's reference frame. What would the moving observer measure for the position $\vec{r}'$ in terms of the stationary observer's values?

2. A paradox is defined in the Merriam-Webster online dictionary as 'an argument that apparently derives self-contradictory conclusions by valid deduction from acceptable premises'. What is paradoxical about the twins paradox?

3. Suppose Ashley and Ryan each throw darts at targets as shown below. Each of them is trying very hard to hit the bulls eye each time. Which one is better? Explain in terms of the average value and uncertainty of their hits.

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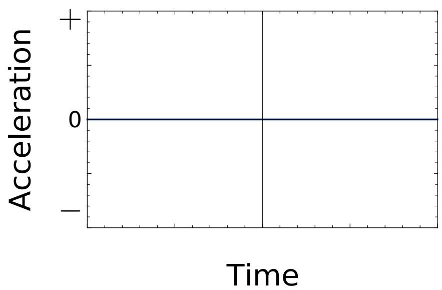
4. Suppose you have a dynamics cart sitting on a track with a motion sensor at one end. The opposite end of the track is tilted upward. You give the cart a shove up the track, remove your hand, and start the motion sensor. Sketch the acceleration versus time graph you would observe until the cart comes to a stop. Explain your reasoning.




5. Use Newton's law of universal gravitation to show that the magnitude of the acceleration due to gravity on an object of mass $m$ at a height $h$ above the surface of the earth is given by the following expression

$$\frac{GM_e}{(R_e + h)^2}$$

where $M_e$ and $R_e$ are the Earth's mass and radius. Hint: Because of the spherical symmetry of the Earth you can treat the mass of the Earth as if it were all concentrated at a point at the Earth's center.

6. Recall the laboratory on the conservation of angular momentum. You studied a rotational 'collision' when you dropped a weight on a rotating disk and determined the angular momentum before and after the collision. Would the procedure you followed change if the weight was moving horizontally at a constant velocity when you dropped it? If it changed, what would be different? Explain your reasoning.

7. Describe a procedure to measure the spring constant of a spring.

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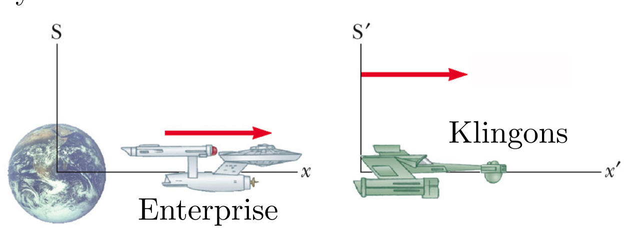
8. A person riding a Ferris wheel at an amusement park moves through the positions at (a) the top, (b) the bottom, and (c) midheight. Rank the three positions according to the magnitude of the net centripetal force on the person. Explain your reasoning.

9. In some motorcycle races, the riders drive over small hills and become airborne for short periods of time. If the motorcycle racer keeps the throttle open (*i.e.* keeps the engine running as maximum power) while leaving the hill and going into the air, the motorcycle tends to nose upward. Why?

10. Consider a rocket in free space with no forces acting on it. After the rocket engine is turned on does the center-of-mass of the system accelerate? Can the speed of the rocket exceed the exhaust speed of the fuel? Explain.

**Problems**. Clearly show all reasoning for full credit. Use a separate sheet for your work.

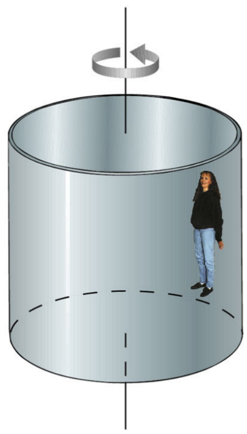
1. 8 pts. A Klingon spacecraft moves away from the Earth at a speed of $0.8c$ (see figure) relative to the Earth. The starship Enterprise pursues the Klingons at a high speed. Observers on the Klingon ship measure the Enterprise overtaking them at a relative speed of $0.4c$. What is the speed of the Enterprise as measured by observers on the Earth?




2. 8 pts. A supertrain (proper length $l_p = 120\ m$) travels at a speed $v = 0.90c$ as it passes through a tunnel of proper length $l_t = 50\ m$. As seen by a trackside observer, is the train ever completely within the tunnel? If so, with how much space to spare? If not, by how much does it miss?

**Problems (continued).** Clearly show all reasoning for full credit.

3. 8 pts. An amusement park ride called the Rotor is made of a large vertical cylinder that spins fast enough so the people inside are pinned against the wall when the floor drops away as shown in the figure. Let $\mu_s$ be the coefficient of static friction between a person and wall, and the radius of the cylinder is $R$. (a) Using Newton's Laws show the maximum period of revolution $T$ necessary to keep a person from falling is $T = 2\pi\sqrt{\mu_s R/g}$. (b) What is the value of $T$ assuming that $R = 3.7\ m$ and $\mu_s = 0.50$. (c) How many revolutions per minute does the cylinder make?

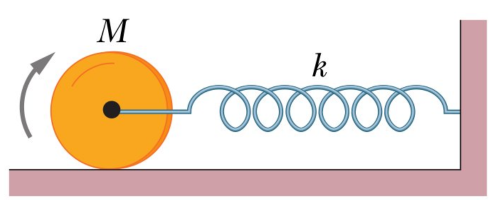


4. 9 pts. A neutron in a nuclear reactor makes an elastic head-on collision with a carbon nucleus initially at rest. (a) Starting from the appropriate conservation laws determine the fraction of the neutron's initial kinetic energy that remains after the collision with the carbon nucleus? (b) Assume the initial kinetic energy of the neutron is $1.5 \times 10^{-13}\ J$. Find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision. The mass of the carbon nucleus is nearly 12.0 times the mass of the neutron.

5. 9 pts. In the figure below a solid cylinder attached to a horizontal spring ($k = 3.0\ N/m$) rolls without slipping along a horizontal surface. If the system is released from rest when the spring is stretched by $x_1 = 0.250\ m$, find (a) the translational kinetic energy and (b) the rotational kinetic energy of the cylinder as it passes through the equilibrium position. (c) Show that under these conditions the cylinder's center of mass executes simple harmonic motion with period

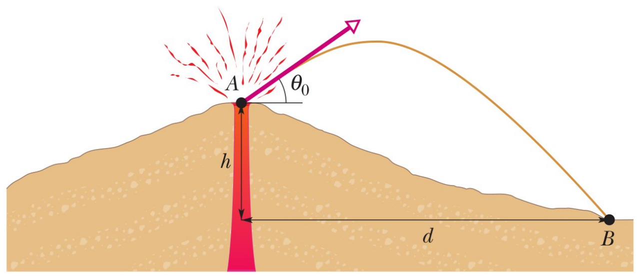
$$T = 2\pi\sqrt{\frac{3M}{2k}}$$

where $M$ is the cylinder mass.



**Problems (continued)**. Clearly show all reasoning for full credit.

6. 9 pts. During a volcanic eruption chunks of rock are blasted out of the volcano. These projectiles are called volcanic bombs. Consider the figure below of Mount Fuji in Japan. A volcanic bomb is launched with an initial speed $v_0 = 257\ m/s$ at an angle $\theta_0$ from point A in the figure and it eventually lands at point B. The mouth of the volcano is a distance $h = 3.3 \times 10^3\ m$ above the landing spot which is a distance $d = 9.4 \times 10^3\ m$ downrange from the mouth of the volcano. Starting from the equations of motion (*i.e.* $x(t)$, $v_x(t)$, $y(t)$, $v_y(t)$ for uniformly accelerated motion) find the launch angle $\theta_0$.




7. 9 pts. Global warming is a cause for concern because even small changes in the Earth's temperature can have significant consequences. For example, if the Earth's polar ice caps were to melt entirely, the resulting additional water in the oceans would flood many coastal cities. Would it appreciably change the length of a day? In other words, would we notice the day getting longer or shorter? Model each polar ice cap as a flat disk of mass $m_i = 1.15 \times 10^{19}\ kg$, radius $R_i = 6 \times 10^5\ m$, and centered on the axis of rotation of the rest of the Earth. Treat the rest of the Earth as a uniform sphere of mass and radius given in the table of constants. An approximate representation of an Arctic polar cap of these dimensions is shown as the white disk at the top of the Earth in the figure below. In our model, there is also a second, Antarctic polar cap that is not visible because of the curve of the Earth, but assume it has the same mass as the Arctic ice cap. After melting assume the water is evenly spread around all the Earth (ignore the continents). The additional depth is small compared to the radius of the Earth.

**Physics 131-1 Final Exam Equations and Constants**

$$\Delta\vec{r} = \vec{r}_{finish} - \vec{r}_{start} \quad \langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} \quad \vec{v} = \frac{d\vec{r}}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$$

$$\Delta\vec{v} = \vec{v}_{finish} - \vec{v}_{start} \quad \langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t} \quad \vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\vec{v}(t+\Delta t)-\vec{v}(t)}{\Delta t}$$

$$x(t) = \frac{1}{2}at^2 + v_0t + y_0 \quad v = at + v_0 \quad a = -g \quad a_c = \frac{v^2}{r} \quad (\vec{v}_c \perp \vec{r}_c \quad \vec{v}_c \perp \vec{a}_c)$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \quad \vec{A}+\vec{B} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} \quad \vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_xB_x + A_yB_y$$

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a} = \frac{d\vec{p}}{dt} \quad \vec{F}_{AB} = -\vec{F}_{BA} \quad \vec{p} = \sum m_i\vec{v}_i \quad \vec{p}_i = \vec{p}_f$$

$$|\vec{F}_k| = \mu_k N \quad |\vec{F}_s| \le \mu_s N \quad |\vec{F}_c| = m\frac{v^2}{r} \quad |\vec{F}_G| = \frac{Gm_1m_2}{r^2} \quad \vec{F}_s(x) = -kx\hat{i} \quad \vec{F}_g(y) = -mg\hat{j}$$

$$W = \int \vec{F}\cdot d\vec{s} = \int |\vec{F}|\ |d\vec{s}|\ \cos\theta = \Delta KE = -\Delta U \quad KE = \frac{1}{2}mv^2 \quad KE_i = KE_f\ (elastic)$$

$$KE_i + U_i = KE_f + U_f \quad KE = KE_{cm} + KE_{rot} \quad KE_{rot} = \frac{1}{2}I\omega^2 \quad U_s(x) = \frac{1}{2}kx^2 \quad U_g(y) = mgy$$

$$d_{Roche} = \left(\frac{12M}{\pi\rho}\right)^{1/3} \quad \rho = \frac{m}{V} \quad \vec{J} = \int_{t_1}^{t_2}\vec{F}dt = \Delta\vec{p}$$

$$\theta = \frac{s}{r} \quad \omega = \frac{v_\perp}{r} = \frac{d\theta}{dt} \quad \alpha = \frac{a_\perp}{r} = \frac{d\omega}{dt} \quad I = \sum m_ir_i^2 = I_{cm} + Md^2 \quad \vec{r}_{cm} = \frac{\sum m_i\vec{r}_i}{\sum m_i}$$

$$\vec{\tau} = rF\sin\phi\,\hat{\theta} = I\vec{\alpha} = \frac{d\vec{L}}{dt} \quad \vec{L} = \sum I_i\vec{\omega}_i \quad \vec{L}_i = \vec{L}_f \quad v_{cm} = r\omega$$

$$x(t) = A\cos(\omega t + \phi) \quad \omega^2 = \frac{k}{m} \quad T = \frac{2\pi}{\omega} = \frac{1}{f} \quad PE = \frac{1}{2}kx^2 \quad ME = \frac{1}{2}kA^2$$

$$\Delta t = \frac{\Delta t'_p}{\sqrt{1-\frac{v^2}{c^2}}} \quad L' = L_p\sqrt{1-\frac{v^2}{c^2}} \quad v'_i = \frac{v_i - v}{1-\frac{v'_iv}{c^2}} \quad v'_i = v_i - v \quad x' = x - vt \quad y' = y$$

$$\frac{dA}{dt} = 0 \quad \frac{dt}{dt} = 1 \quad \frac{dt^2}{dt} = 2t \quad \frac{d}{d\theta}\cos\theta = -\sin\theta \quad \frac{d}{d\theta}\sin\theta = \cos\theta \quad \frac{df(x)}{du} = \frac{df(x)}{dx}\frac{dx}{du}$$

$$\int f(x)dx = \lim_{\Delta x\to 0}\sum f(x_i)\Delta x \quad \int dx = x + c \quad \int xdx = \frac{x^2}{2} + c \quad \left(1-\frac{v^2}{c^2}\right)^{\pm 1/2} \approx 1 \mp \frac{1}{2}\frac{v^2}{c^2}$$

$$\sin\theta = \frac{opp}{hyp} \quad \cos\theta = \frac{adj}{hyp} \quad \tan\theta = \frac{opp}{adj} \quad \cos^2\theta + \sin^2\theta = 1 \quad x^2+y^2+z^2 = R^2 \quad V = \frac{4}{3}\pi r^3$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad C = 2\pi r \quad Area = \pi r^2 \quad Area = \frac{1}{2}bh \quad Area = 4\pi r^2 \quad V = \pi r^2\ l$$

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|---|---|---|---|
| Speed of Light ($c$) | $2.9979 \times 10^8\ m/s$ | proton/neutron mass | $1.67 \times 10^{-27}\ kg$ |
| $R$ | $8.31 J/K - mole$ | $g$ | $9.8\ m/s^2$ |
| Gravitation constant | $6.67 \times 10^{-11}\ N - m^2/kg^2$ | Earth radius | $6.37 \times 10^6\ m$ |
| Earth-Moon distance | $3.84 \times 10^8\ m$ | Earth mass | $5.9742 \times 10^{24}\ kg$ |
| Electron mass | $9.11 \times 10^{-31}\ kg$ | Moon mass | $7.3477 \times 10^{22}\ kg$ |
| 1 newton | $0.2248\ lbs - force$ | Moon radius | $1.74 \times 10^6\ m$ |
| Solar radius | $6.96 \times 10^8\ m$ | Solar mass | $1.99 \times 10^{30}\ kg$ |
| Earth-Sun distance | $1.50 \times 10^{11}\ m$ | 1 u | $1.661 \times 10^{-27}\ kg$ |

**Moments of Inertia**