Magnetic Effects in Matter

Consider the following, semi-classical model of an atom. An unpaired electron follows a circular orbit around the nucleus with a speed v (Figure 1). An external field $\vec{B}$ is applied perpendicular to the place of the electron's orbit (Figure 2).

1. What is the magnetic moment associated with the orbital motion before the external field is turned on in terms of e, R, and m_e?
2. What is it after the external field is turned on?
3. What is the magnetic susceptibility of metals in this model? Use the results in the table for comparison.

Material	Susceptibility
Bismuth	-16×10^{-5}
Gold	-3.4×10^{-5}
Silver	-2.4×10^{-5}
Copper	-0.97×10^{-5}
Hydrogen	-0.0002×10^{-5}

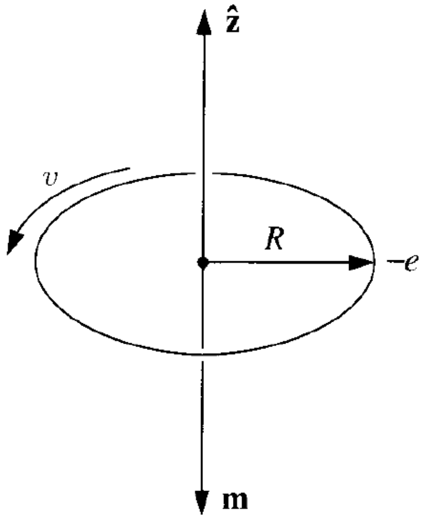


Figure 1.

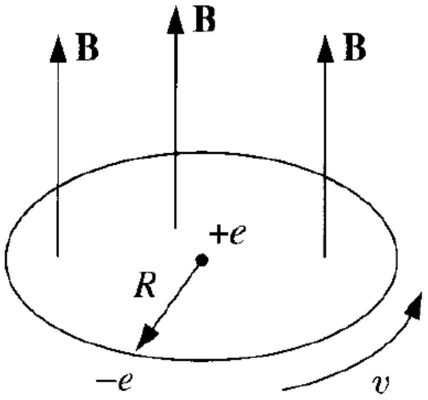


Figure 2.

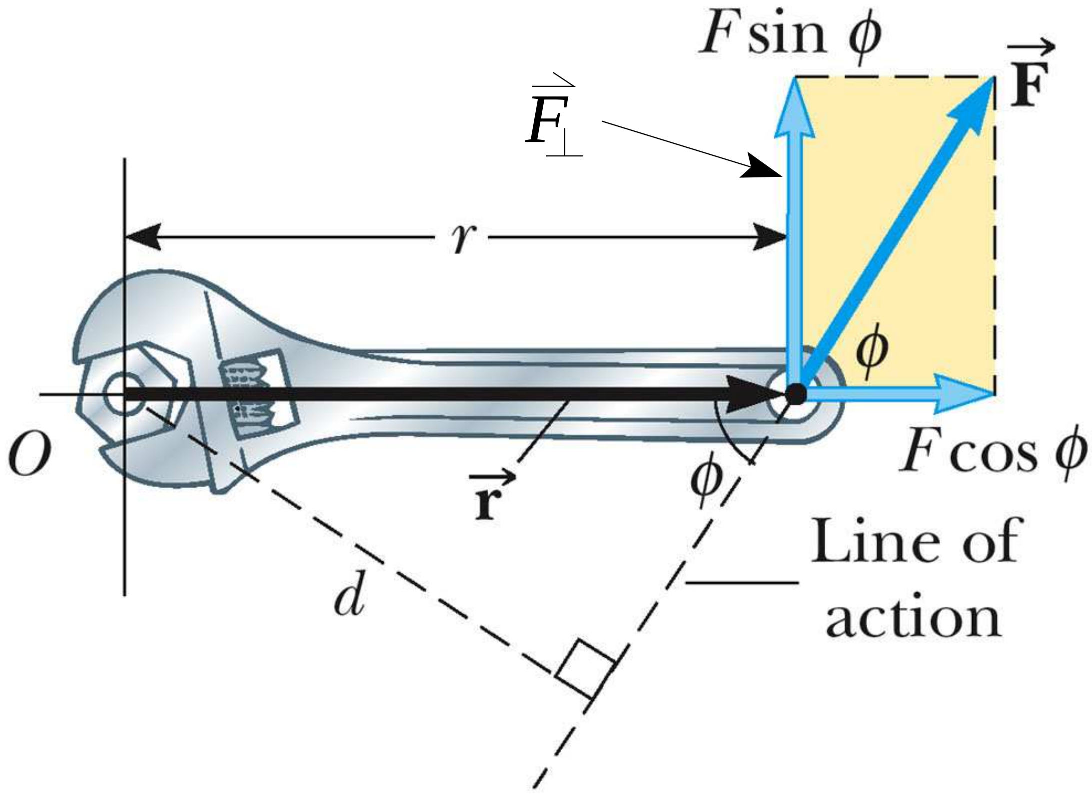
Linear → Rotational Quantities

Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v	$v = r\omega$	$\omega = \frac{v}{r} = \frac{d\theta}{dt}$
a	$a = r\alpha$	$\alpha = \frac{a}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$dW = \vec{F} \cdot d\vec{s}$		$dW = \vec{\tau} \cdot d\vec{\theta}$
$\vec{F} = m\vec{a}$	$\vec{N} = \vec{\tau} = \vec{r} \times \vec{F} = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$

$$I = \int r^2 dm = \int r^2 \rho d\tau$$

Torque - Rotational Equivalent of Force

$$\vec{F} = m\vec{a} \longrightarrow \vec{\tau} = \vec{r} \times \vec{F} = r\vec{F}_{\perp}$$



Moments of Inertia

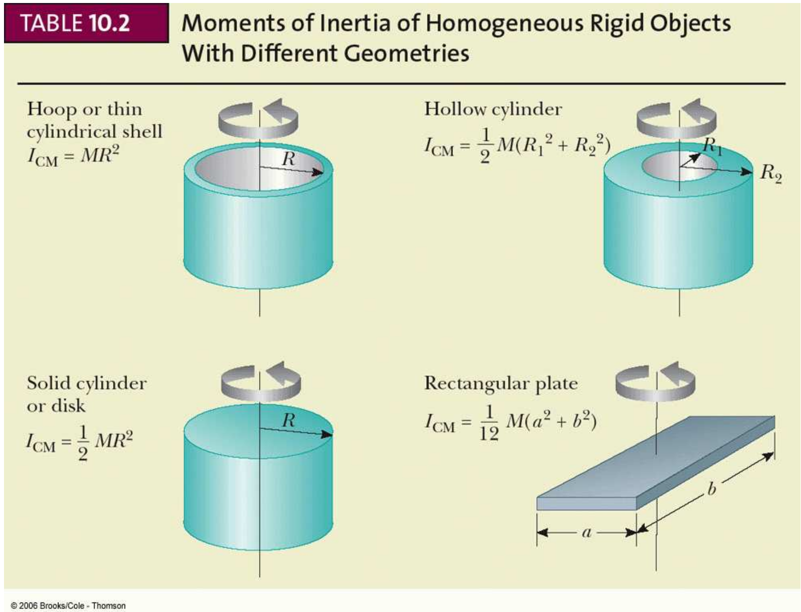


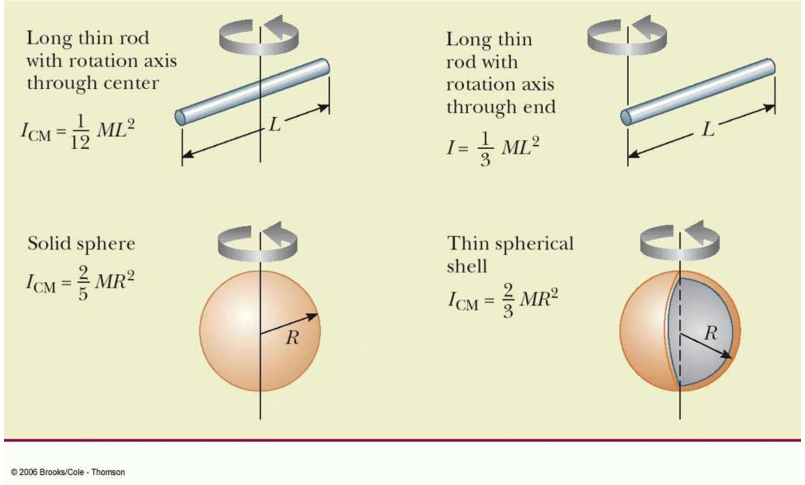
TABLE 10.2 Moments of Inertia of Homogeneous Rigid Objects With Different Geometries

Hoop or thin cylindrical shell
$I_{CM} = MR^2$

Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$

Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$

Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



Long thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$

Long thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$

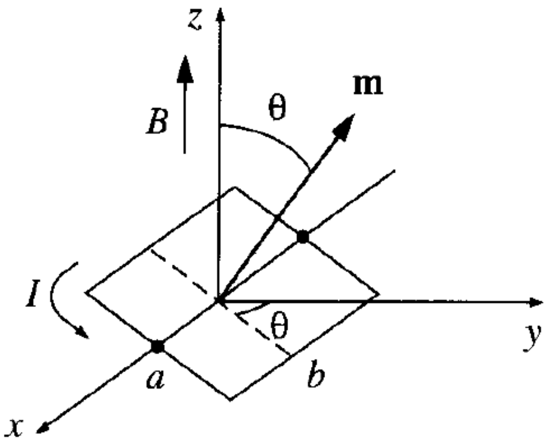
Solid sphere
$I_{CM} = \frac{2}{5}MR^2$

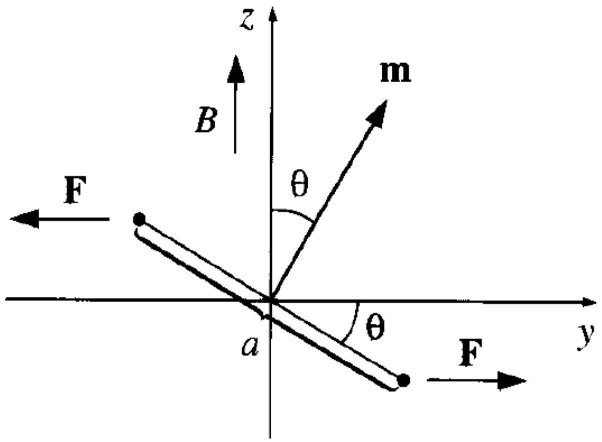
Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$

Torque on a Rectangular Current Loop

The rectangular current loop shown below is immersed in a uniform magnetic field $B = B_0\hat{z}$ with current I flowing through it in the direction shown. The loop has width a and length b.

1. What is the force on each straight section of the loop?
2. What is the torque exerted on each section of the loop?
3. What is the net torque?





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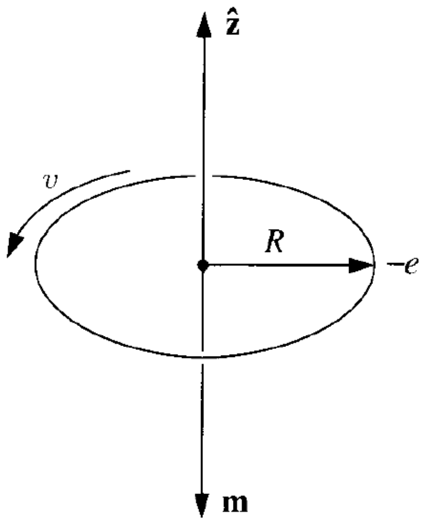

Figure 1.

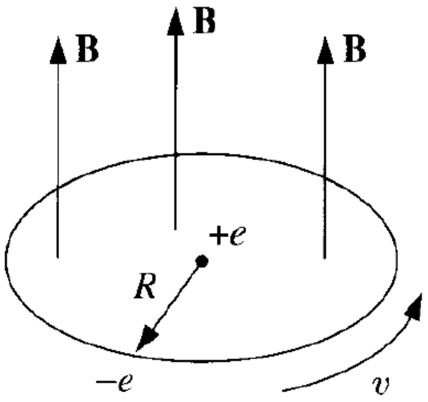

Figure 2.

Vector Identities from Griffith's Inside Cover

(2)
$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

(3)
$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

(4)
$$\nabla(fg) = f\nabla g + g\nabla f$$

(5)
$$\nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A}$$

(6)
$$\nabla \cdot (f\vec{A}) = f(\nabla \cdot \vec{A}) + (\vec{A} \cdot (\nabla f)$$

(7)
$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

(8)
$$\nabla \times (f\vec{A}) = f(\nabla \times \vec{A}) - \vec{A} \times (\nabla f)$$

(9)
$$\nabla \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} + \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A})$$

(10)
$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

(11)
$$\nabla \times (\nabla f) = 0$$

(12)
$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

Magnetic Effects in Matter

Material	χ_m (measured)	χ_m (calculated)
Bismuth	-16×10^{-5}	-4.3×10^{-5}
Gold	-3.4×10^{-5}	-4.6×10^{-5}
Silver	-2.4×10^{-5}	-4.6×10^{-5}
Copper	-0.97×10^{-5}	-5.2×10^{-5}
Hydrogen	-0.0002×10^{-5}	-8.5×10^{-5}

Magnetic Effects in Matter (and Electric)

Material	χ_m (measured)	χ_m (calculated)
Bismuth	-16×10^{-5}	-4.3×10^{-5}
Gold	-3.4×10^{-5}	-4.6×10^{-5}
Silver	-2.4×10^{-5}	-4.6×10^{-5}
Copper	-0.97×10^{-5}	-5.2×10^{-5}
Hydrogen	-0.0002×10^{-5}	-8.5×10^{-5}

Gas	$\chi_e^\dagger$ (measured)	χ_e (calculated)
Hydrogen	2.5×10^{-4}	2.3×10^{-4}
Helium	0.65×10^{-4}	0.71×10^{-4}
Neon	1.3×10^{-4}	1.4×10^{-4}
Argon	5.2×10^{-4}	5.6×10^{-4}

[†] For $1\ atm$, 20° C.