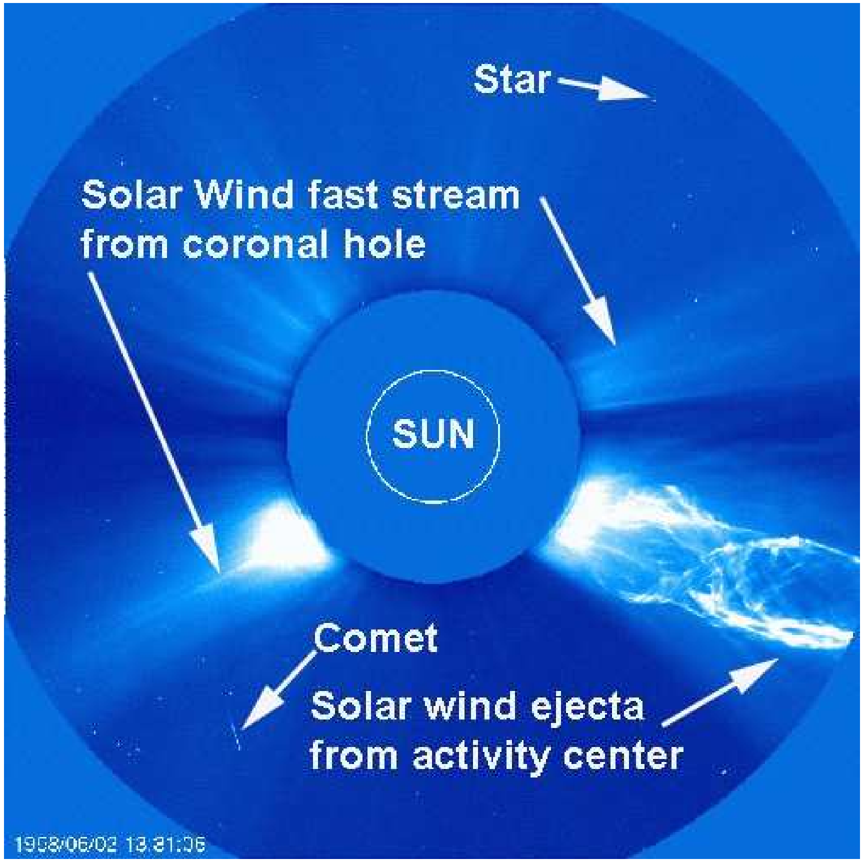
The Solar Wind

1. The solar wind is a stream of charged particles - a plasma - from the upper atmosphere of the sun consisting of electrons and protons with energies of $\approx 1\ keV$.
2. The particles escape the Sun's gravity because of the high temperature of the corona, and also through a process that is not well-understood.



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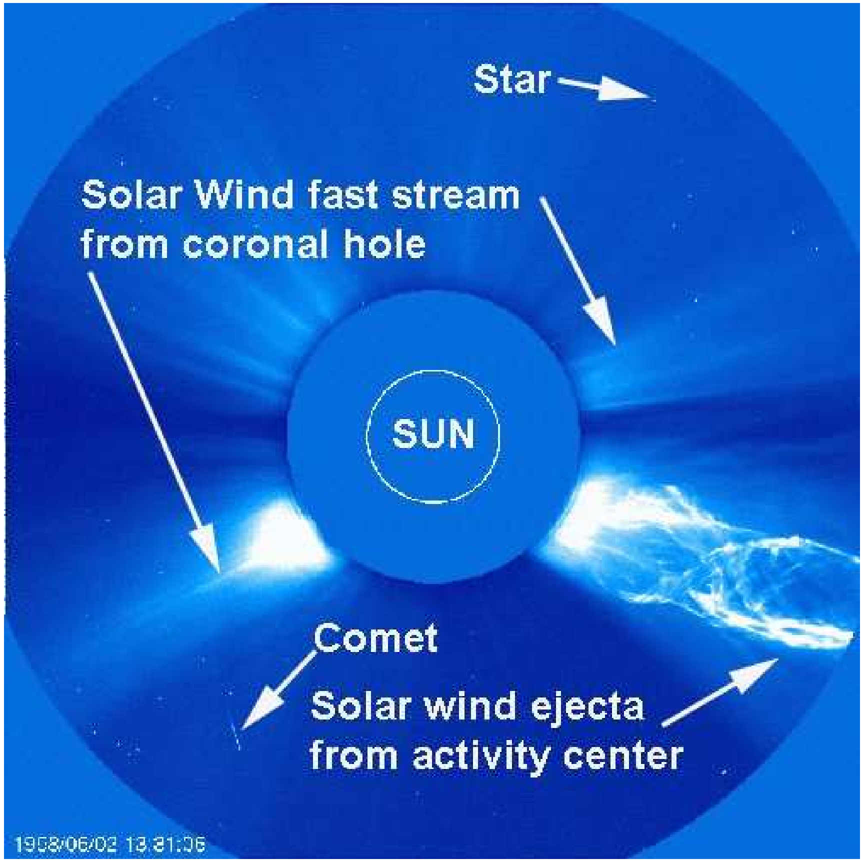

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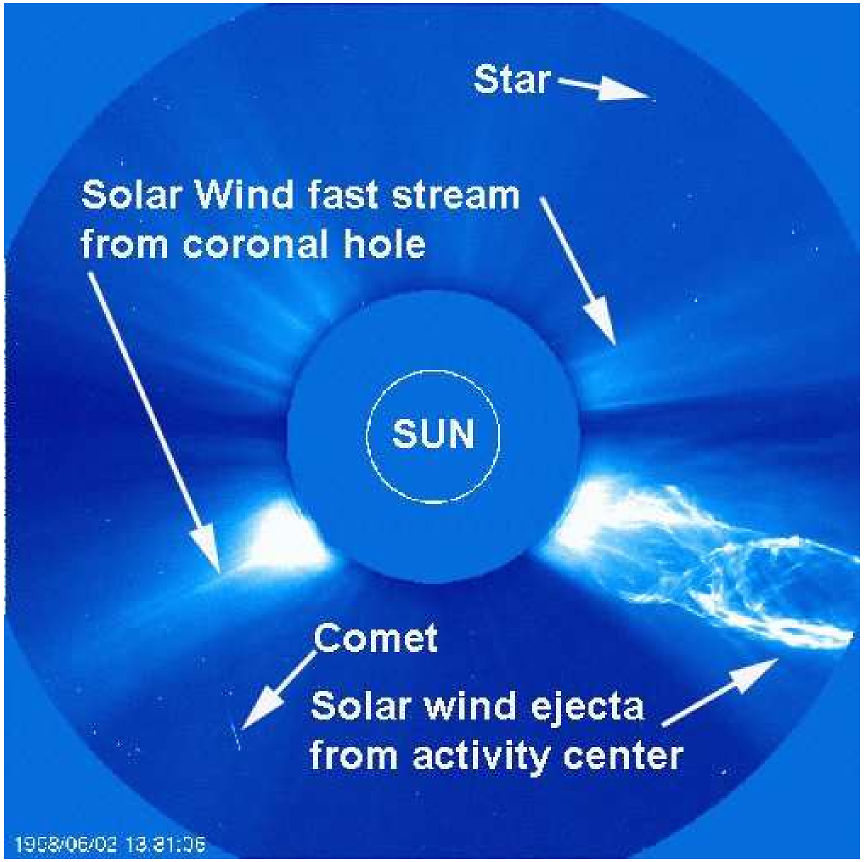
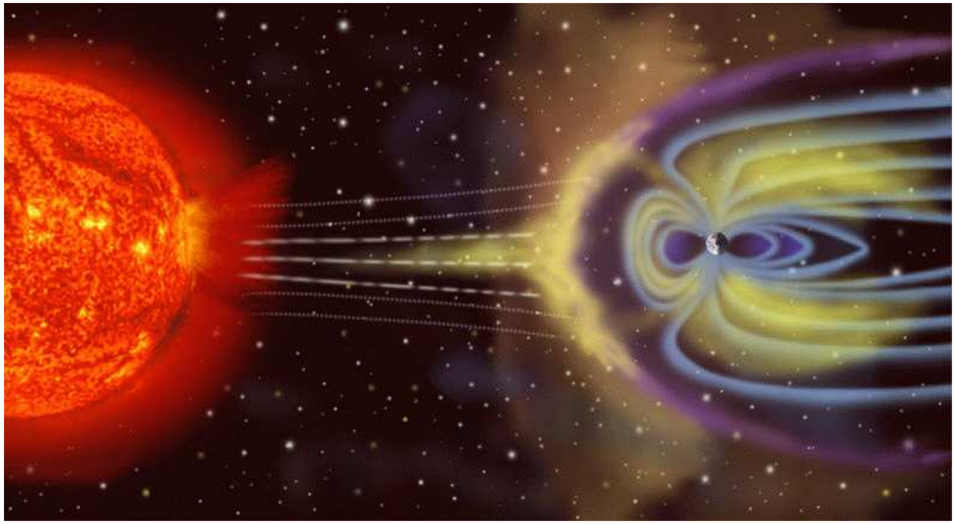


Why don't we all have cancer and cataracts?

The Magnetic Field of the Earth

The Earth's magnetic field deflects most of the solar wind particles away from the surface and effectively shields us from this ionizing radiation. Treat the Earth as a rotating sphere with a uniform distribution of charge.

1. What is the magnetic moment of the Earth in this model?
2. What is the vector potential $\vec{A}$ at Richmond, VA?
3. The magnetic field in Richmond has a magnitude of about $2 \times 10^{-9}\ T$. What is the current and charge required to produce this field?
4. Consider a proton in the solar wind with a kinetic energy $KE = 10^{-3}\ MeV$ moving towards the Earth along the x-axis. It reaches the edge of the Earth's magnetosphere at $R_p = 7 \times 10^7\ m$ and $\theta_p = 52.45°$ and $\phi_p = 10°$. At this point which force is greater on the proton, the electrical or magnetic force?

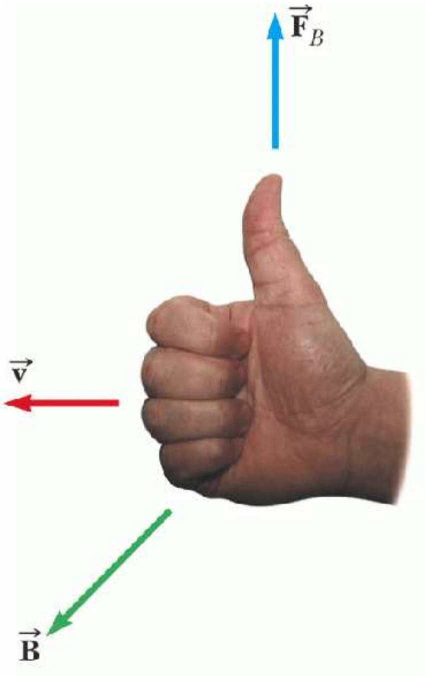


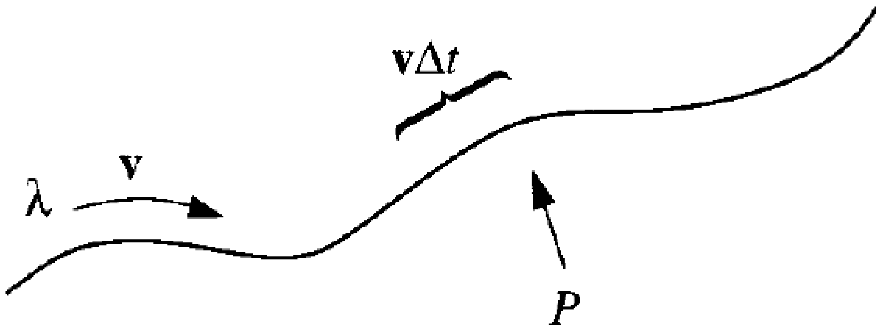
The Magnetic Force Law

Magnetic fields exert forces on moving charges (*i.e.* currents), but not on stationary charges. The force is called the Lorentz force and is

$$\vec{F}_{mag} = Q\vec{v} \times \vec{B} = \int I \left(d\vec{l} \times \vec{B} \right)$$

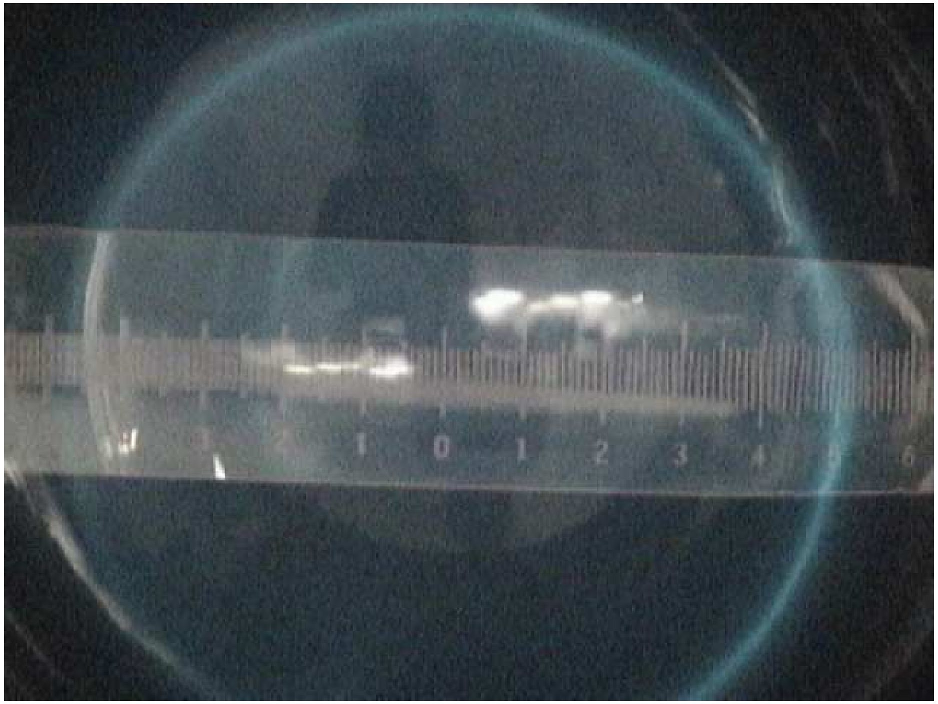
where Q is the charge, $\vec{v}$ is the velocity vector, $\vec{B}$ is the magnetic field, I is the electric current, and $d\vec{l}$ is an infinitesimally short section of electric current and points in the direction of the current.





The Magnetic Force Law - Evidence

The Pasco e/m experiment measures the electron's charge to mass ratio by bending a beam of electrons into a circle.



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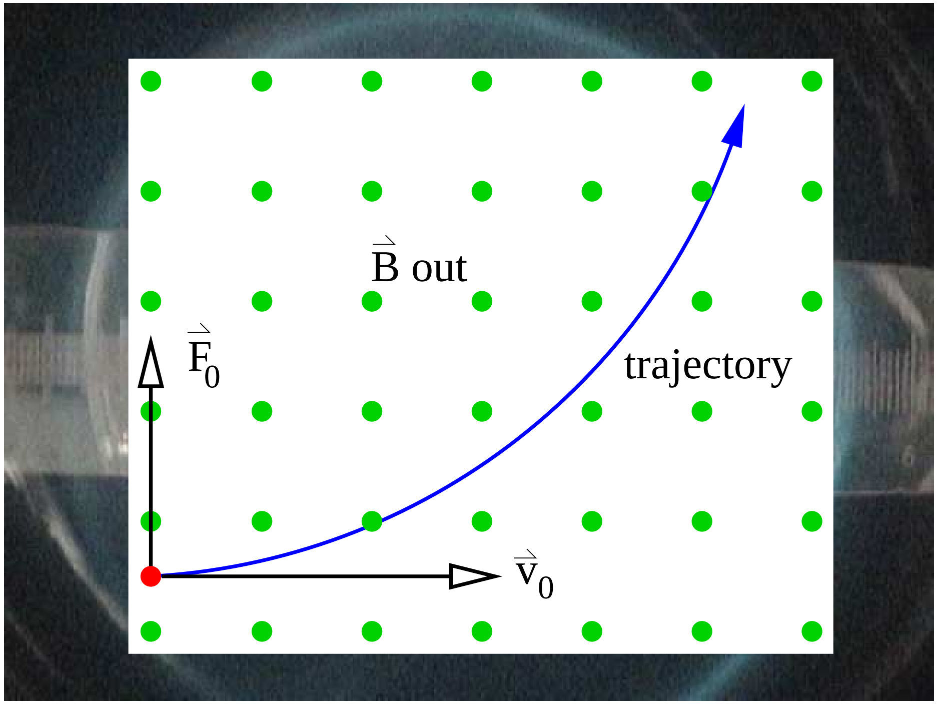
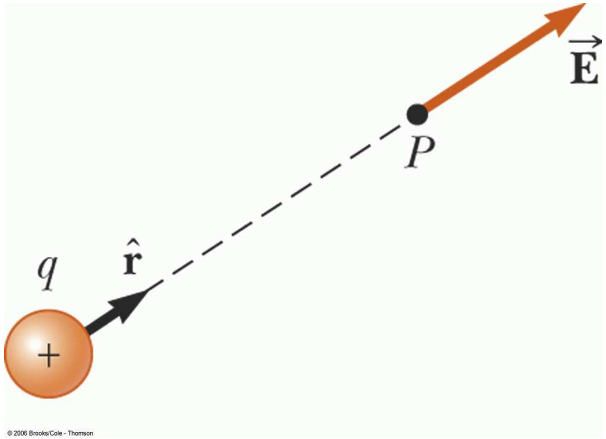


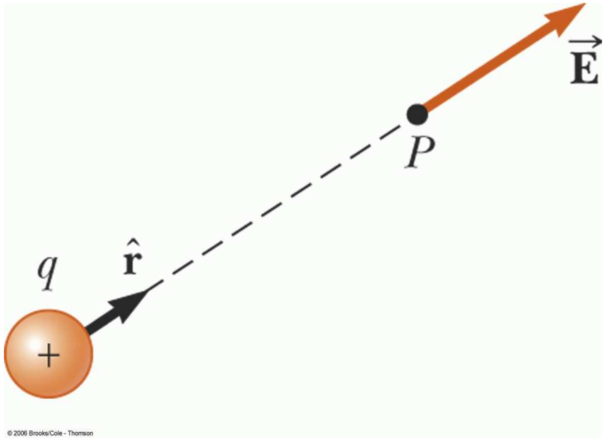
The Electric and Magnetic Fields

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq\,\hat{\boldsymbol{\imath}}}{\boldsymbol{\imath}^2}$$

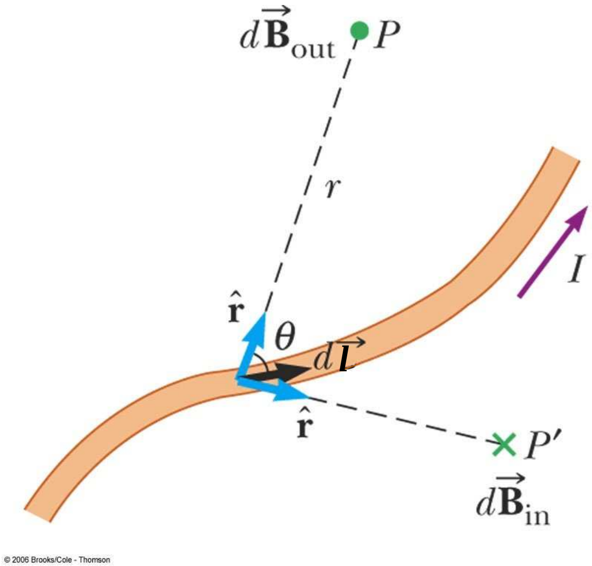


The Electric and Magnetic Fields

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq\,\hat{\mathscr{r}}}{\mathscr{r}^2}$$

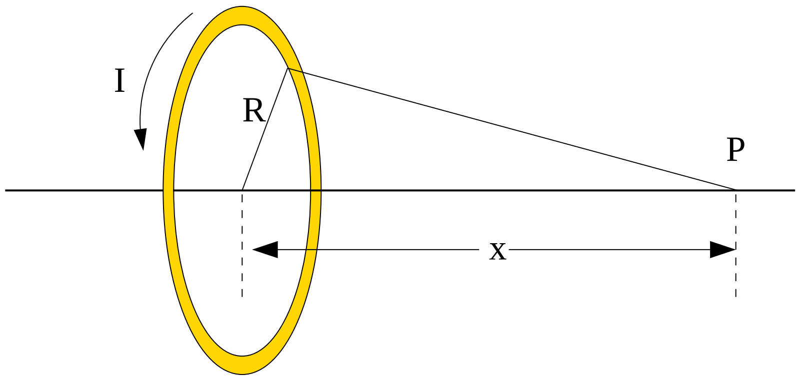


$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$



Evidence - The Magnetic Field of a Current Loop

Consider a circular loop of radius R in the $y - z$ plane and carrying a steady current I. What is the magnetic field at an axial point P a distance x from the center of the loop in terms of I, R, x, and any other constants?



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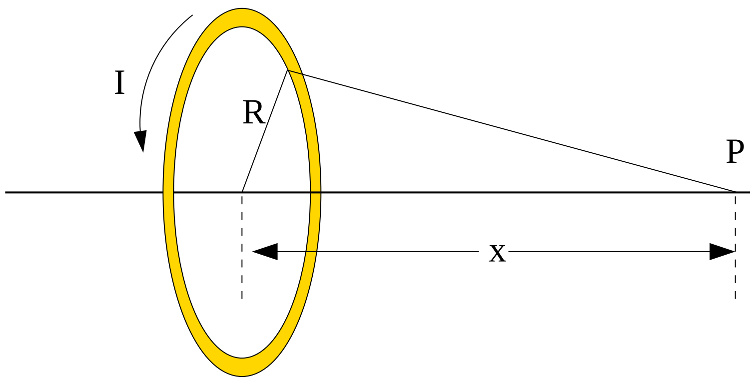

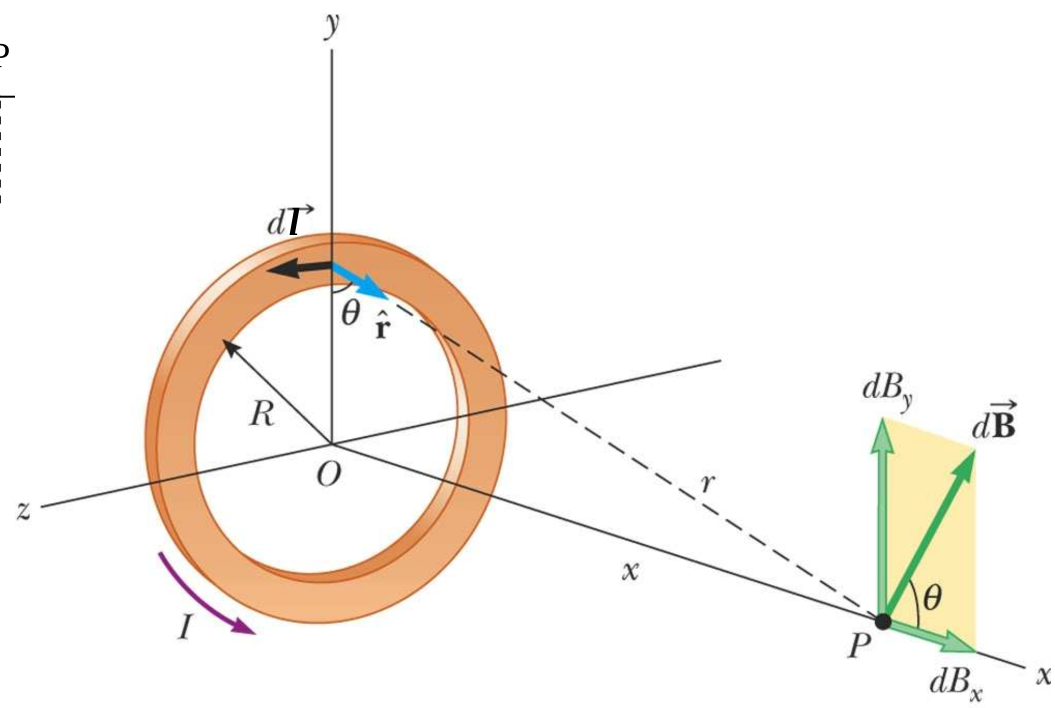

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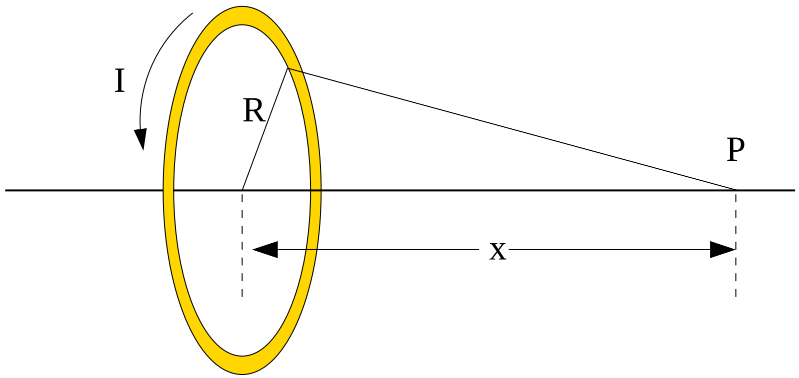
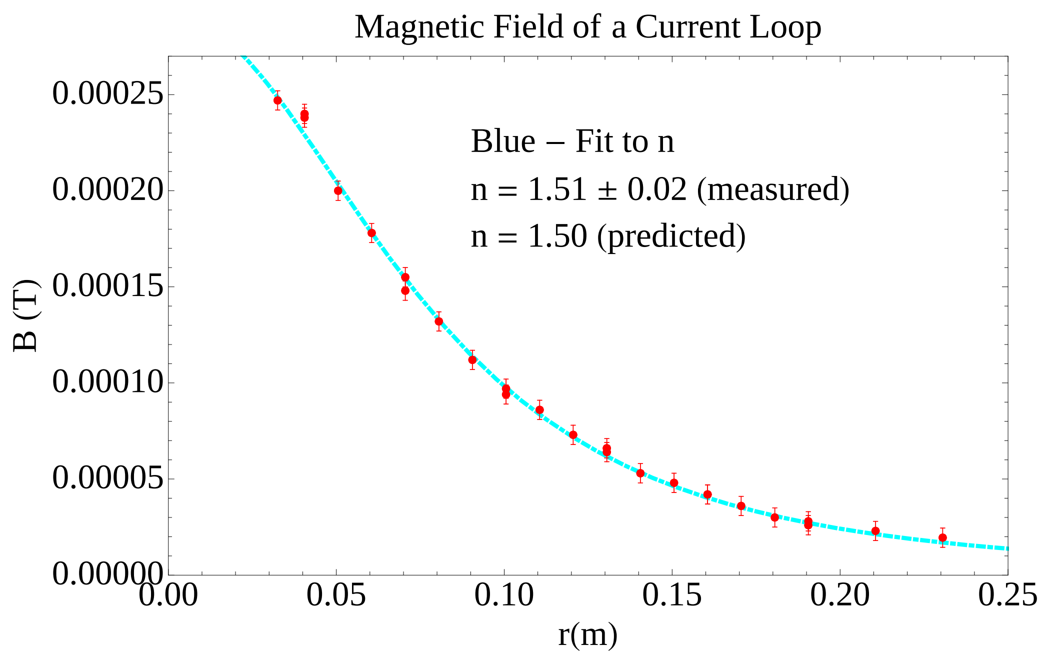
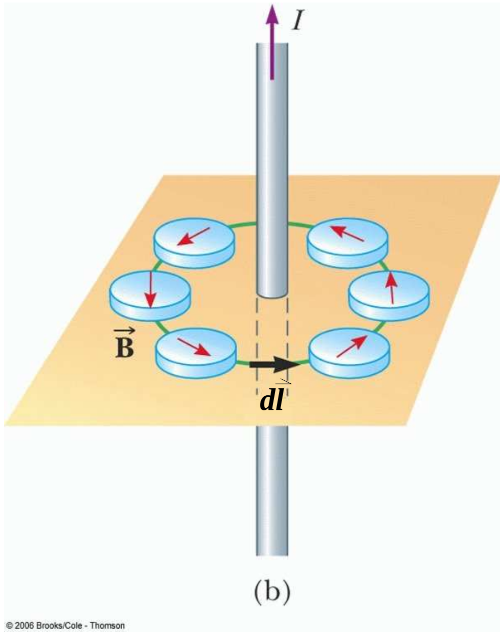




Ampere's Law

Consider a long, straight wire carrying a current I. The magnetic field lines form rings centered on the wire as shown below.



What is the magnetic field at a distance s from a long, straight wire? What is the integral of $\vec{B}$ around a complete, circular path centered on the wire? In other words calculate the following integral.

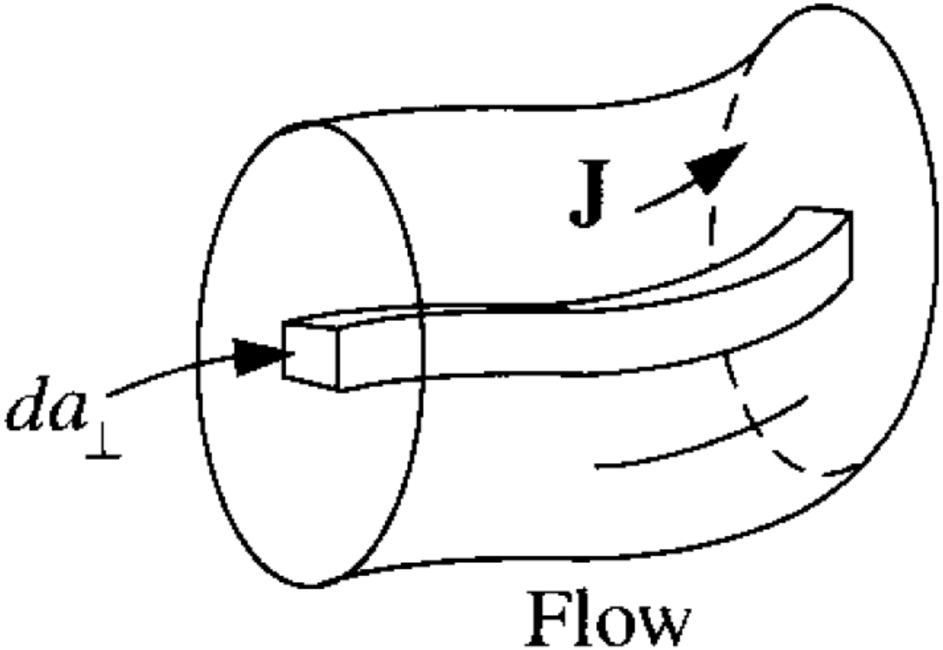
$$\oint \vec{B} \cdot d\vec{l}$$

The Current Density

The current density $\vec{J}$ is defined as

$$\vec{J} \equiv \frac{d\vec{I}}{da_{\perp}}$$

where $\vec{I}$ is the current and $da_{\perp}$ is the cross sectional area perpendicular to the current flow.

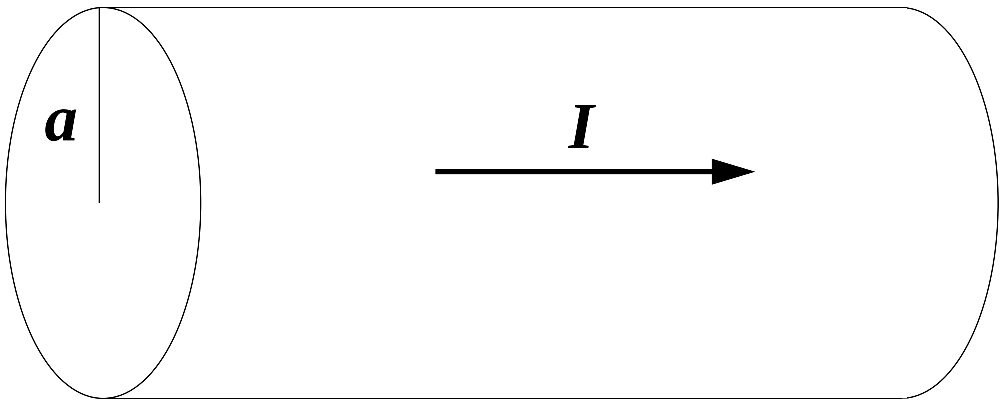


Ampere's Law - an Example

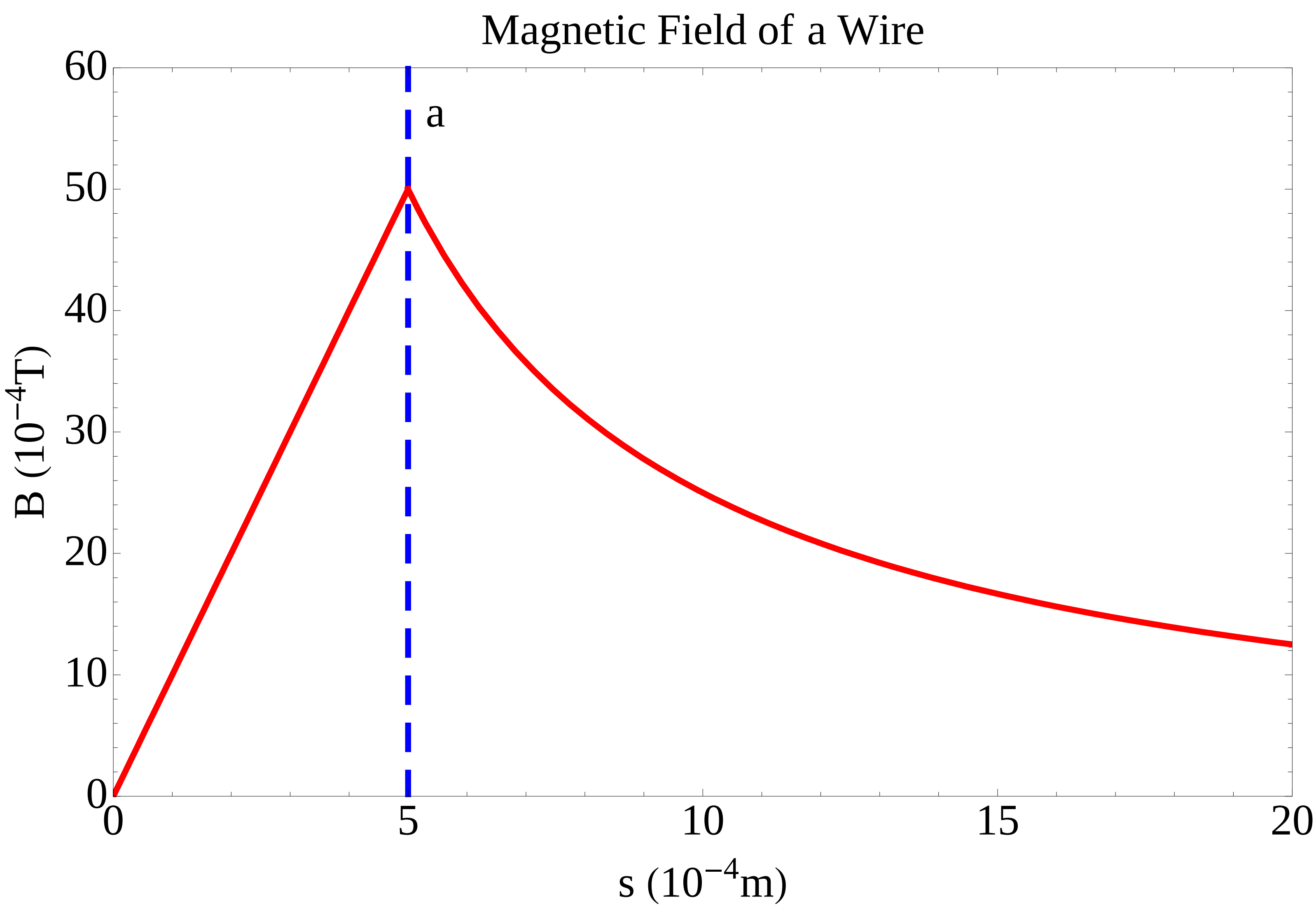
Consider a long, straight cylindrical conductor (a wire) carrying a current uniformly distributed over its cross section with a current density

$$J = \frac{I}{\pi a^2}$$

where a is the radius of the wire and I is the current. What is the magnetic field inside and outside the wire?



Ampere's Law - The Results



Vector Potential $\vec{A}$

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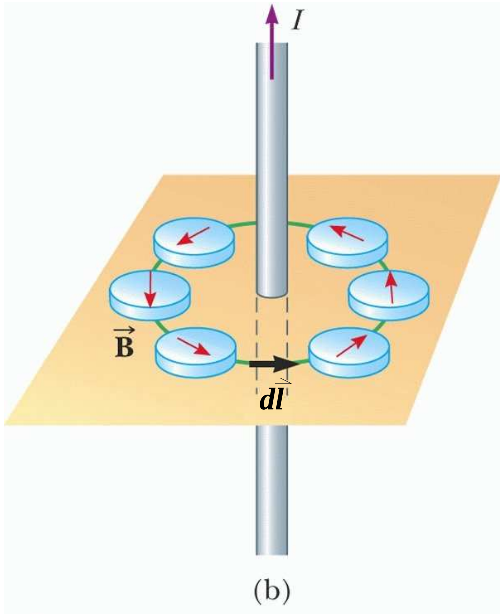

What is the vector potential at a distance s from a long, straight wire? Is your result consistent with our previous result for the magnetic field of the wire $\vec{B} = (\mu_0 I/2\pi s)\hat{\phi}$?

Vector Identities from Griffith's Inside Cover

(1) $$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

(2) $$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

(3) $$\nabla(fg) = f\nabla g + g\nabla f$$

(4) $$\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A}$$

(5) $$\nabla\cdot(f\vec{A}) = f(\nabla\cdot\vec{A}) + (\vec{A}\cdot(\nabla f)$$

(6) $$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$$

(7) $$\nabla\times(f\vec{A}) = f(\nabla\times\vec{A}) - \vec{A}\times(\nabla f)$$

(8) $$\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{A})$$

(9) $$\nabla\cdot(\nabla\times\vec{A}) = 0$$

(10) $$\nabla\times(\nabla f) = 0$$

(11) $$\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$$

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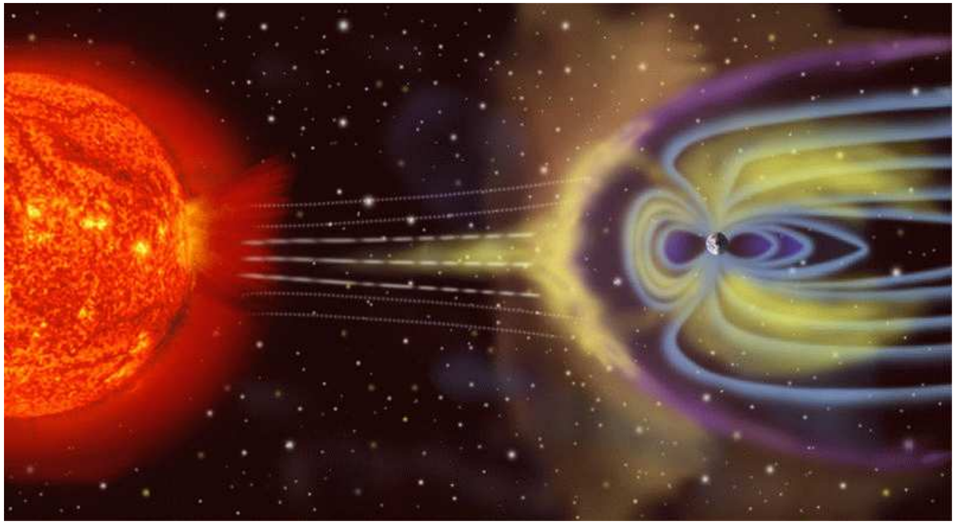

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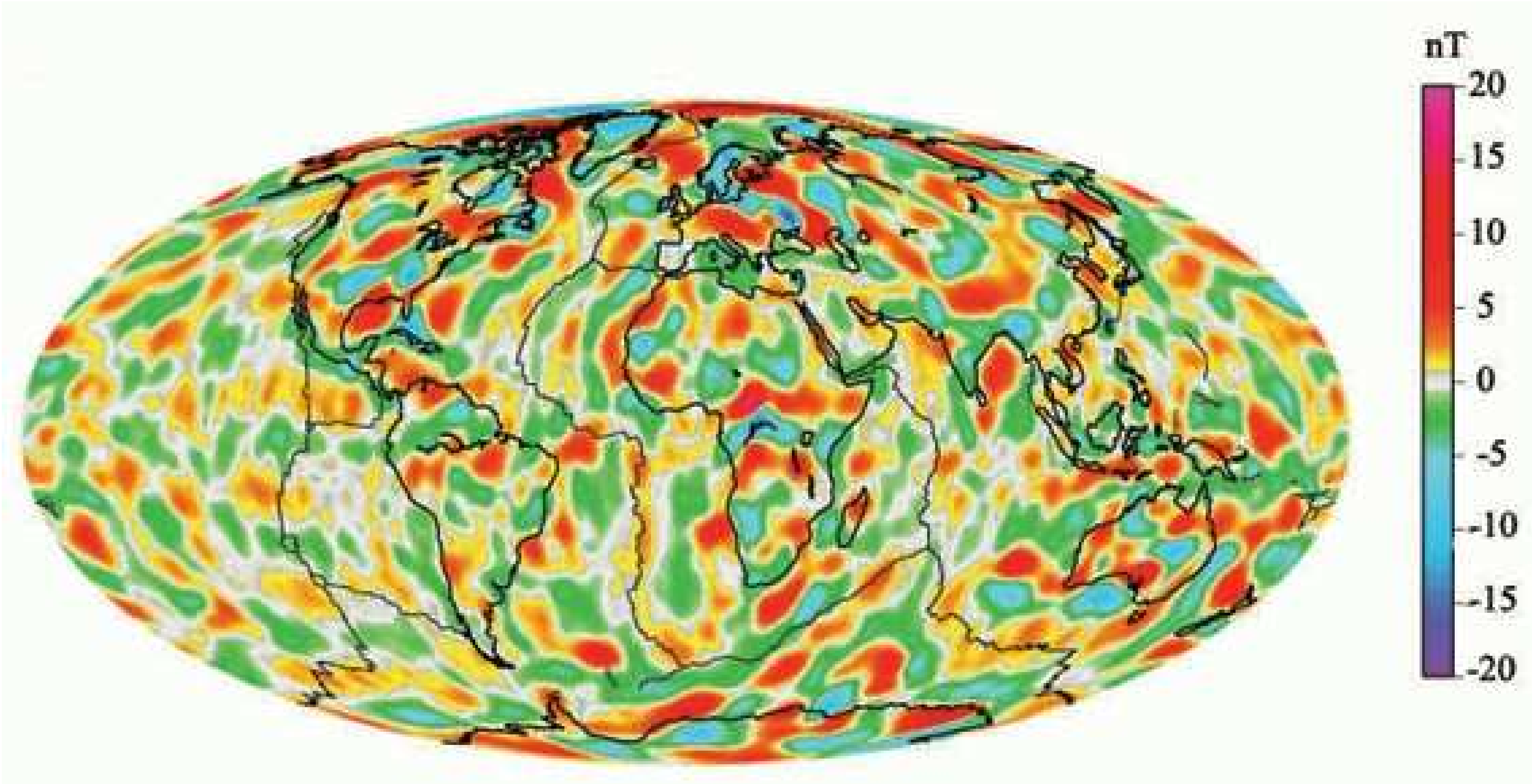

Source:

`http://www.nasa.gov/centers/goddard/news/topstory/2004/0517magnet.html`