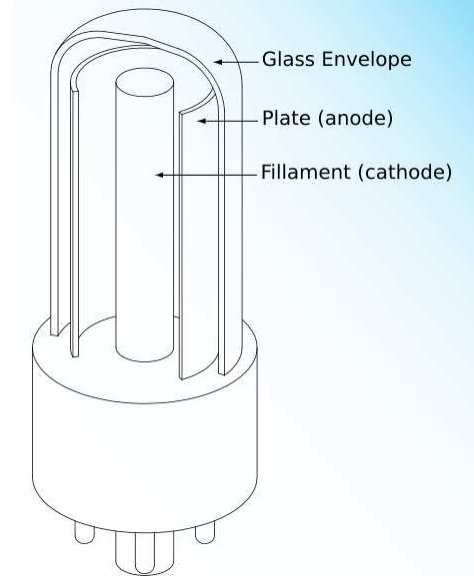


# How Electronics Started! And JLab Hits the Wall!

In electronics, a vacuum diode or tube is a device used to amplify, switch, otherwise modify, or create an electrical signal by controlling the movement of electrons in a low-pressure space. Almost all depend on the thermal emission of electrons. The figure shows the components of the device. A demonstration of how they work is [here](#).

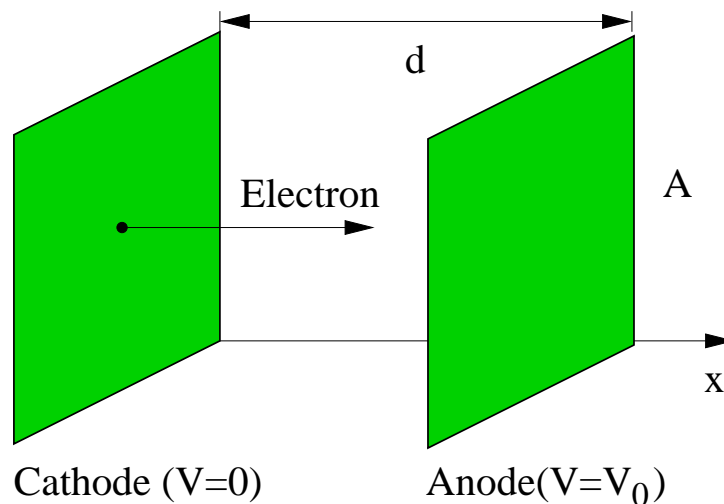
The physics here also determines the limits on the amount of beam that can be produced in an accelerator.



# The Child-Langmuir Law

In a vacuum diode, electrons are boiled off a hot cathode at zero potential and accelerated across a gap to the anode at a positive potential  $V_0$ . The cloud of moving electrons within the gap (called the space charge) builds up and reduces the field at the cathode surface to zero. From then on a steady current flows between the plates.

Suppose two plates are large relative to the separation ( $A \gg d^2$  in the figure) so that edge effects can be ignored and  $V$ ,  $\rho$ , and the electron speed  $v$  are functions of only  $x$ .

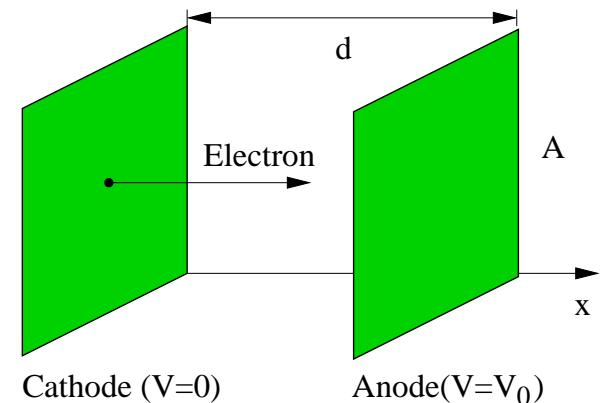


# The Child-Langmuir Law

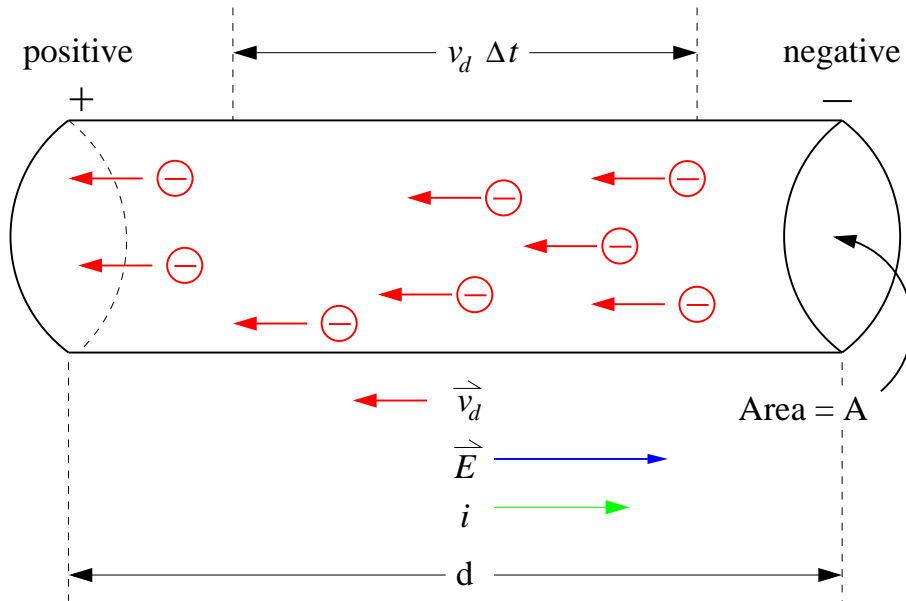
1. What is Poisson's equation for the region between the plates?
2. Assuming the electrons start from rest at the cathode, what is their speed at point  $x$ ?
3. In the steady state  $I$ , the current, is independent of  $x$ . How are  $\rho$  and  $v$  related?
4. Now generate a differential equation for  $V$  by eliminating  $\rho$  and  $v$  and solve this equation for  $V$  as a function of  $x$ ,  $V_0$ , and  $d$ . Make a plot to compare  $V(x)$  and the potential without the space charge.
5. What are  $\rho$  and  $v$  as functions of  $x$ ?
6. Show that

$$I = KV_0^{3/2}$$

and find  $K$ . This is the Child-Langmuir law.



# Why Should You Care? The Physics 132 Picture

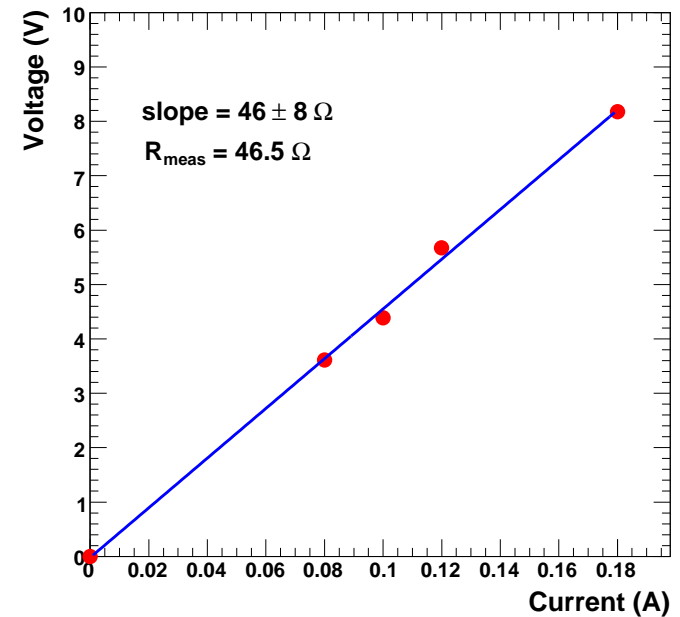


$$I = JA = nqv_d A = \frac{A}{\rho d} V$$

$$I = \frac{1}{R} V$$

$$V = IR$$

Ohm's Law



2007-10-16 11:35:02

# Why Should You Care, Part Deux?

The image is a screenshot of the MSNBC website. At the top left is the MSNBC logo. To its right is a search bar and the text "MSN Home". Below the logo, there are navigation links for "Today Show", "Nightly News", "Dateline", and "Meet the Sports". A red banner across the top contains the text "BREAKING NEWS: Bush says he finds Obama to be 'very smart and engaging'". Below this is a breadcrumb trail: "Technology & science / Space". On the left side, there is a "Categories" menu with links for "U.S. news", "World news", "Politics", "Business", "Sports", "Entertainment", "Tech & science", "Space", "Science", "Tech and gadgets", "Games", "Wireless", "Security", "Innovation", and "Health". The main content area features an article titled "Beam weapons almost ready for battle" with the sub-headline "Directed energy could revolutionize warfare, expert says". The article includes an artistic rendering of a satellite in space firing a red laser beam towards Earth. To the right of the image is an "INTERACTIVE" section with a "Launch" button and the text "Combat in the cosmos: The militarization of space". At the bottom of the image, the date "January 11, 2006" is displayed.

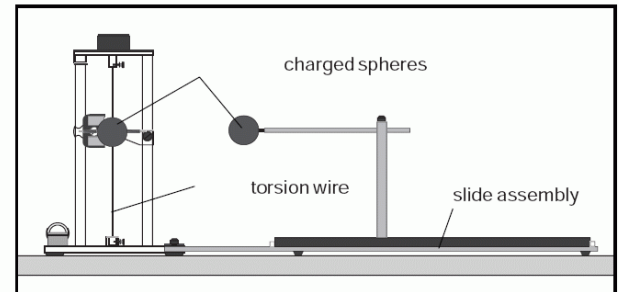
January 11, 2006

# Coulomb's Law and Superposition

---

- Measuring the Electrostatic Force of Charges.

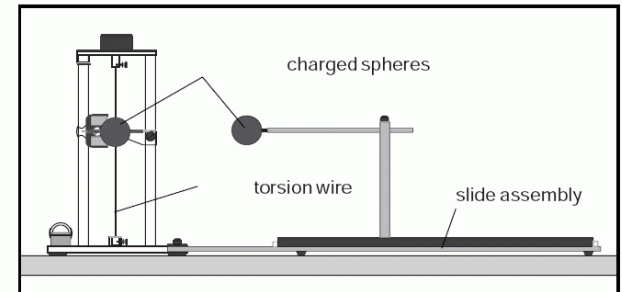
Use a torsion pendulum and charged spheres.



# Coulomb's Law and Superposition

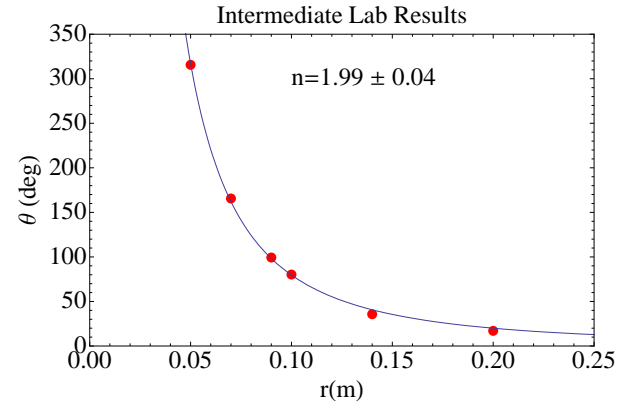
## ● Measuring the Electrostatic Force of Charges.

Use a torsion pendulum and charged spheres.



## ● Evidence for Coulomb's Law.

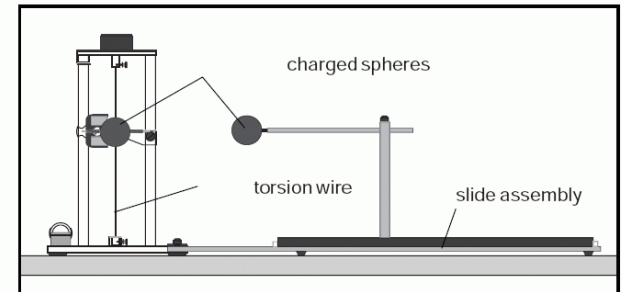
Fix charge and vary distance.



# Coulomb's Law and Superposition

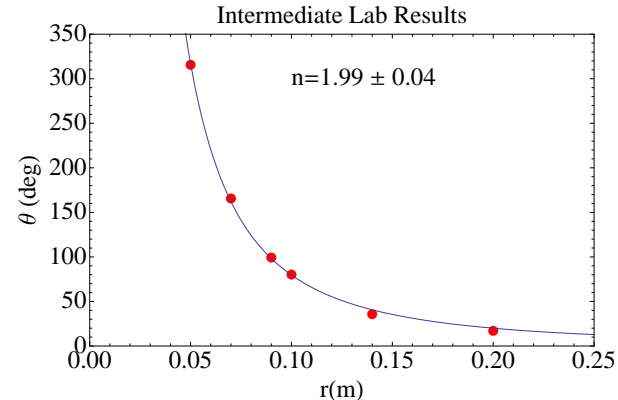
## Measuring the Electrostatic Force of Charges.

Use a torsion pendulum and charged spheres.



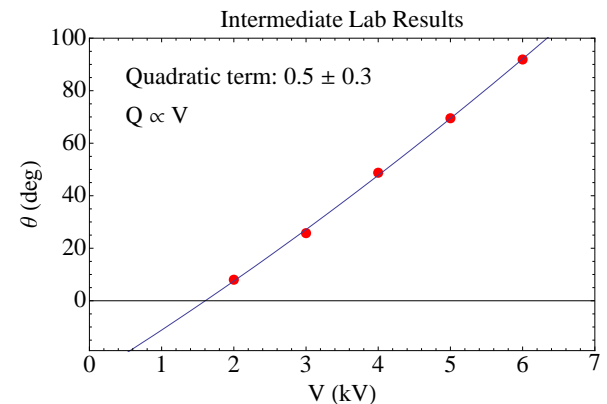
## Evidence for Coulomb's Law.

Fix charge and vary distance.



## Evidence for Superposition.

Fix distance and vary charge on one sphere.

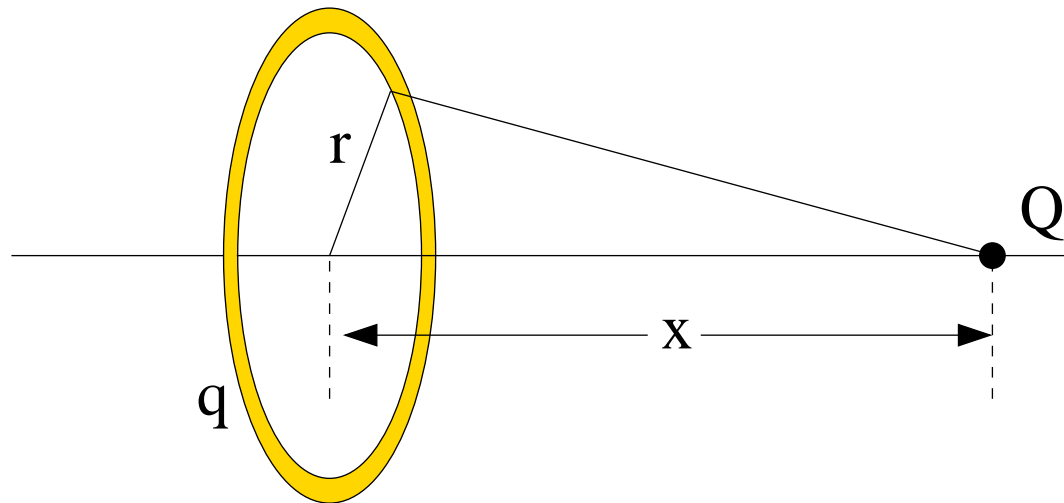




# Electric Field of a Ring

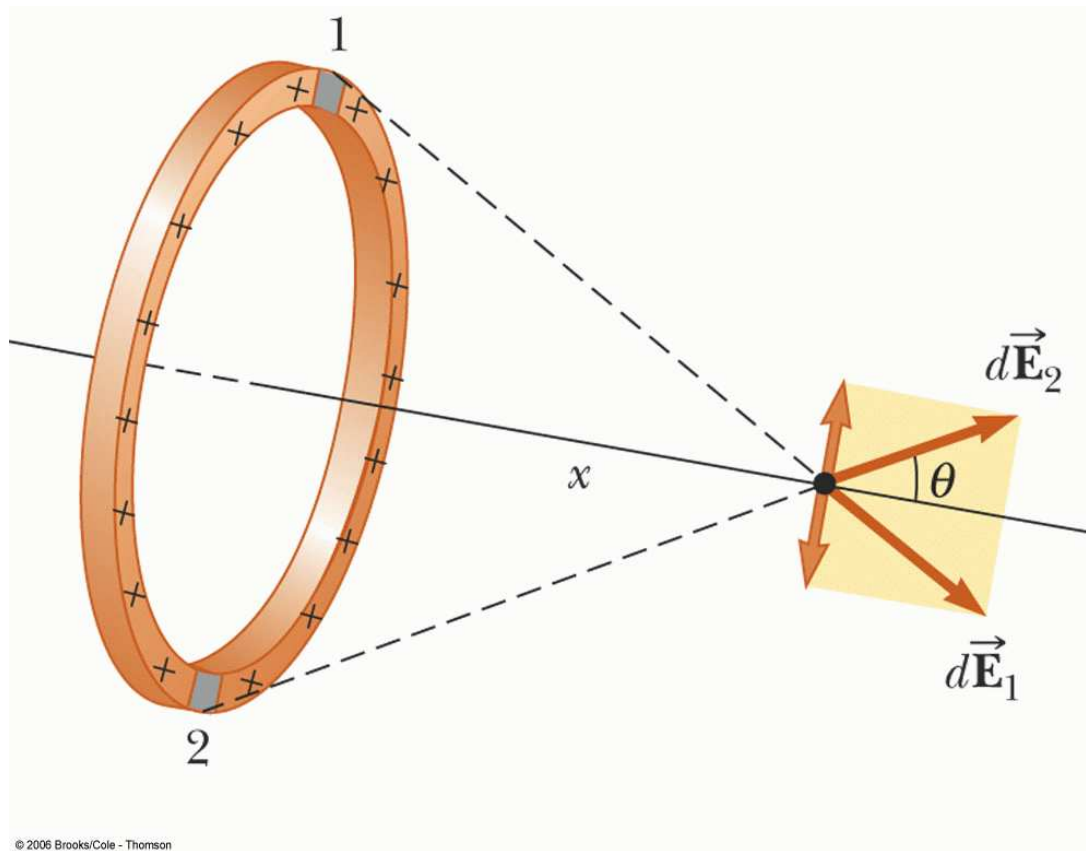
---

A ring of radius  $r$  as shown in the figure has a positive charge distribution per unit length with total charge  $q$ . Calculate the electric field  $\vec{E}$  along the axis of the ring at a point lying a distance  $x$  from the center of the ring. Get your answer in terms of  $r$ ,  $x$ ,  $q$ . What happens as  $x \rightarrow \infty$ ?



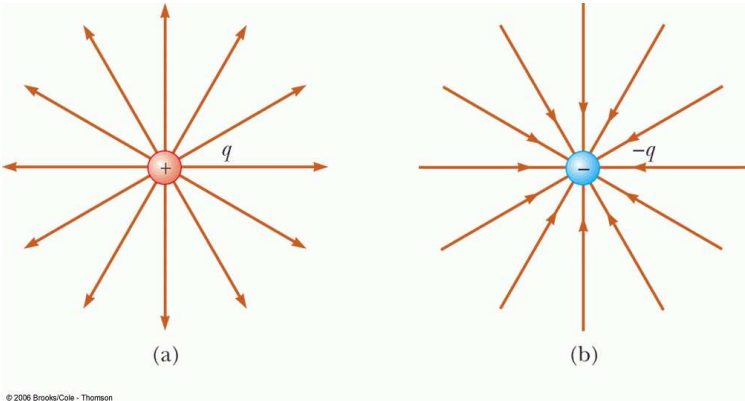
# Electric Field of a Ring

A ring of radius  $r$  as shown in the figure has a positive charge distribution per unit length with total charge  $q$ . Calculate the electric field  $\vec{E}$  along the axis of the ring at a point lying a distance  $x$  from the center of the ring. Get your answer in terms of  $r$ ,  $x$ ,  $q$ . What happens as  $x \rightarrow \infty$ ?

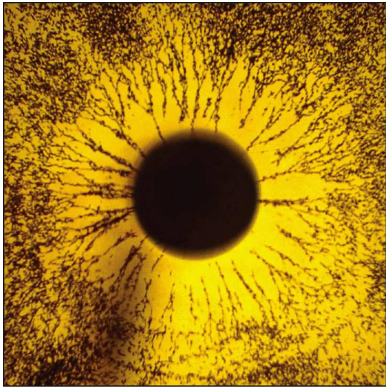


# Electric Field Lines

Point Charges

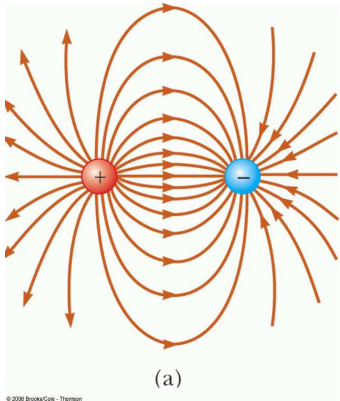


© 2006 Brooks/Cole - Thomson

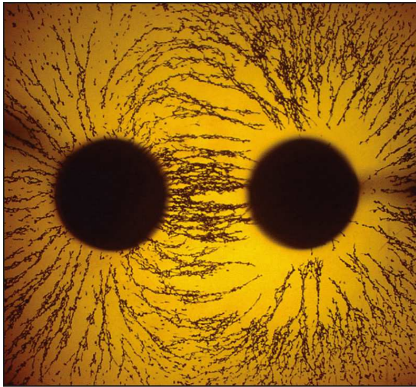


© 2006 Brooks/Cole - Thomson

Electric Dipole

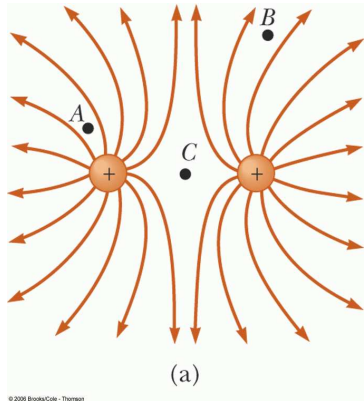


© 2006 Brooks/Cole - Thomson

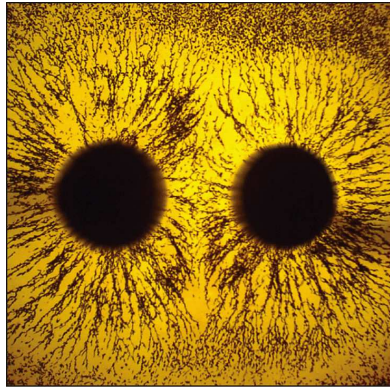


© 2006 Brooks/Cole - Thomson

Positive Charges



© 2006 Brooks/Cole - Thomson



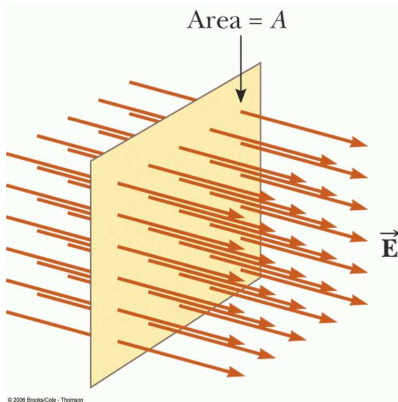
© 2006 Brooks/Cole - Thomson

# Properties of Electric Field Lines

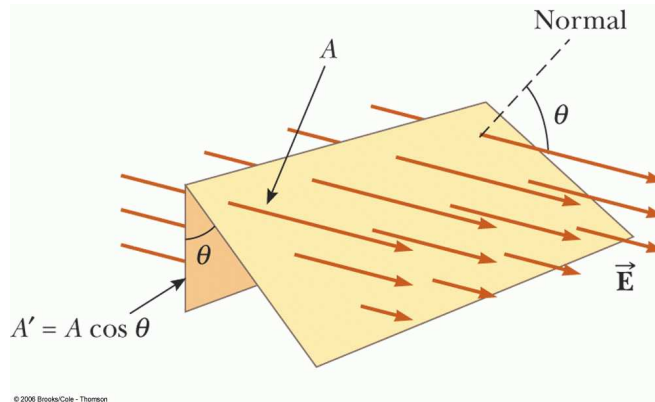
---

- Field lines start from positive charges (sources) and end at negative ones (sinks).
- Are symmetrical around point charges.
- Density of field lines is related to strength of field.
- Direction of field is tangent to the field line.
- Field lines never cross!

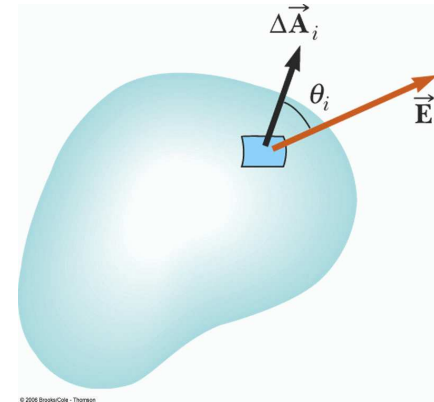
# Electric Flux



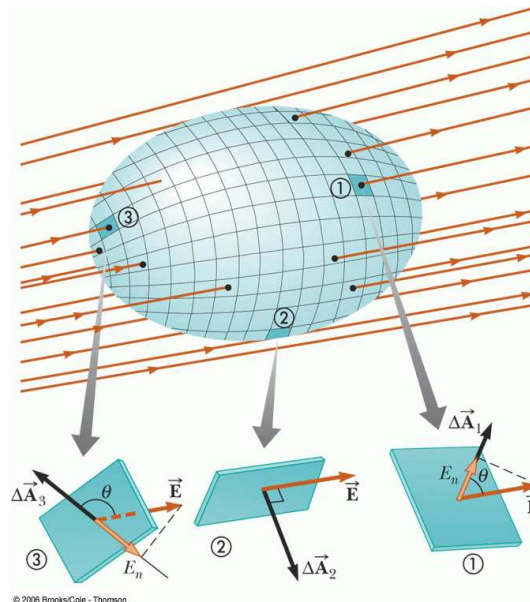
$$\Phi_E = EA$$



$$\Phi_E = EA \cos \theta = \vec{E} \cdot \vec{A}$$



$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

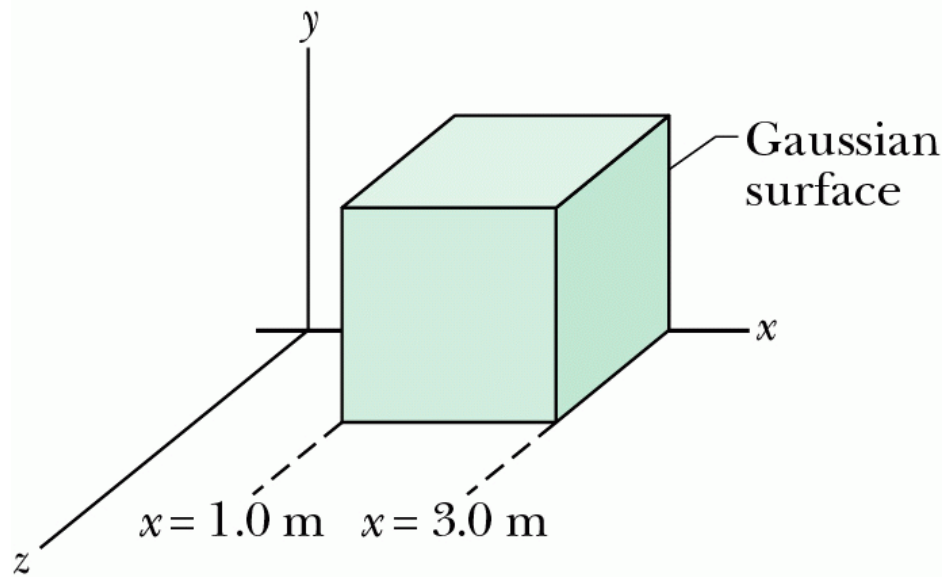


# An Example of Electric Flux

A nonuniform magnetic field is described by

$$\vec{E} = (3.0 \text{ N/C} - m)x\hat{x} + (4.0 \text{ N/C})\hat{y}$$

pierces the Gaussian cube shown in the figure. What is the flux through the cube? Note the orientation of the coordinate system.



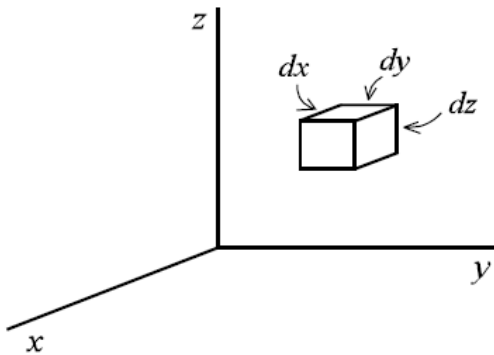
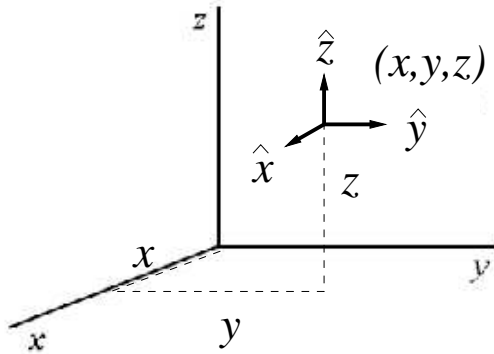
# Gauss's Law

---

What is the flux from a point charge  $q$  at the origin through a sphere centered at the origin of radius  $r$ ?

# Differential Surface and Volume Elements

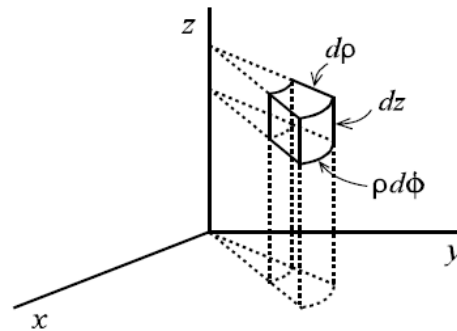
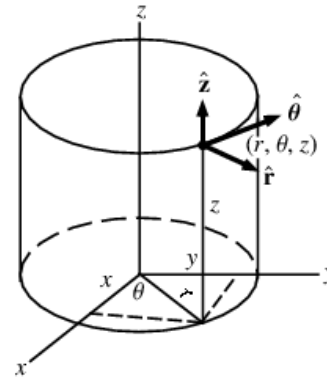
Cartesian Coordinates



$$dA = dx dy$$

$$d\tau = dx dy dz$$

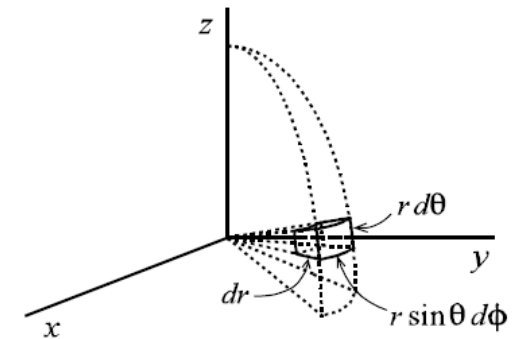
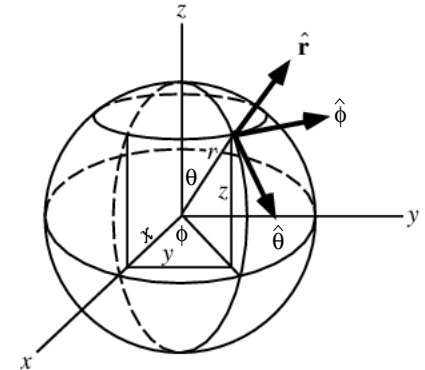
Cylindrical Coordinates



$$dA = \rho d\phi dz$$

$$d\tau = \rho d\phi dz d\rho$$

Spherical Coordinates



$$dA = r^2 d\cos\theta d\phi$$

$$d\tau = r^2 d\cos\theta d\phi dr$$



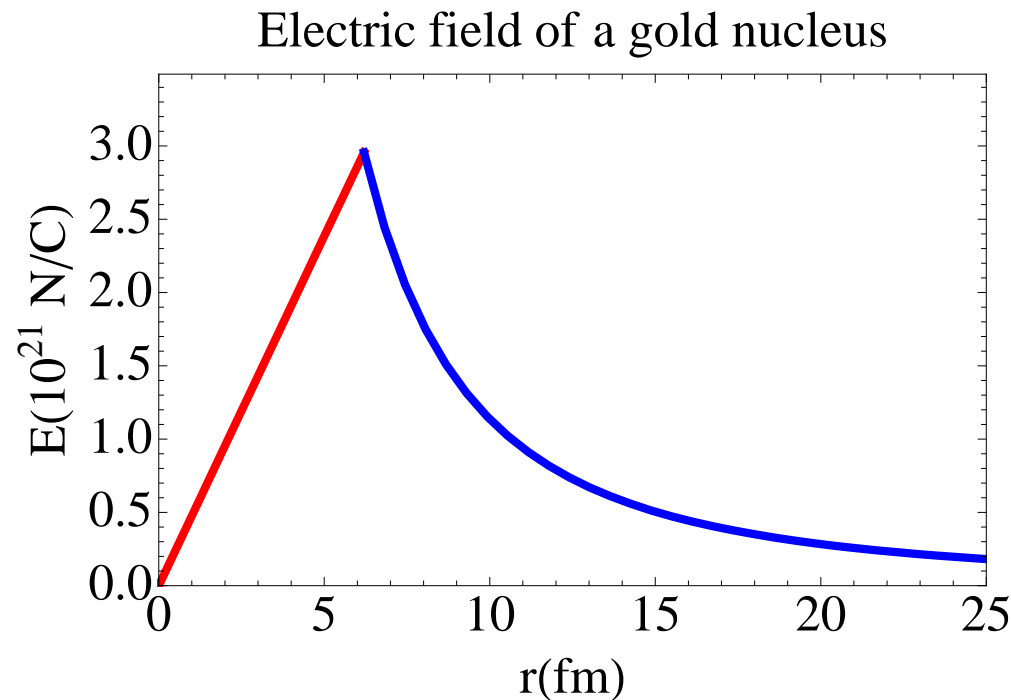
# Applying Gauss's Law: Nuclear $\vec{E}$

---

The nucleus of a gold atom has a radius  $R = 6.2 \times 10^{-15} \text{ m}$  and a positive charge  $q = Ze$  where  $Z = 79$  is the atomic number. Assume the gold nucleus is spherical and the charge is uniformly distributed in the volume. What is the electric field for any  $r$ ?

# Applying Gauss's Law: Nuclear $\vec{E}$

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# Applying $V$

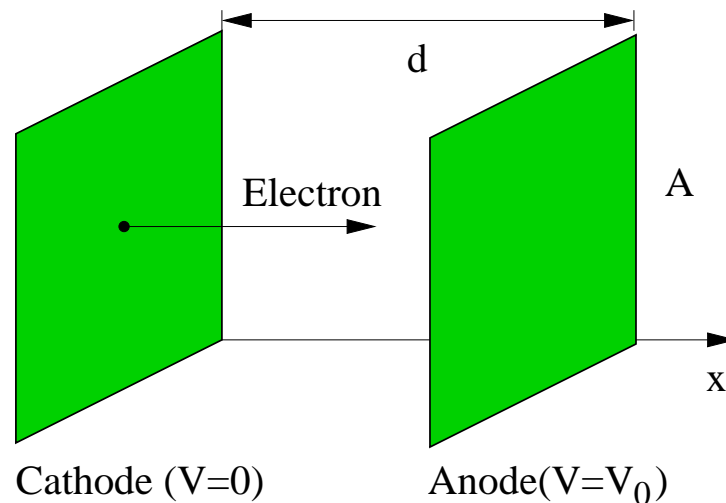
---

1. What is the potential energy of a point charge?
2. What is the potential inside and outside a uniformly charged sphere of total charge  $q$  and radius  $R$ ?
3. What is the electric field for the previous question?

# The Child-Langmuir Law

In a vacuum diode, electrons are boiled off a hot cathode at potential zero and accelerated across a gap to the anode at a positive potential  $V_0$ . The cloud of moving electrons within the gap (called the space charge) builds up and reduces the field at the cathode surface to zero. From then on a steady current flows between the plates.

Suppose two plates are large relative to the separation ( $A \gg d^2$  in the figure) so that edge effects can be ignored and  $V$ ,  $\rho$ , and the electron speed  $v$  are functions of only  $x$ .

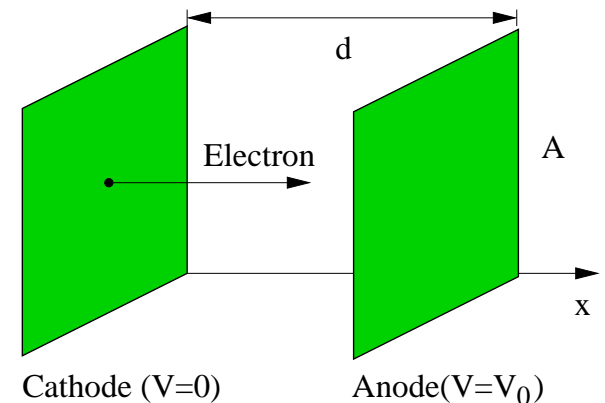


# The Child-Langmuir Law

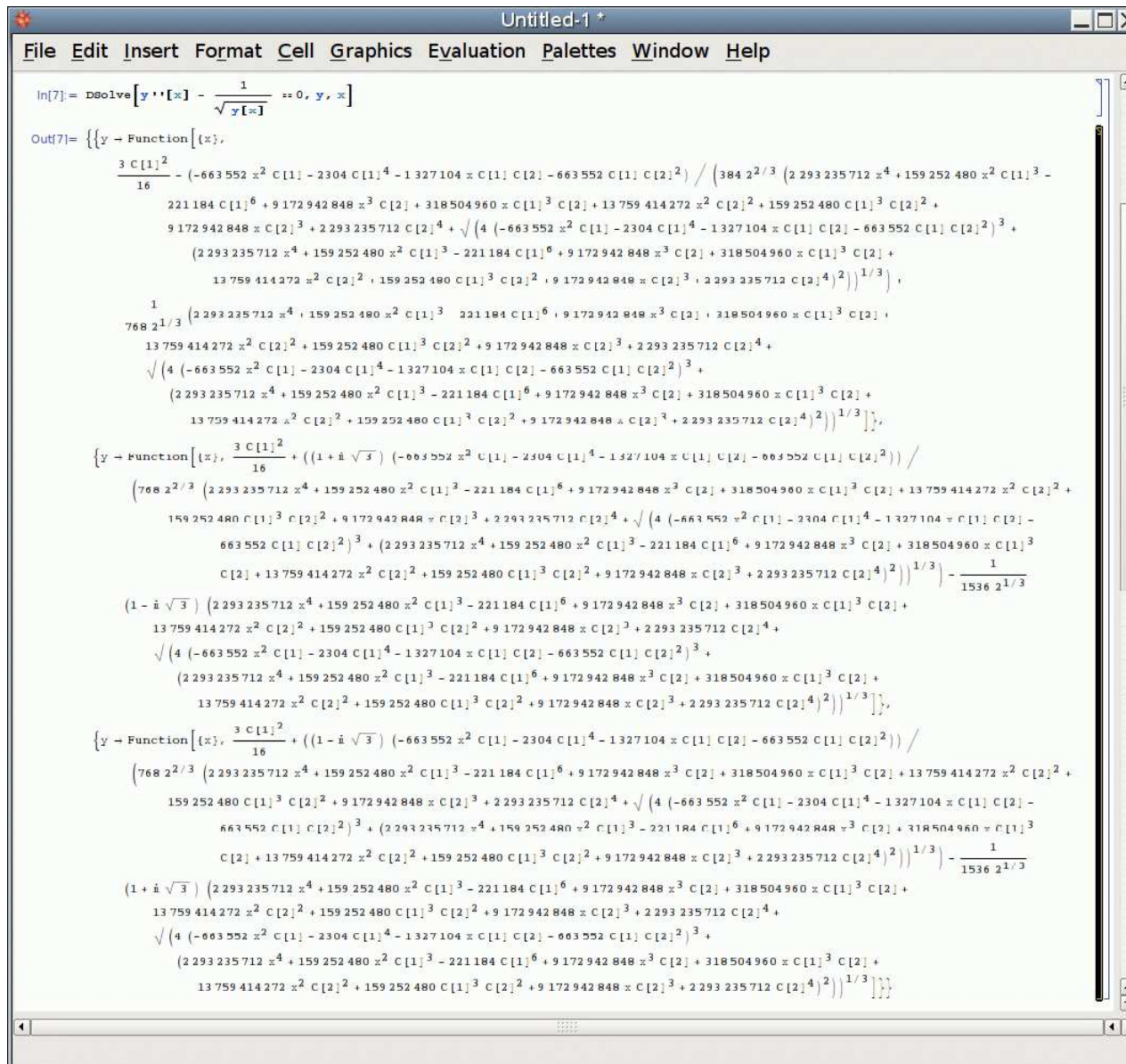
1. What is Poisson's equation for the region between the plates?
2. Assuming the electrons start from rest at the cathode, what is their speed at point  $x$ ?
3. In the steady state  $I$ , the current, is independent of  $x$ . How are  $\rho$  and  $v$  related?
4. Now generate a differential equation for  $V$  by eliminating  $\rho$  and  $v$  and solve this equation for  $V$  as a function of  $x$ ,  $V_0$ , and  $d$ . Make a plot to compare  $V(x)$  and the potential without the space charge.
5. What are  $\rho$  and  $v$  as functions of  $x$ ?
6. Show that

$$I = KV_0^{3/2}$$

and find  $K$ . This is the Child-Langmuir law.



# Using *Mathematica* To Solve the DE



The screenshot shows the Mathematica interface with a window titled "Untitled-1". The menu bar includes File, Edit, Insert, Format, Cell, Graphics, Evaluation, Palettes, Window, and Help. The input field contains the command:

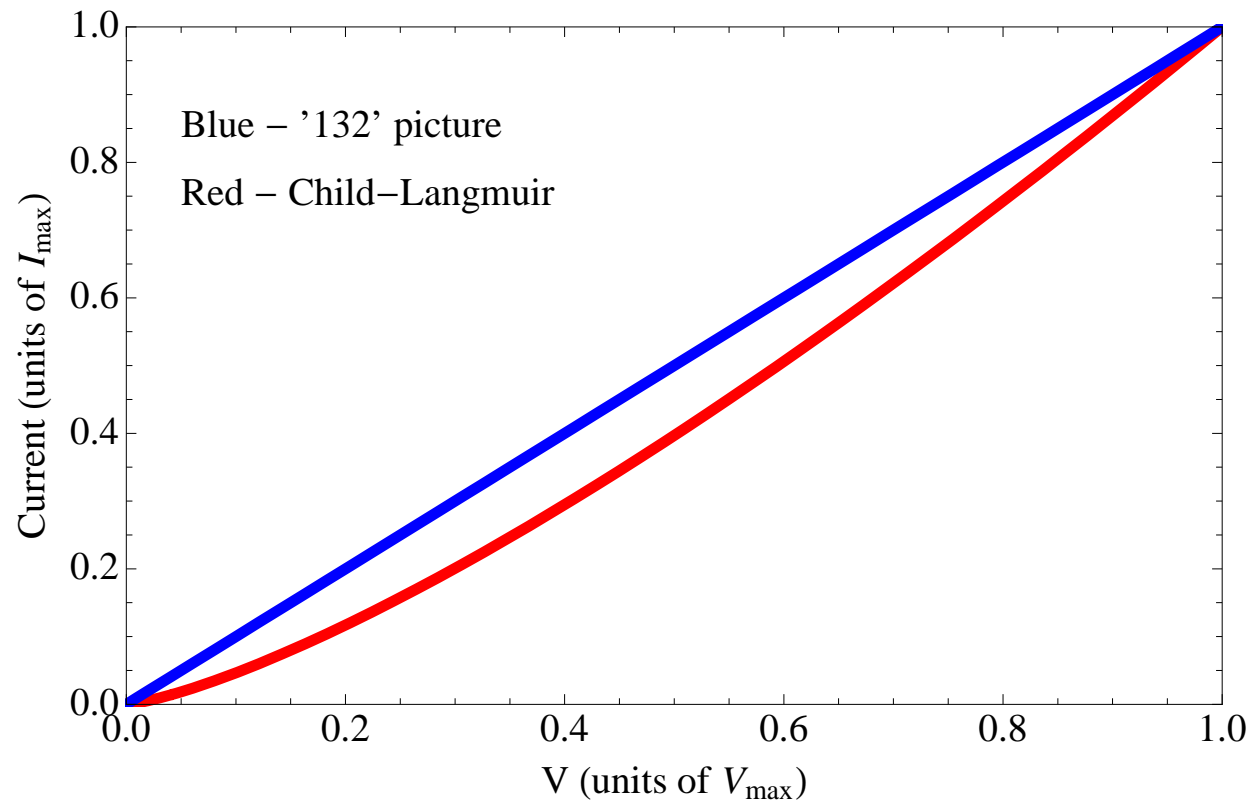
```
In[7]:= DSolve[y''[x] - 1/Sqrt[y[x]] == 0, y, x]
```

The output field displays the solution:

```
Out[7]= {{y -> Function[x],  
 3 C[1]^2 / 16 - (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] - 663 552 C[1] C[2]^2) / (384 2^(2/3) (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 -  
 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] + 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 +  
 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4) + Sqrt[4 (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] - 663 552 C[1] C[2]^2)^3 +  
 (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4)^2]^(1/3)},  
 1 / (768 2^(1/3) (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4) +  
 Sqrt[4 (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] - 663 552 C[1] C[2]^2)^3 +  
 (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4)^2]^(1/3)}],  
 {y -> Function[x], 3 C[1]^2 / 16 + ((1 + I Sqrt[3]) (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] - 663 552 C[1] C[2]^2)) /  
 (768 2^(2/3) (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] + 13 759 414 272 x^2 C[2]^2 +  
 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4) + Sqrt[4 (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] -  
 663 552 C[1] C[2]^2)^3 + (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 C[2] + 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4)^2]^(1/3)) - 1 / (1536 2^(1/3)  
 (1 - I Sqrt[3]) (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4) +  
 Sqrt[4 (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] - 663 552 C[1] C[2]^2)^3 +  
 (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4)^2]^(1/3)}],  
 {y -> Function[x], 3 C[1]^2 / 16 + ((1 - I Sqrt[3]) (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] - 663 552 C[1] C[2]^2)) /  
 (768 2^(2/3) (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] + 13 759 414 272 x^2 C[2]^2 +  
 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4) + Sqrt[4 (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] -  
 663 552 C[1] C[2]^2)^3 + (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 C[2] + 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4)^2]^(1/3)) - 1 / (1536 2^(1/3)  
 (1 + I Sqrt[3]) (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4) +  
 Sqrt[4 (-663 552 x^2 C[1] - 2304 C[1]^4 - 1327 104 x C[1] C[2] - 663 552 C[1] C[2]^2)^3 +  
 (2 293 235 712 x^4 + 159 252 480 x^2 C[1]^3 - 221 184 C[1]^6 + 9 172 942 848 x^3 C[2] + 318 504 960 x C[1]^3 C[2] +  
 13 759 414 272 x^2 C[2]^2 + 159 252 480 C[1]^3 C[2]^2 + 9 172 942 848 x C[2]^3 + 2 293 235 712 C[2]^4)^2]^(1/3)}]}
```

# The Child-Langmuir Law: Results

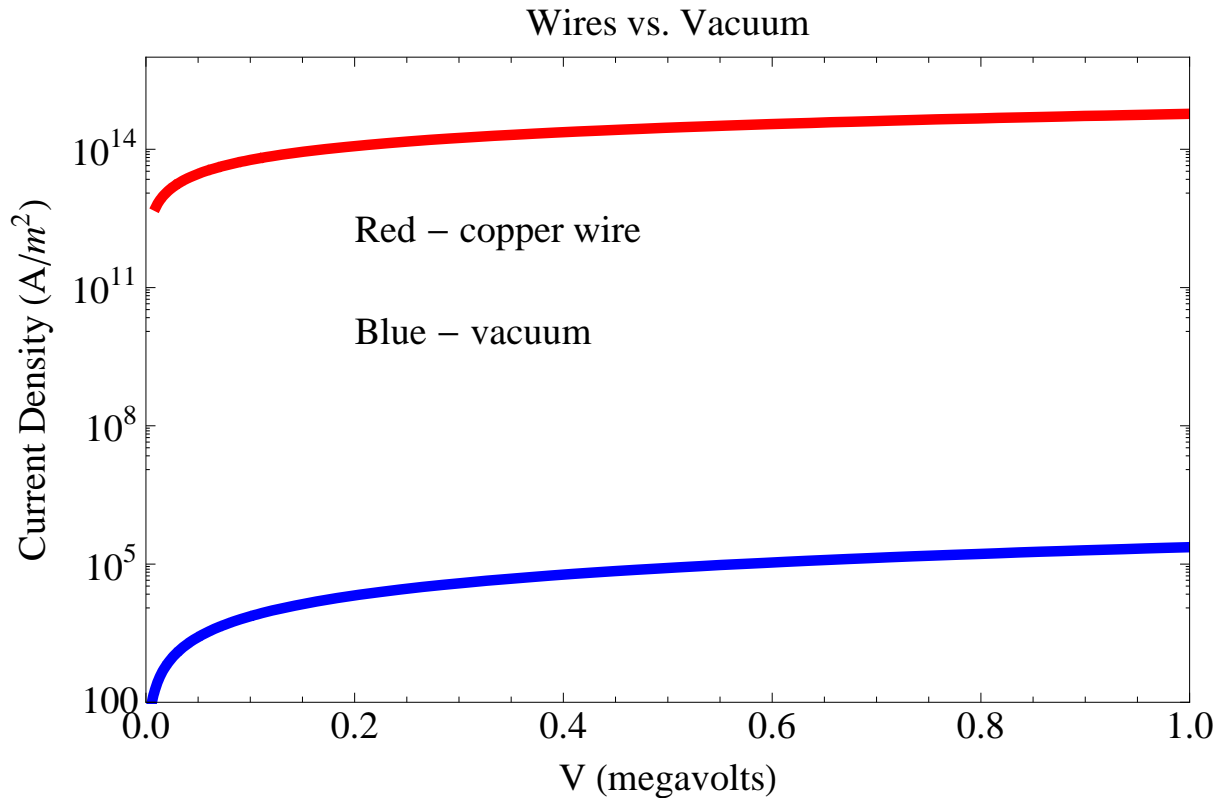
Comparing the Child-Langmuir Law with the no-space-charge solution.





# More Child-Langmuir Law Results

Comparing the Child-Langmuir Law in vacuum with a copper wire.



$$J_{CU} = \frac{1}{\rho} \frac{V}{d}$$

$$\rho = 1.7 \times 10^{-8} \Omega\text{-m}$$

$$J_{CL} = \frac{4\epsilon_0}{9d^2} \sqrt{\frac{2e}{m_e}} V^{3/2}$$

$$d = 0.1 \text{ m}$$