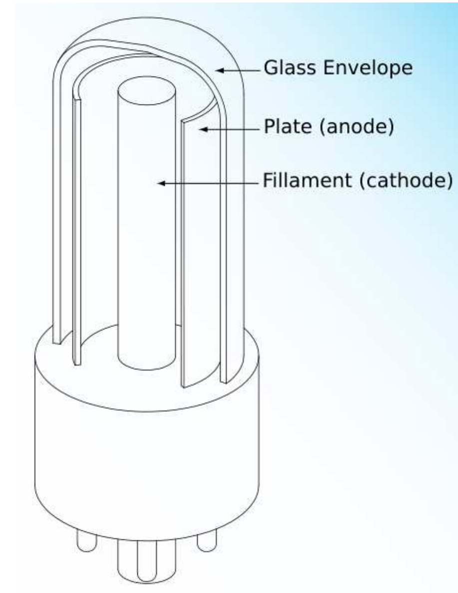
How Electronics Started! And JLab Hits the Wall!

In electronics, a vacuum diode or tube is a device used to amplify, switch, otherwise modify, or create an electrical signal by controlling the movement of electrons in a low-pressure space. Almost all depend on the thermal emission of electrons. The figure shows the components of the device. A demonstration of how they work is here.

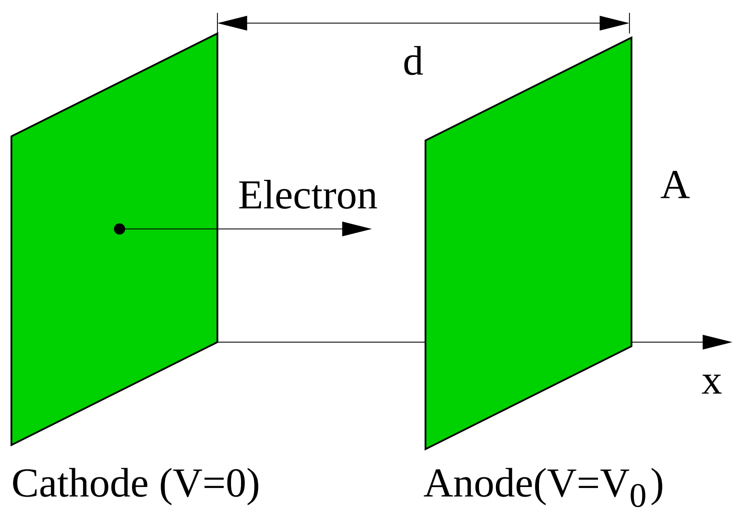


The physics here also determines the limits on the amount of beam that can be produced in an accelerator.

The Child-Langmuir Law

In a vacuum diode, electrons are boiled off a hot cathode at zero potential and accelerated across a gap to the anode at a positive potential V_0. The cloud of moving electrons within the gap (called the space charge) builds up and reduces the field at the cathode surface to zero. From then on a steady current flows between the plates.

Suppose two plates are large relative to the separation ($A >> d^2$ in the figure) so that edge effects can be ignored and V, ρ, and the electron speed v are functions of only x.



The Child-Langmuir Law

1. What is Poisson's equation for the region between the plates?

2. Assuming the electrons start from rest at the cathode, what is their speed at point x?

3. In the steady state I, the current, is independent of x. How are ρ and v related?

4. Now generate a differential equation for V by eliminating ρ and v and solve this equation for V as a function of x, V_0, and d. Make a plot to compare $V(x)$ and the potential without the space charge.

5. What are ρ and v as functions of x?

6. Show that

$$I = KV_0^{3/2}$$

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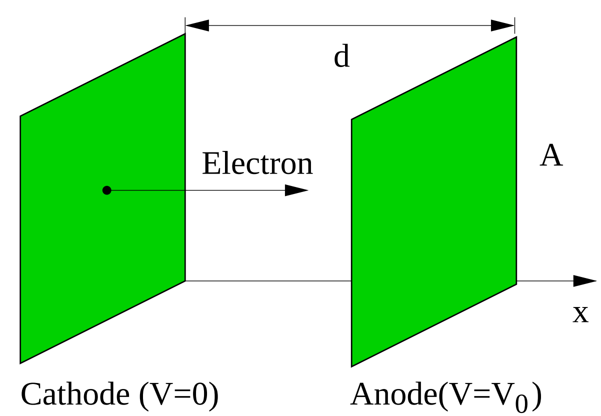
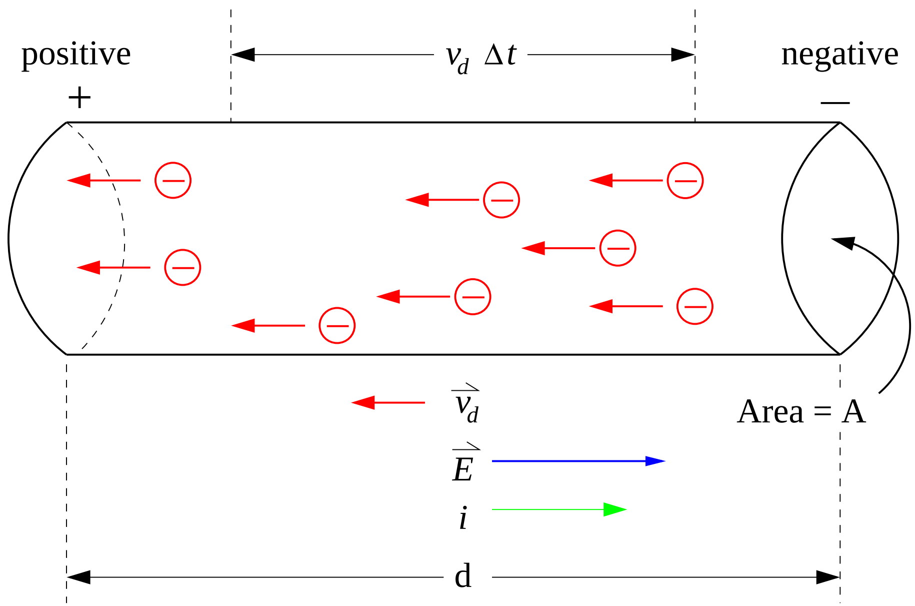


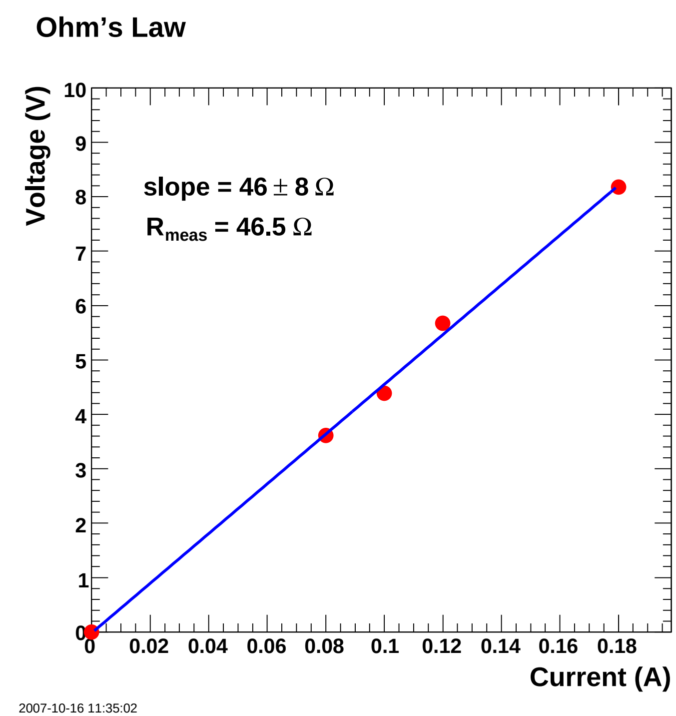
Why Should You Care? The Physics 132 Picture



$$I = JA = nqv_dA = \frac{A}{\rho d}V$$

$$I = \frac{1}{R}V$$

$$V = IR$$



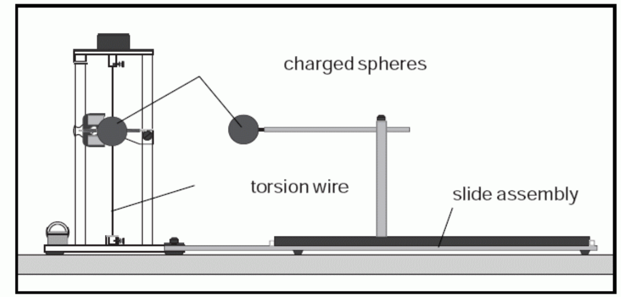
Why Should You Care, Part Deaux?

January 11, 2006

Coulomb’s Law and Superposition

- Measuring the Electrostatic Force of Charges.

  Use a torsion pendulum and charged spheres.



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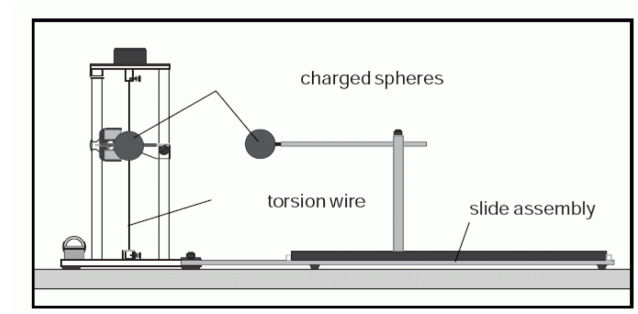
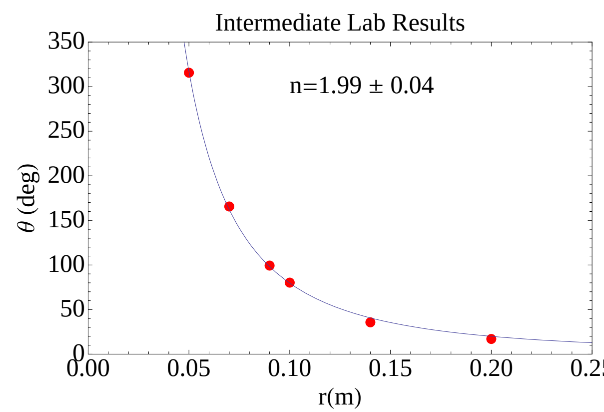


- Evidence for Coulomb's Law.

  Fix charge and vary distance.



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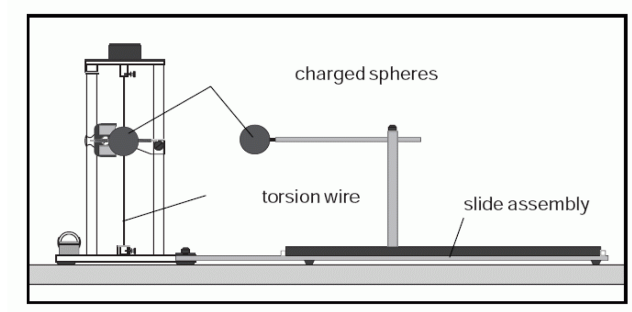

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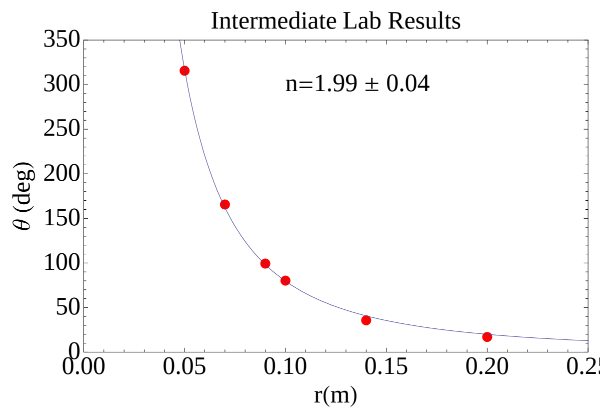
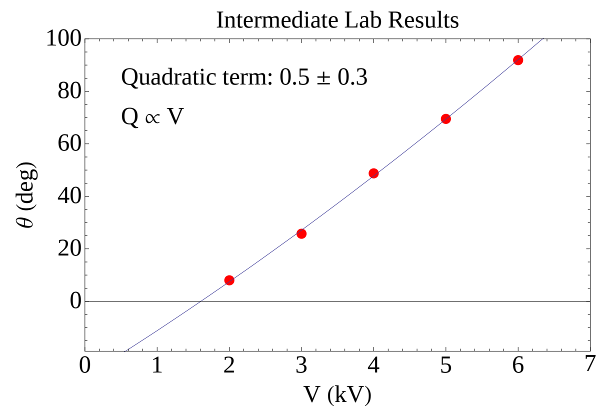


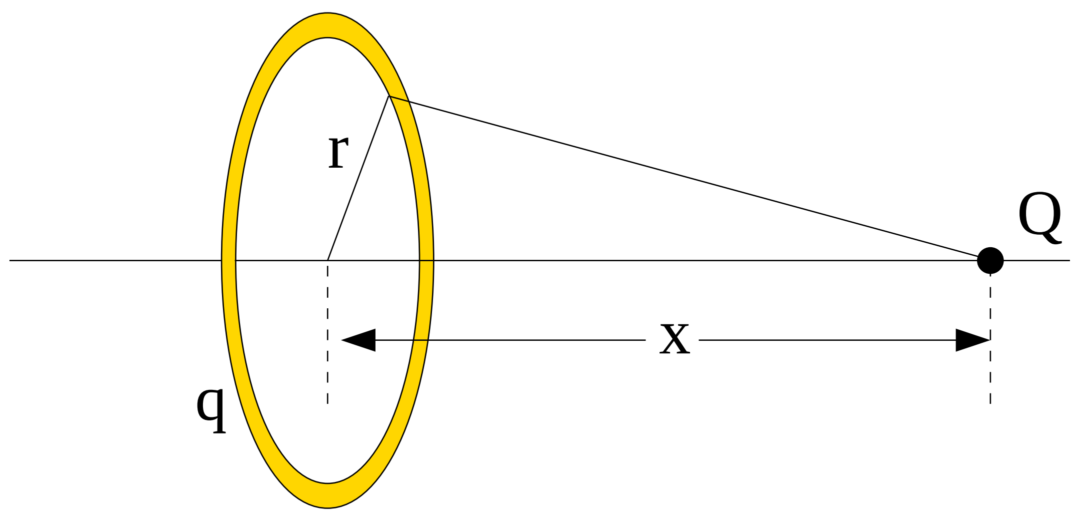
- Evidence for Superposition.

  Fix distance and vary charge on one sphere.



Electric Field of a Ring

A ring of radius r as shown in the figure has a positive charge distribution per unit length with total charge q. Calculate the electric field $\vec{E}$ along the axis of the ring at a point lying a distance x from the center of the ring. Get your answer in terms of r, x, q. What happens as $x \to \infty$?



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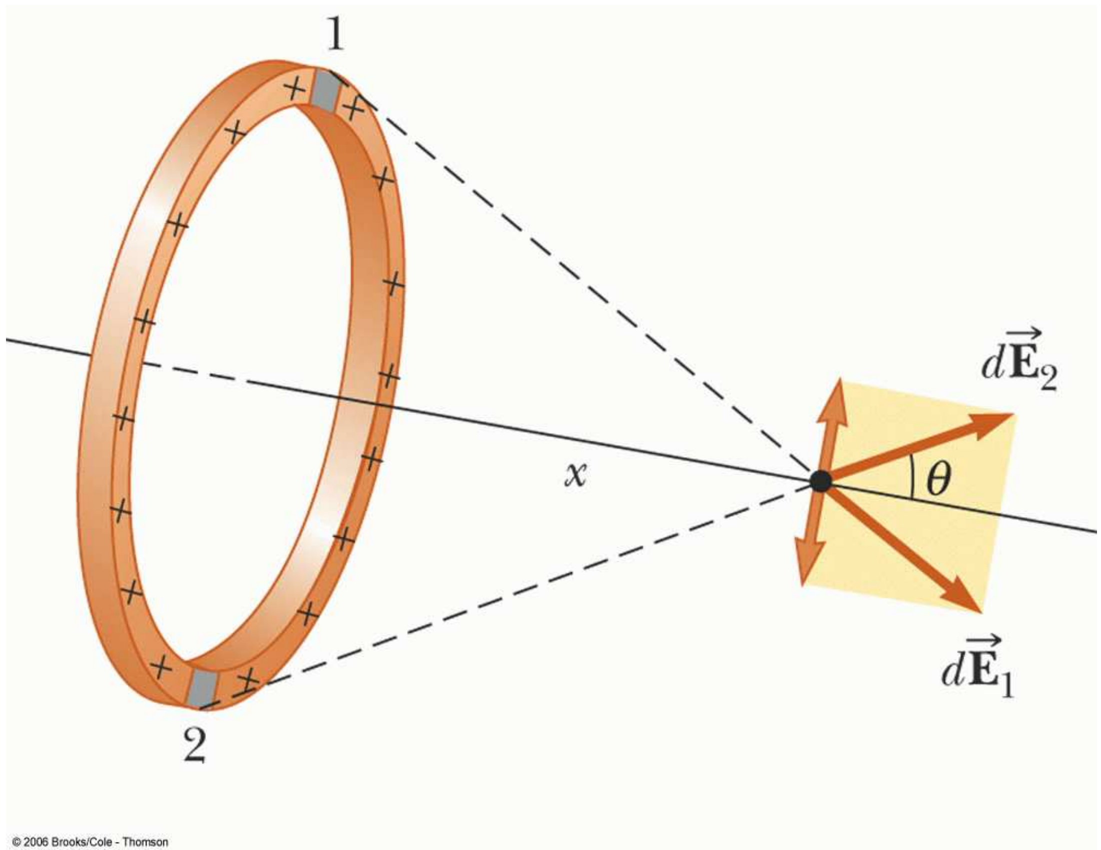
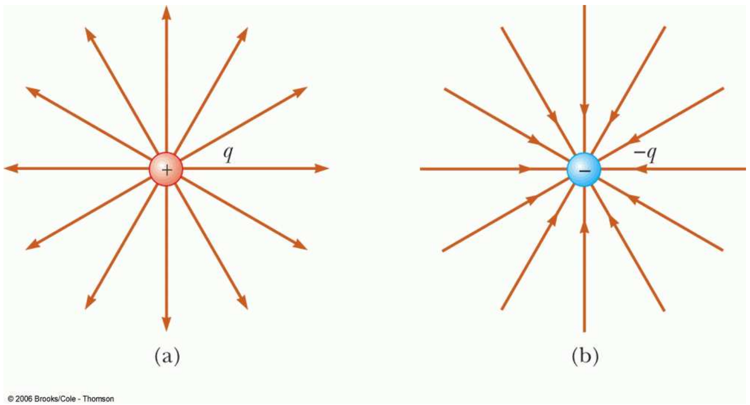
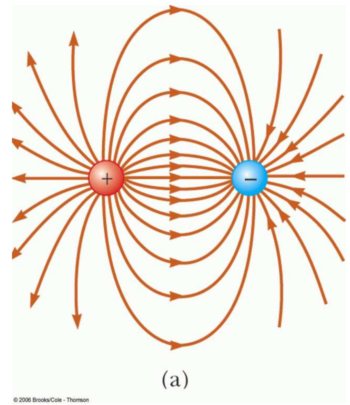


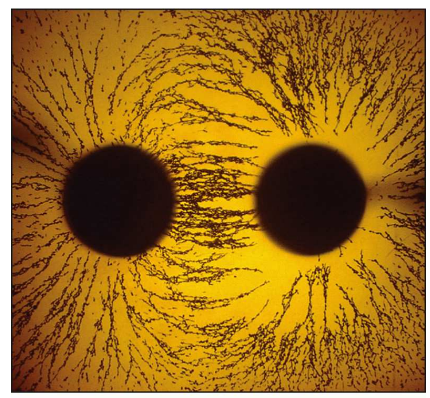
Electric Field Lines

Point Charges

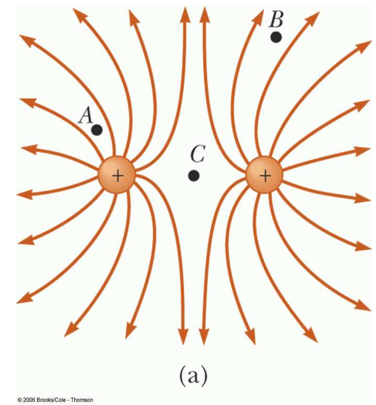


Electric Dipole





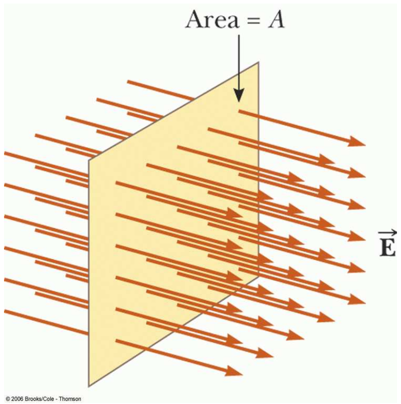
Positive Charges

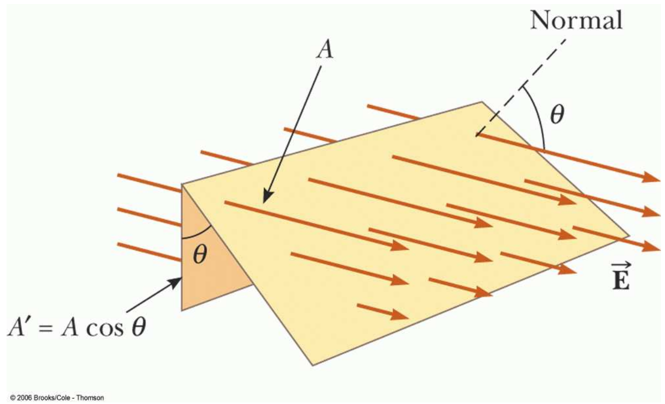


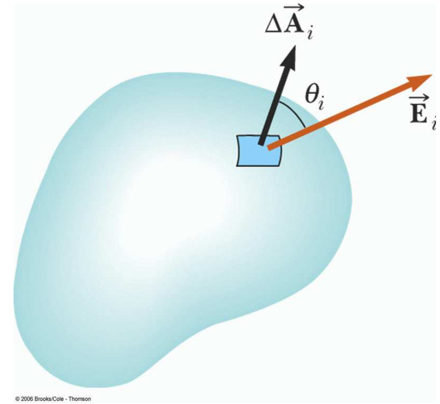
Properties of Electric Field Lines

- Field lines start from positive charges (sources) and end at negative ones (sinks).
- Are symmetrical around point charges.
- Density of field lines is related to strength of field.
- Direction of field is tangent to the field line.
- Field lines never cross!

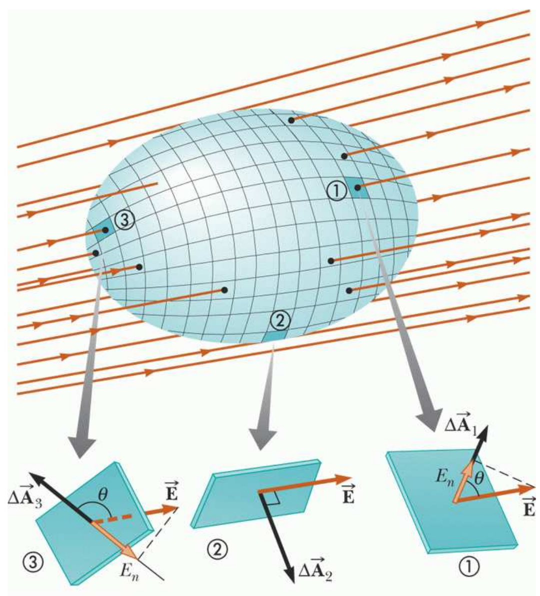
Electric Flux







$$\Phi_E = EA \qquad \Phi_E = EA\cos\theta = \vec{E}\cdot\vec{A} \qquad \Phi_E = \oint \vec{E}\cdot d\vec{A}$$

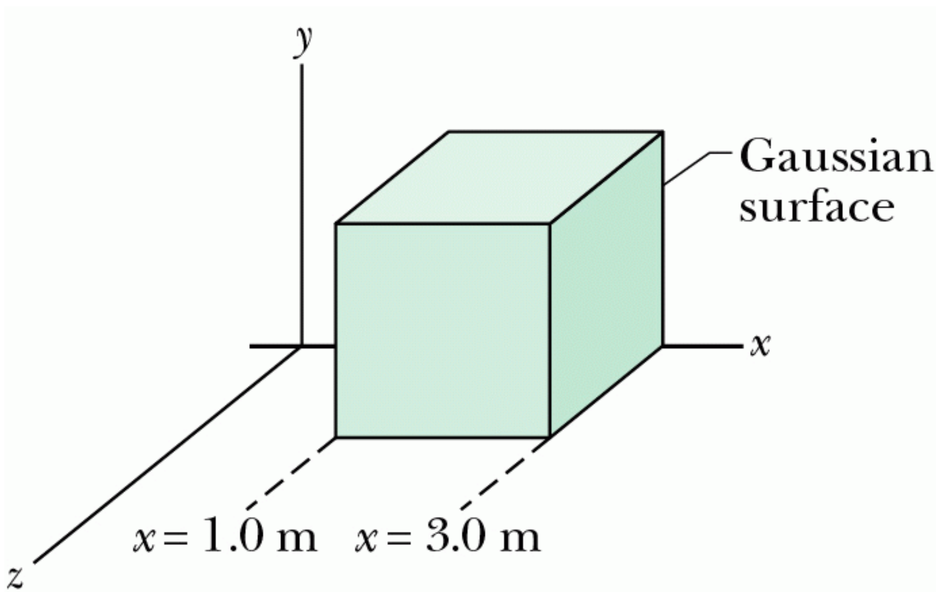


An Example of Electric Flux

A nonuniform magnetic field is described by

$$\vec{E} = (3.0\ N/C - m)x\hat{x} + (4.0\ N/C)\hat{y}$$

pierces the Gaussian cube shown in the figure. What is the flux through the cube? Note the orientation of the coordinate system.

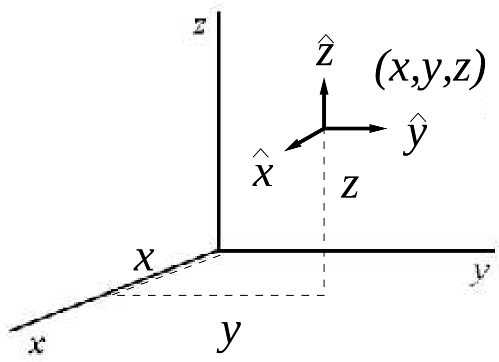


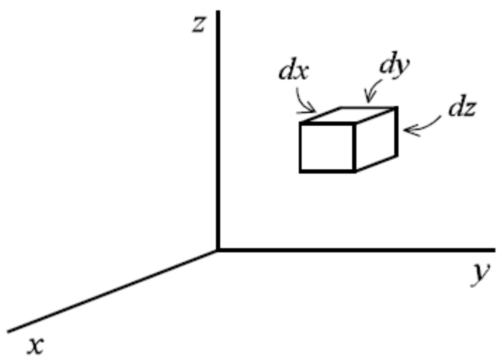
Gauss's Law

What is the flux from a point charge q at the origin through a sphere centered at the origin of radius r?

Differential Surface and Volume Elements

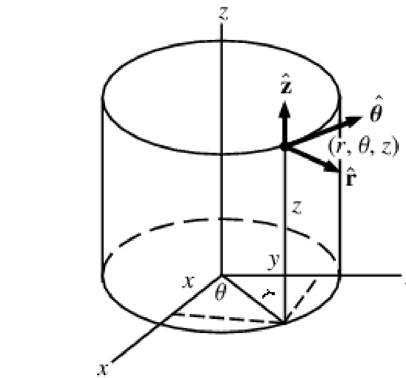
Cartesian Coordinates

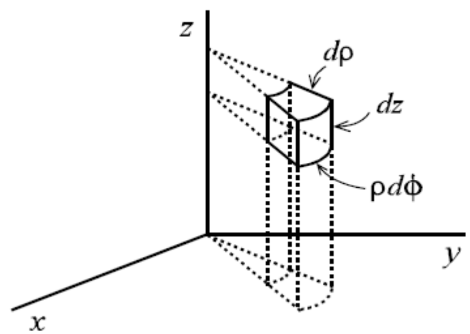




$$dA = dxdy$$
$$d\tau = dxdydz$$

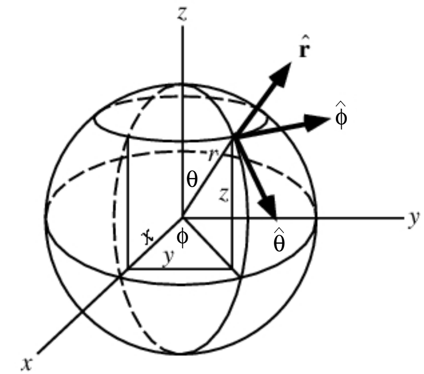
Cylindrical Coordinates

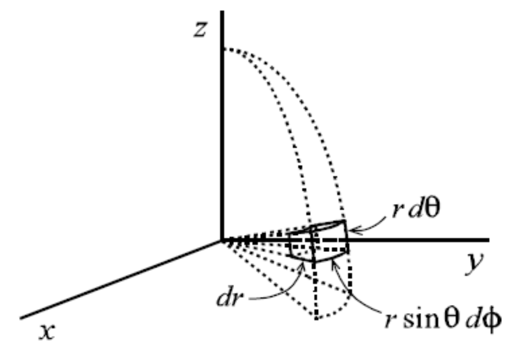




$$dA = \rho d\phi dz$$
$$d\tau = \rho d\phi dz d\rho$$

Spherical Coordinates





$$dA = r^2 d\cos\theta d\phi$$
$$d\tau = r^2 d\cos\theta d\phi dr$$

Applying Gauss's Law: Nuclear $\vec{E}$

The nucleus of a gold atom has a radius $R = 6.2 \times 10^{-15}\ m$ and a positive charge $q = Ze$ where $Z = 79$ is the atomic number. Assume the gold nucleus is spherical and the charge is uniformly distributed in the volume. What is the electric field for any r?

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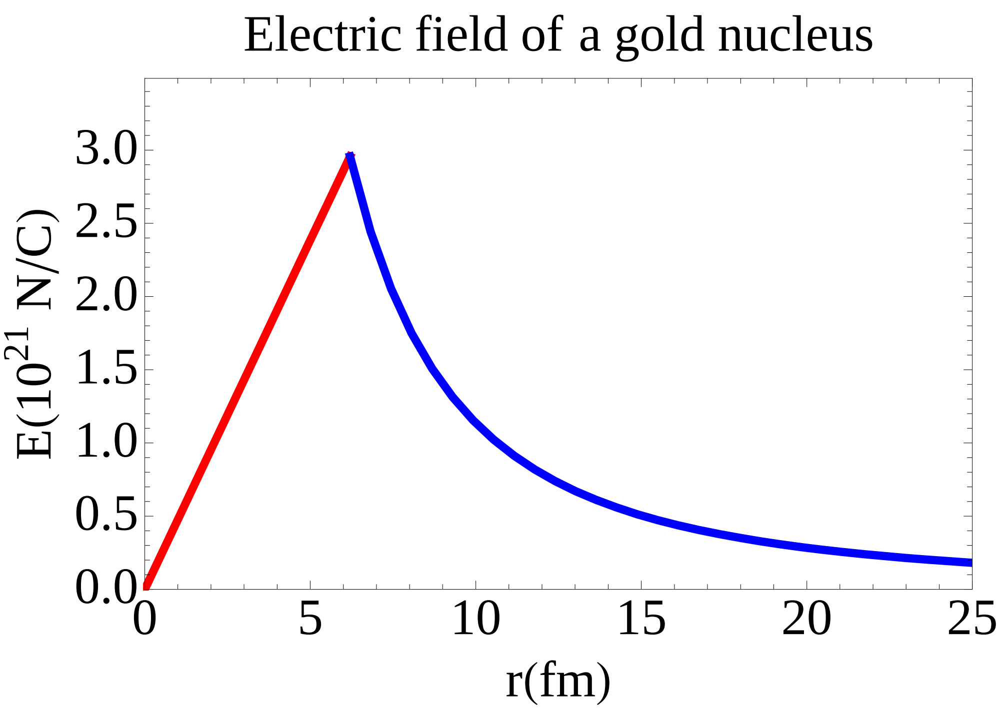

An Example of Applying $\nabla \times \vec{E}$

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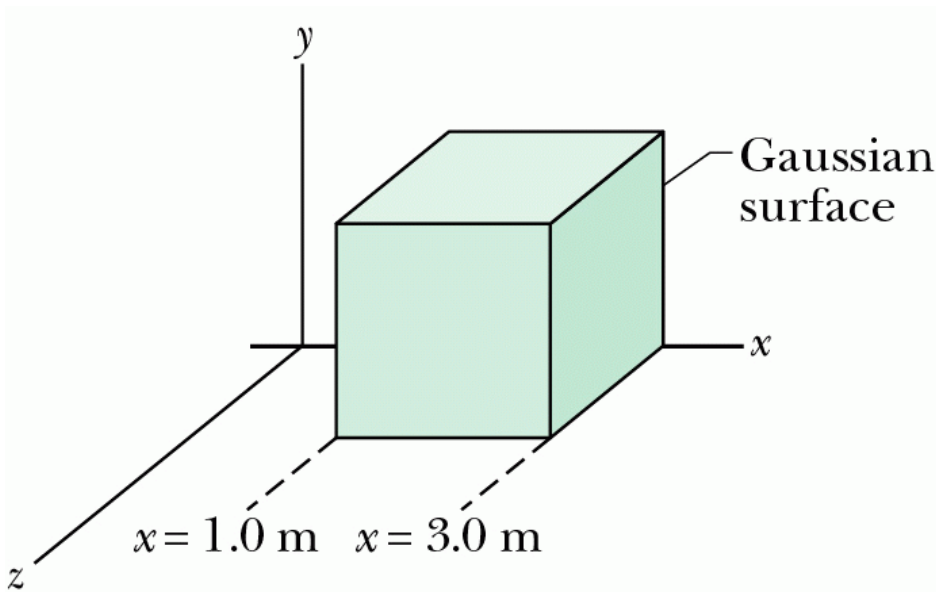
pierces the Gaussian cube shown in the figure. Is this a conservative field?



Applying V

1. What is the potential energy of a point charge?
2. What is the potential inside and outside a uniformly charged sphere of total charge q and radius R?
3. What is the electric field for the previous question?

The Child-Langmuir Law

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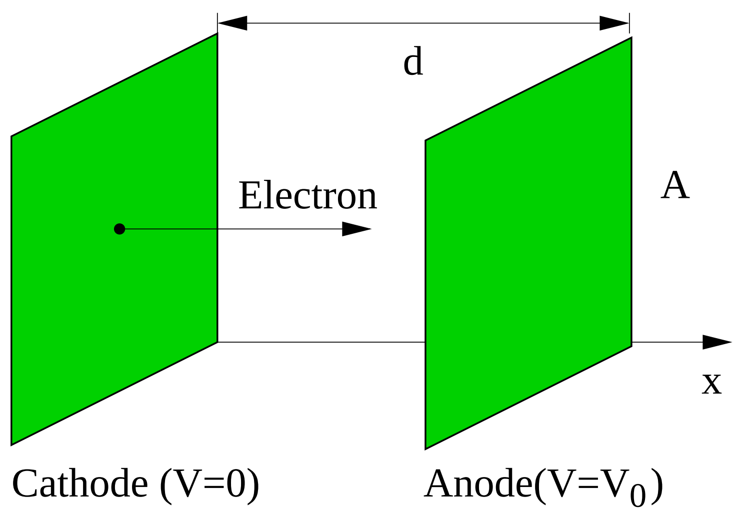

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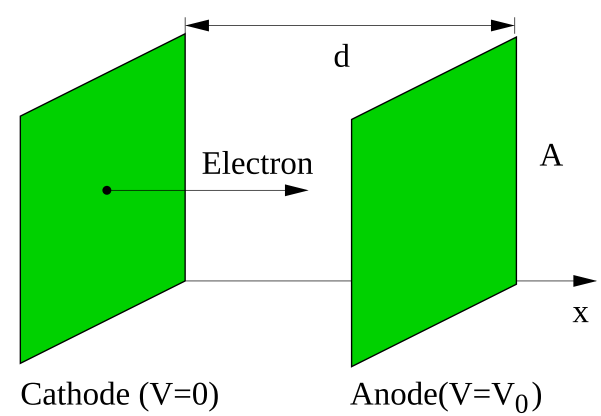
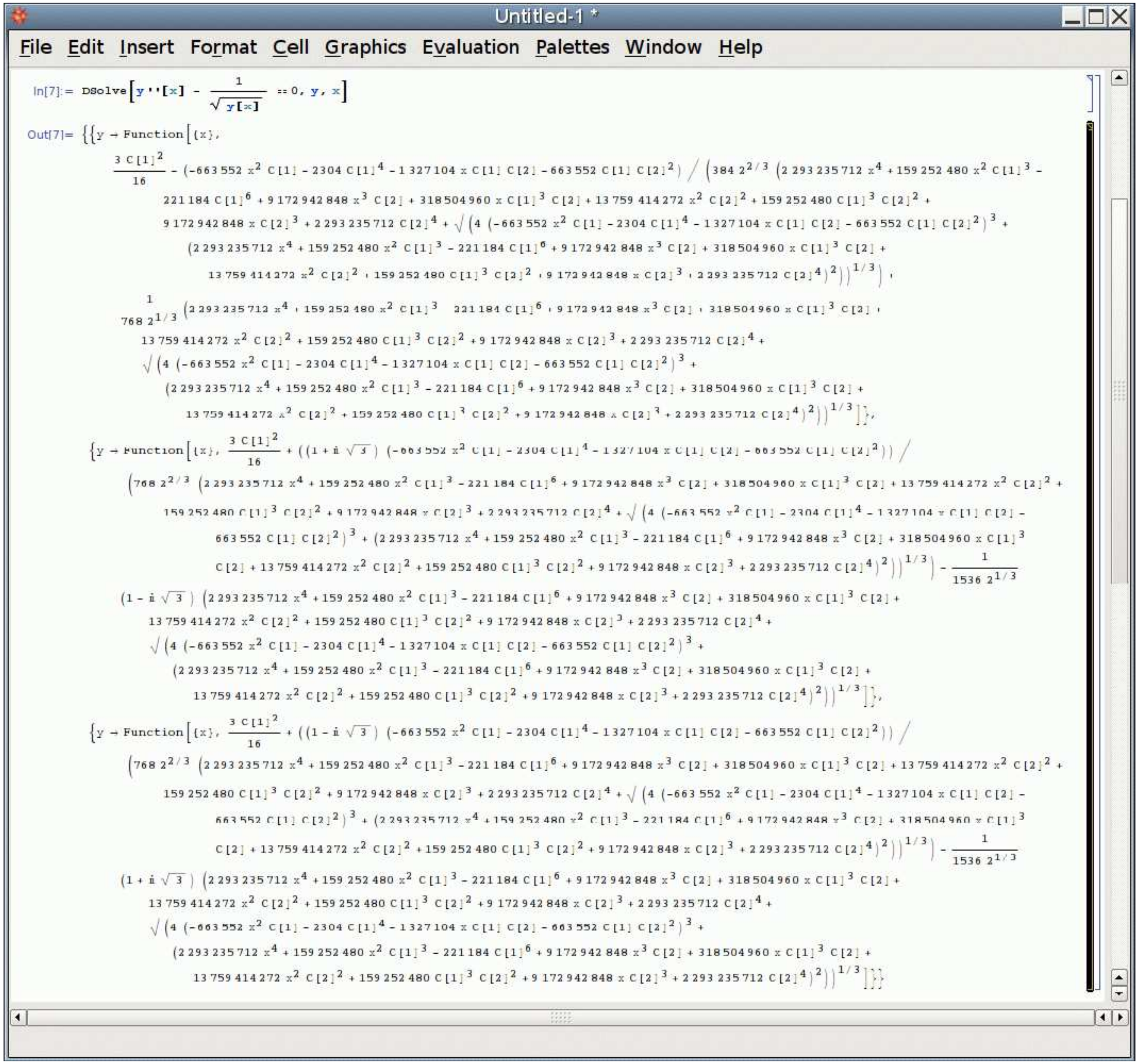
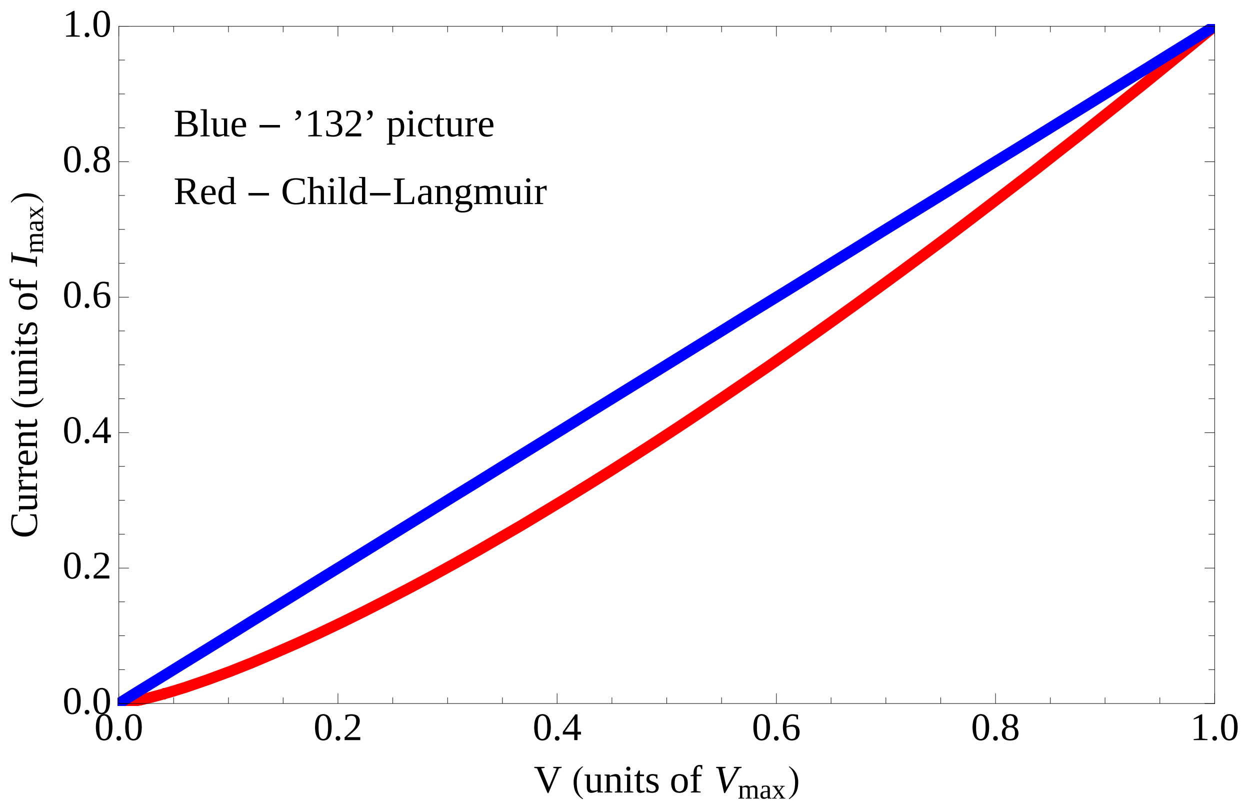


Using *Mathematica* To Solve the DE



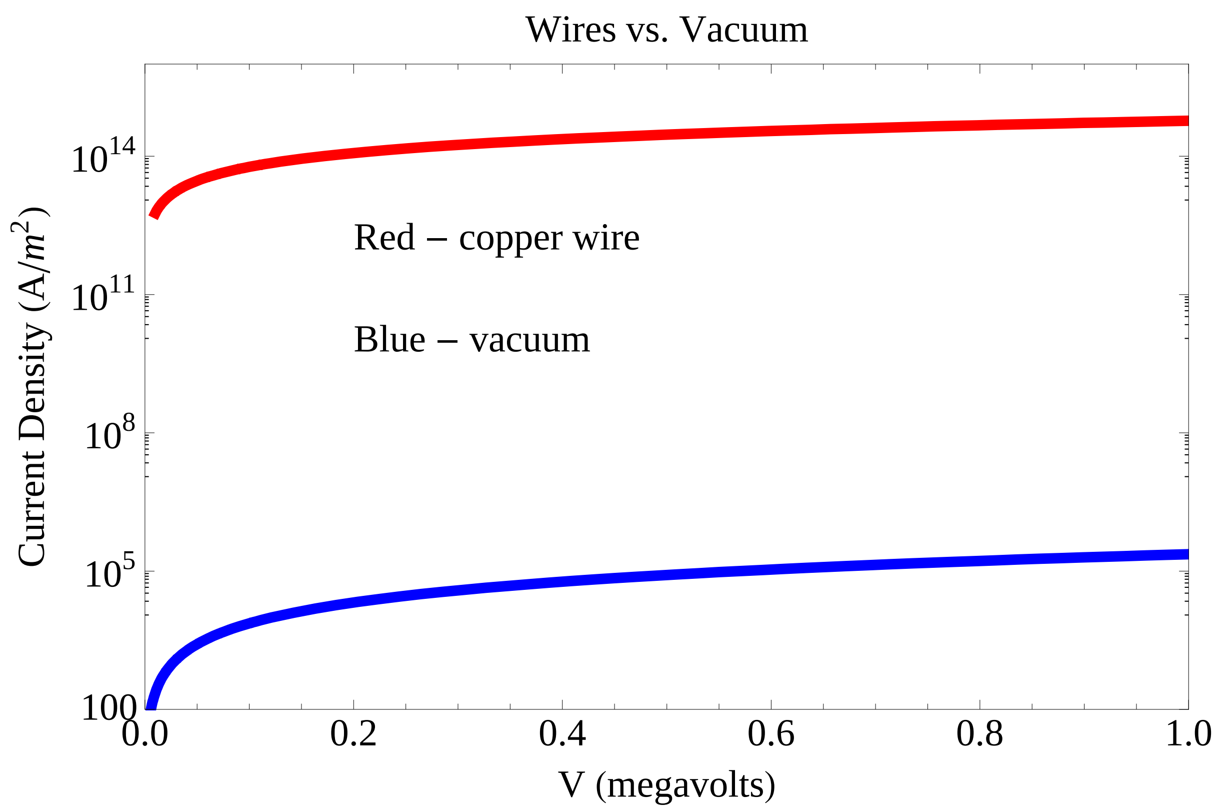
The Child-Langmuir Law: Results

Comparing the Child-Langmuir Law with the no-space-charge solution.



More Child-Langmuir Law Results

Comparing the Child-Langmuir Law in vacuum with a copper wire.



$$J_{CU} = \frac{1}{\rho}\frac{V}{d} \qquad \rho = 1.7\times 10^{-8}\,\Omega{-}m \qquad J_{CL} = \frac{4\epsilon_0}{9d^2}\sqrt{\frac{2e}{m_e}}V^{3/2} \qquad d = 0.1\,m$$