Physics 303 Test 2

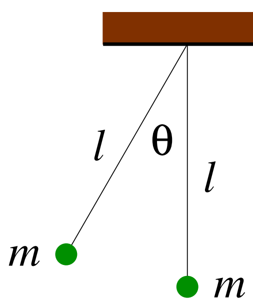
I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Signature ____

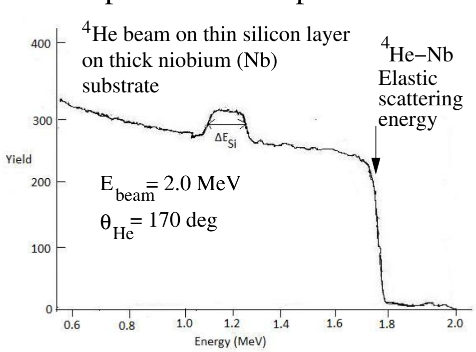
Questions (5 pts. apiece) Answer questions 1-5 in complete, well-written sentences WITHIN the spaces provided.

1. What advantages are there, if any, to using the center-of-mass frame in solving the Rutherford scattering problem.

2. Two balls of clay of equal mass m are hanging side-by-side from identical, massless strings as shown in the figure. One ball is pulled to an angle θ to the vertical and released. It swings down, strikes the other ball, and they stick together. Is momentum conserved in this collision? Is kinetic energy conserved? Explain both answers.



3. The spectrum of a Rutherford backscattering measurement is shown in the figure. There is a broad, gently sloping plateau in the yield that abruptly drops to zero at the Rutherford elastic scattering energy for $^4\mathrm{He} - \mathrm{Nb}$ elastic scattering. What process creates this plateau? Ignore the bump on the broad plateau labelled ΔE_{Si}.



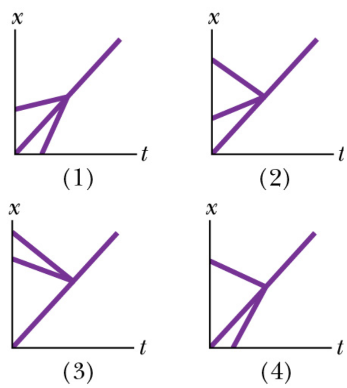
Do not write below this line.

4. The eigenvectors for the CO_2 molecule we studied in class are $\vec{X}_1 = (1,1,1)$, $\vec{X}_2 = (-1,0,1)$, and $\vec{X}_3 = (1,-2m_O/m_C,1)$ where $m_O = 16\ u$ is the oxygen atom mass and $m_C = 12\ u$ is the carbon atom mass. Suppose a CO_2 molecule is excited with the following initial conditions at $t = 0$.

$$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0 \quad v_1 = -v_0 \quad v_2 = \frac{8}{3}v_0 \quad v_3 = -v_0$$

What eigenvectors to you expect to be excited in the solution? Explain.

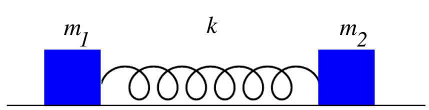
5. The figure below shows four plots of position versus time for two bodies and their center-of-mass. The two bodies undergo a perfectly inelastic, one-dimensional collision while moving along the x axis. Which graphs correspond to physically impossible situations? Explain.



Problems (1-3). Clearly show all reasoning for full credit. Use a separate sheet to show your work.

1. 20 pts. A boat of mass $m_b = 80\ kg$ (uniformly distributed along its length) and length $l = 5\ m$ is at rest in quiet water. A person of mass $m_p = 65\ kg$ walks from the bow to the stern. What distance did the boat move and in what direction? Neglect water resistance.

2. 25 pts. Two masses m_1 and m_2 are connected by a spring of rest length l and spring constant k as shown in the figure. The system slides without friction on a horizontal surface in the direction of the spring's length. What is the Lagrangian of the motion? What are the equations of motion (*i.e.* differential equations) the system must satisfy?



3. 30 pts. A neutron of mass $m_n = 1\,u$ and known kinetic energy K_0 is scattered through an angle $\theta = 90°$ in an elastic collision with a deuteron of mass $m_d = 2\,u$ that is initially at rest. Starting from the conservation of momentum, derive an expression for the ratio of the scattered neutron's final kinetic energy K_n to the kinetic energy of the recoiling deuteron K_d in terms of known quantities, *e.g.* the masses m_n, m_d. Hint: Make early use of the neutron scattering angle $\theta = 90°$.

Equations, Conversions, and Constants

$$\vec{F} = m\vec{a} = \dot{\vec{p}} = -\frac{dV}{dx} \quad \vec{F}_G = -\frac{Gm_1m_2}{r^2}\hat{r} \quad \vec{F}_C = \frac{kq_1q_2}{r^2}\hat{r} \quad \vec{F}_g = -mg\hat{y} \quad \vec{F}_s = -kr\hat{r}$$

$$|\vec{F}_{cent}| = m\frac{v^2}{r} \quad \vec{F}_f = -bv\hat{v} \quad \int \frac{df}{dx} = \int df \quad \ddot{y} + A\dot{y} + By = 0 \Rightarrow y = Ce^{\lambda t}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad \ddot{y} + \omega_0^2 y = 0 \Rightarrow y = A\sin(\omega_0 t + \phi) = \alpha_1 e^{i\omega_0 t} + \alpha_2 e^{-i\omega_0 t}$$

$$\ddot{y} + \omega_0^2 y = \omega_0^2 l \Rightarrow y = C + A\sin(\omega_0 t + \phi) \quad \omega_0^2 = \frac{k}{m} \quad \omega_0^2 = \frac{g}{l} \quad \mu = \frac{m_1m_2}{m_1+m_2}$$

$$V = -\int_{x_s}^{x} \vec{F}(\vec{r}\,') \cdot d\vec{r}\,' \quad E = K + V \quad V_s = \frac{kx^2}{2} \quad V_g = mgy \quad V_G = -\frac{Gm_1m_2}{r} \quad V_C = \frac{kq_1q_2}{r}$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad \mathscr{L} = K - V \quad \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} = 0$$

$$\vec{L} = \vec{r}\times\vec{p} \quad \vec{N} = \frac{d\vec{L}}{dt} \quad l = \mu r^2\dot{\theta} \quad V_{cent} = \frac{L^2}{2\mu r^2} \quad \vec{p} = m\vec{v} \quad \vec{p}_i = \vec{p}_f \quad K_i = K_f \quad e = \frac{|\vec{v}_{2f} - \vec{v}_{1f}|}{|\vec{v}_{2i} - \vec{v}_{1i}|}$$

$$\frac{1}{r} = \frac{\mu\alpha}{l^2}\left(1 + \epsilon\cos(\theta - \theta_0)\right) \quad \epsilon = \sqrt{1 + \frac{2El^2}{\mu\alpha^2}} \quad \sin\left(\frac{\theta_s}{2}\right) = \frac{1}{\epsilon}$$

$$\alpha = -ke^2 Z_1 Z_2 \qquad ke^2 = \frac{\hbar c}{137} = 1.44\,MeV{-}fm \quad V_{eff}(r) = \frac{l^2}{2\mu r^2} + V(r) \quad \vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$\sin A = \cos(A - \pi/2) \quad \cos A = -\sin(A - \pi/2) \quad \arccos(-x) = \pi - \arccos(x)$$

More Equations, Conversions, and Constants

Speed of light (c)	$3.0 \times 10^8\ m/s$	g	$9.8\ m/s^2$
Gravitation constant (G)	$6.67 \times 10^{-11}\ N - m^2/kg^2$	Earth's radius	$6.37 \times 10^6\ m$
Coulomb constant (k_e)	$8.99 \times 10^9 \frac{N-m^2}{C^2}$	Earth's mass	$5.97 \times 10^{24}\ kg$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	Proton/Neutron mass	$1.67 \times 10^{-27}\ kg$
Planck's constant (h)	$6.626 \times 10^{-34}\ J - s$	Proton/Neutron mass	$932 \times 10^6\ eV/c^2$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} \frac{kg^2}{N-m^2}$	Electron mass	$9.11 \times 10^{-31}\ kg$
Permeability constant (μ_0)	$4\pi \times 10^{-7} N/A^2$	Electron mass	$0.55 \times 10^6\ MeV/c^2$
1 MeV	$10^6\ eV$	1.0 eV	$1.6 \times 10^{-19}\ J$
1 kg	$931.5\ MeV/c^2$	1 u	$1.67 \times 10^{-27}\ kg$

$$\int \frac{dx}{x^2 - a^2} = -\frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \qquad \int \tanh x dx = \ln\left[\cosh x\right] \qquad \int \coth x dx = \ln[\sinh x]$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} \qquad \int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1} \frac{x}{a} \qquad \frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \sec x = \sec x \tan x \qquad \frac{d}{dx} \ln ax = \frac{1}{x} \qquad \frac{d}{dx} \cot x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) \qquad \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2 - a^2}}{2} + \frac{a^2}{2} \ln\left[x + \sqrt{x^2 - a^2}\right]$$

$$\int \tanh^2(x) dx = x - \tanh x \qquad \int \tanh^3(x) dx = \ln\left[\cosh x\right] + \frac{\operatorname{sech}^2(x)}{2}$$

$$\int \sqrt{\tanh x} dx = -\tan^{-1}\left[\sqrt{\tanh x}\right] - \frac{1}{2} \ln\left[1 - \sqrt{\tanh x}\right] + \frac{1}{2} \ln\left[1 + \sqrt{\tanh x}\right]$$

$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx} \coth x = -\operatorname{csch}^2 x \qquad \frac{d}{dx} \sinh x = \cosh x$$

$$\int \frac{1}{\sqrt{ar^4 + br^3 - r^2}} dr = \frac{r\sqrt{-1 + br + ar^2} \arctan\left(\frac{-2 + br}{2\sqrt{-1 + br + ar^2}}\right)}{\sqrt{r^2(-1 + br + ar^2)}}$$

$$\int \frac{1}{\sqrt{ax^2 + bx + c}} dx = -\frac{1}{\sqrt{-a}} \arcsin\left(\frac{2ax + b}{\sqrt{b^2 - 4ac}}\right)$$

$$\int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}} \arctan\left(\frac{2ax + b}{\sqrt{4ac - b^2}}\right)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Even More Equations, Conversions, and Constants

Pythagorean identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Reciprocal identities:

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Even - odd identities:

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Product to sum formulas:

$$\sin x \cdot \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

$$\cos x \cdot \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin x \cdot \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\cos x \cdot \sin y = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$$

Sum to product:

$$\sin x \pm \sin y = 2\sin\left(\frac{x \pm y}{2}\right)\cos\left(\frac{x \mp y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Double - angle formulas:

$$\sin 2\theta = 2 \cdot \sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Co - function identities:

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

Periodicity identities:

$$\sin(x \pm 2\pi) = \sin x$$

$$\cos(x \pm 2\pi) = \cos x$$

$$\tan(x \pm \pi) = \tan x$$

$$\cot(x \pm \pi) = \cot x$$

$$\sec(x \pm 2\pi) = \sec x$$

$$\csc(x \pm 2\pi) = \csc x$$

Sum and difference formulas:

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Half - angle formulas:

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{(1 - \cos x)}{\sin x}$$

Law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

Area of triangle:

$$\frac{1}{2}ab\sin C$$

$$\sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2}(a+b+c)$