Physics 303 Final

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Signature ____________________________________

Questions (4 pts. apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. What is the ultimate fate of the Universe? Explain and cite observations and/or calculations to support your statement.

2. Approximately how big is the atomic nucleus?

3. What observations did Rutherford make to establish the size of the atomic nucleus?

4. What are the advantages, if any, of using the center-of-mass frame to study collisions?

5. Consider a particle of energy E undergoing one-dimensional motion with potential energy $V(x)$. Where are the 'allowed' and 'forbidden' regions of of the particle's motion? How does one locate them?

6. We have treated small oscillations about the equilibrium position of a particle as harmonic oscillations where the spring constant k is

$$k = \frac{d^2V(x_e)}{dx^2}$$

where x_e is the equilibrium position and $V(x)$ is the potential energy. When is this approximation accurate?

7. The gravitational potential due to a point mass m at a distance r is

$$\Phi(r) = -\frac{Gm}{r} \qquad .$$

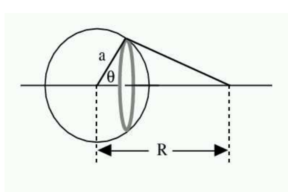
Consider a thin, spherical shell of total mass M and radius a and a ring within that shell of mass dM that is concentric with an axis going through the center of the shell (see figure). The surface mass density of the shell is

$$\sigma = \frac{M}{4\pi a^2} \qquad .$$

Show the contribution to the gravitational potential by the thin ring at a distance R from the center of the shell along the axis shown in the figure is

$$d\Phi(r) = \frac{GM}{2}\frac{\sin\theta d\theta}{r}$$

where r is the distance from the ring to the point along the axis.



Problems. Clearly show all reasoning for full credit. Use a separate sheet to show your work. See the last page for a table of integrals and constants.

1. 12 pts. In a collision with a nucleus of unknown mass, an α particle (a ^{4}He nucleus) scatters directly backwards and loses 64 percent of its original energy. If the scattering is elastic, then what is the mass of the nucleus?

2. 14 pts. What is the r dependence of the mass density $\rho(r)$ of a planet for which the gravitational force has constant magnitude throughout its interior?

3. 14 pts. Two atoms in a diatomic molecule with masses m_1 and m_2 interact through a potential energy

$$V(r) = \frac{a}{r^4} - \frac{b}{r^3}$$

where r is the separation of the atoms and a and b are positive constants. Assume circular orbits of angular momentum l for all the questions below.

   (a) If the atoms move in circular orbits, then what is the equilibrium separation of the two atoms in terms of the constants above?

   (b) What is the maximum equilibrium separation the atoms can have before breaking up? Your answer should be in terms of a, b, and μ.

   (b) What is the maximum angular momentum the molecule can have without breaking up? Your answer should be in terms of a, b, and μ.

4. 16 pts. Observations of a distant star's velocity reveal a sinusoidal 'wobble' with a period of 7.2 days and an amplitude of $40\ \mathrm{m/s}$. If the wobble is due to the motion of an unseen, planetary companion, then what is the mass of the unseen planet and what is the planet-star distance? The mass of the star is $M_s = 1.5 \times 10^{30} kg$.

See next page.

5. 16 pts. Consider an object falling from rest at $t = 0$ through a viscous medium (like the ill-fated Lieutenant Chisov of the Soviet air force). The friction force is found to be related to the velocity by the expression

$$F_f = c(e^{\lambda v} - 1)$$

where c and λ are positive constants and v is the velocity in the vertical direction.

(a) What is the differential equation that relates velocity and time?

(b) How are velocity v and time t related to one another? In other words, solve the differential equation from the previous part.

(c) What is the particular solution for the object's motion?

Hint: You might find the table of integrals below useful. Some of them differ from the indefinite integrals in Schaum's *Mathematical Handbook*.

Some Useful Integrals.

$$\int e^x dx = e^x \qquad \int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int e^{-x} dx = -e^{-x} \qquad \int \frac{dx}{(p+qe^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p+qe^{ax})} - \frac{1}{ap^2} \ln(p + qe^{ax})$$

$$\int \frac{dx}{1+e^x} = \ln\left(\frac{e^x}{1+e^x}\right) \qquad \int \frac{dx}{a+be^{px}} = \frac{x}{a} - \frac{1}{ap} \ln\left(|a + be^{px}|\right)$$

$$\int \frac{xe^x dx}{(1+x)^2} = \frac{e^x}{1+x} \qquad \int xe^{ax} dx = e^{ax}\left(\frac{x}{a} - \frac{1}{a^2}\right)$$

$$\int x^2 e^{ax} dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) \qquad \int \frac{dx}{pe^{ax}+qe^{-ax}} = \frac{1}{a\sqrt{pq}} \arctan\left(\sqrt{\frac{p}{q}} e^{ax}\right)$$

Some Useful Constants.

1 fm	10^{-15} m	Boltzmann's constant(k_B)	$1.38 \times 10^{-23} J/K$
1 Å	10^{-10} m	Avogadro's number (N_A)	6.023×10^{23}
$\mathrm{ke}^2 = \frac{\hbar c}{137}$	1.44 MeV − fm	Speed of light(c)	$3.0 \times 10^8\ m/s$
Coulomb's constant ($\frac{1}{4\pi\epsilon_0}$)	$8.99 \times 10^9 \frac{N-m^2}{kg^2}$	Electron mass	$9.11 \times 10^{-31}\ kg$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	Proton mass	$1.67 \times 10^{-27}\ kg$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} F/m$	1.0 eV	$1.6 \times 10^{-19}\ J$
1.0 u	$1.67 \times 10^{-27}\ kg$	Gravitational constant (G)	$6.67 \times 10^{-11}\ \frac{N-m^2}{kg^2}$