Physics 303

One-Dimensional Free Fall

1. In solving Newton's Second Law for free fall through a resistive medium we used the following expression to represent the drag force

$$\vec{F}_f = -\frac{1}{2} C_D \rho S v^2 \hat{v} \tag{1}$$

where C_D is the drag coefficient, ρ is the density, S is the cross-sectional area, v is velocity, and $\hat{v}$ is a unit vector in the direction of $\vec{v}$. We found that v and t are related by

$$\frac{1}{2v_t} \ln\left(\frac{v_t + v}{v_t - v}\right) = -\frac{g}{v_t^2} t \tag{2}$$

where

$$v_t = \frac{2mg}{C_D \rho S} \tag{3}$$

and m is the mass and g is the acceleration of gravity. Show that Equation 2 can be written in the following form.

$$v(t) = -v_t \left[\frac{1 - \exp(-\frac{2gt}{v_t})}{1 + \exp(-\frac{2gt}{v_t})}\right] \tag{4}$$

2. Show that Equation 4 can be written in the following form

$$v(t) = -v_t \tanh\left(\frac{gt}{v_t}\right) \tag{5}$$

where $\tanh x$ means the hyperbolic tangent of x defined in the following way.

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{6}$$

3. Apply a direct integration to Equation 5 to get the general solution for $y(t)$ and then use the initial conditions that at $t = 0$, $y = y_0$ to find the particular solution.

4. Use the solution from Problem 3 to show that time it takes for Lt. Chisov to hit the ground can be written in the following way

$$t_{hit} = \frac{v_t}{g} \text{ArcCosh}\left[\exp\left(\frac{g y_0}{v_t^2}\right)\right] \tag{7}$$

where ArcCosh$x$ means the inverse hyperbolic cosine of x.