Phil 251 - Elementary Symbolic Logic

Final Exam - Extra Derivations

For each of the following construct a proof demonstrating that the validity claim holds.

1. $\{(\forall x)(\sim Fx \vee \sim Gx), Fa\} \vdash \sim (\forall x)(\sim Fx \vee Gx)$

2. $\{(\forall x)(Fx \supset Gx), Fm \vee Fn\} \vdash (\exists x)Gx$

3. $\{(\forall x)(Gx \supset (\exists y) \sim Kxy), (\exists x)(Fx \& (\forall y)Kxy)\} \vdash (\exists x)(Fx \& \sim Gx)$

4. $\{(\forall x)[(Fx \equiv Hx) \supset Gx]\} \vdash (\forall x)(\sim Fx \& \sim Hx) \supset (\forall x)Gx$

5. $\{(\forall x)[Ax \supset (\forall y)(By \supset Cxy)], Am \& Bn\} \vdash Cmn$

6. $\{(\forall y)(My \supset Ay), (\exists y)(Cy \& My)\} \vdash (\exists y)(Cy \& Ay)$

7. $\{(\forall x)(Ax \vee Bx), (\forall x)(Bx \supset Ax)\} \vdash (\forall x)Ax$

8. $\vdash (\forall x)(\forall y)[(Fx \supset Fy) \equiv (Fy \vee \sim Fx)]$

9. $\{(\forall y)(Jy \supset (Ky \& Ly)), (\exists x) \sim Kx\} \vdash (\exists z) \sim Jz$

10. $\{(\exists x)Fx \supset (\exists x)(Bx \& Cx), (\exists x)(Cx \vee Dx) \supset (\forall x)Ax\} \vdash (\forall x)(Fx \supset Ax)$

11. $\{(\forall x)Fx \supset (\exists x)Gx, (\forall x) \sim Gx\} \vdash (\exists x) \sim Fx$

12. $\{(\forall x)(Ax \& \sim Bx) \supset (\exists x)Cx, \sim (\exists x)(Cx \vee Bx)\} \vdash \sim (\forall x)Ax$

13. $\{(\exists x)(\exists y)Rxy, (\forall x)(\forall y)(Rxy \supset (\forall z)Rxz), (\forall x)((\forall z)Rxz \supset (\forall y)Ryx)\} \vdash (\forall x)(\forall y)Rxy$

14. $\{(\forall x)Ax \equiv (\exists x)(Bx \& Cx), (\forall x)(Cx \supset Bx)\} \vdash (\forall x)Ax \equiv (\exists x)Cx$

15. $\{(\exists x) \sim Ax \vee (\exists x) \sim Bx, (\forall x)Bx\} \vdash \sim (\forall x)Ax$

16. $\vdash (\forall x)(Ax \& (\exists y) \sim Bxy) \equiv \sim (\exists x)[\sim Ax \vee (\forall y)(Bxy \& Bxy)]$

17. $\{(\forall x)[(Ax \& Bx) \supset Cx], \sim (\forall x)(Ax \supset Cx)\} \vdash \sim (\forall x)Bx$

18. $\{(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy, (\exists x)(\forall y) \sim Bxy\} \vdash (\exists x)(\forall y) \sim Axy$

19. $\{(\exists x)[Ax\&(\forall y)(By \supset Cxy)]\} \vdash (\forall x)[Bx \supset (\exists y)(Ay\&Cyx)]$

20. $\{(\exists x)[Ax\&(\forall y)(By \supset Cxy)], (\exists x)Ax \supset Bj\} \vdash (\exists x)Cxj$

21. $\{(\forall x)[(\exists y)Rxy \supset (\exists z) \sim Wz], (\exists y)(\exists z)(Ryz\&Hz), (\forall x)(\sim Hx \supset Wx)\} \vdash (\exists z)(\sim Wz\&Hz)$

22. $\vdash [(\exists x)Ax \supset ((\exists y)By \supset (\forall z)Cz)] \supset (\forall x)(\forall y)(\forall z)((Ax\&By) \supset Cz)$

23. $\{(\exists z)Qz \supset (\forall w)(Lww \supset\sim Hw), (\exists x)Bx \supset (\forall y)(Ay \supset Hy)\}$
$\vdash (\exists w)(Qw\&Bw) \supset (\forall y)(Lyy \supset\sim Ay)$

24. $\{(\forall x)(\forall y)((Ax\&By) \supset Cxy), (\exists y)(Ey\&(\forall w)(Hw \supset Cyw)), (\forall x)(\forall y)(\forall z)[(Cxy\&Cyz) \supset Cxz],$
$(\forall w)(Ew \supset Bw)\} \vdash (\forall z)(\forall w)[(Az\&Hw) \supset Czw]$