Distance of Closest Approach for `millepede`

The goal is to obtain an equation for the distance of closest approach (DOCA) between the track point on an SVT sensor and the nearest silicon strip. This result is used to calculate the derivatives needed for `millepede`.

Outline

1. Equation for a straight, cosmic-ray track from the `coatjava` reconstruction code.
2. Intersection of the cosmic-ray track and the sensor plane (track point).
3. Perpendicular distance from the track point to the silicon strip.

Cosmic-Ray Track Equation

To obtain the distance of closest approach between a straight, cosmic-ray track and a strip on the SVT start with the equation of the track from the reconstruction code.

$$\vec{V}_{trk} = (x_t, y_t, z_t) = (m_{yx}y + b_{yx}, y, m_{yz}y + b_{yz}) \tag{1}$$

where y is the independent variable. This expression can be rewritten in terms of the point on the track $\vec{l}_0$ which intersects the $y = 0$ plane and a unit vector in the direction of the track $\hat{l}$

$$\vec{l}_0 = (b_{yx}, 0, b_{yz}) \qquad \hat{l} = \frac{(m_{yx}, 1, m_{yz})}{\sqrt{m_{yx}^2 + 1 + m_{yz}^2}} \tag{2}$$

so the track can now be written as

$$\vec{V}_{trk}(d_{tl}) = d_{tl}\hat{l} + \vec{l}_0 \tag{3}$$

where d_{tl} is the distance along the track from the starting point at $\vec{l}_0$.

Track-Sensor Intersection

The relevant parameters in the plane transverse to the beam are shown in the figure.

The equation of a plane (the sensor) in three dimensions is
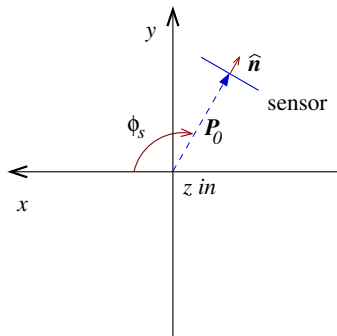
$$ax + by + cz + d = 0 \qquad (4)$$

with normal

$$\vec{n} = (a, b, c) \qquad (5)$$

which can be rewritten as

$$(\vec{P} - \vec{P}_0) \cdot \hat{n} = 0 \qquad (6)$$

where $\vec{P}$ is any vector, $\vec{P}_0$ is a vector to a point in the plane (see the figure), and $\hat{n}$ is a unit vector normal to the plane.

Track-Sensor Intersection

Now apply these ideas to the SVT. Let $\vec{P}_0$ be a vector of length R_l from the origin to the center of sensor. We want to use Equation 3 and find the value of d_{tl} for a point on the track that intersects the sensor so

$$(\vec{V}_{trk}(d_{tp}) - \vec{P}_0) \cdot \hat{n} = 0 \tag{7}$$

where d_{tp} is the distance from $\vec{l}_0$ to the plane/sensor. So

$$(d_{tp}\hat{l} + \vec{l}_0 - \vec{P}_0) \cdot \hat{n} = 0 \tag{8}$$

$$d_{tp}\hat{l} \cdot \hat{n} + (\vec{l}_0 - \vec{P}_0) \cdot \hat{n} = 0 \tag{9}$$

$$d_{tp} = -\frac{(\vec{l}_0 - \vec{P}_0)}{\hat{l} \cdot \hat{n}} \tag{10}$$

and

$$\vec{V}_{trk}(d_{tp}) = d_{tp}\hat{l} + \vec{l}_0 \quad \text{at the intersection with sensor} \tag{11}$$

We need to express the result in the appropriate parameters. In the ideal SVT geometry $\vec{P}_0 = (R_I \cos\phi_s, R_I \sin\phi_s, 0)$ so

$$\hat{n} = \frac{\vec{P}_0}{|\vec{P}_0|} = (\cos\phi_s, \sin\phi_s, 0) \tag{12}$$

Strip - Track Point Distance

We now focus on getting the perpendicular distance from the point where the cosmic-ray track intersects the plane (see Equations 2, 10, and 11) to the silicon strip. The endpoints of the strip are

$$\vec{s}_u = (x_u, y_u, z_u) \qquad \vec{s}_d = (x_d, y_d, z_d) \tag{13}$$

where the subscript u refers to the upstream endpoint and d refers to the downstream one. The equation for the strip line is

$$\vec{S}_s = d_{st} \frac{\vec{s}_u - \vec{s}_d}{|\vec{s}_u - \vec{s}_d|} + \vec{s}_u = d_{st}\hat{s} + \vec{s}_u \tag{14}$$

where $\vec{s}_u$ and $\vec{s}_d$ are known from the geometry, but d_{st} is, as yet, unknown. We go back to Equation 3 for $V_{trk}(d_{tl})$, the equation of the intersection of the track line with the sensor, and use the upstream endpoint $\vec{s}_u$ which lies in the plane. Using an expression from Ref. 1 for the vector from the intersection of the track line and the plane/sensor to a point on a line

$$D\vec{OCA} = (\vec{s}_u - \vec{V}_{trk}(d_{tp})) - ((\vec{s}_u - \vec{V}_{trk}(d_{tp})) \cdot \hat{s})\hat{s} \tag{15}$$

and we're done!

Getting the Sign

We have to assign a sign to the *DOCA* so the derivatives will behave properly. Start with a vector $\hat{d}$ which is a unit vector in the direction of $\vec{DOCA}$ so

$$\hat{d} = \frac{\vec{DOCA}}{|\vec{DOCA}|} \tag{16}$$

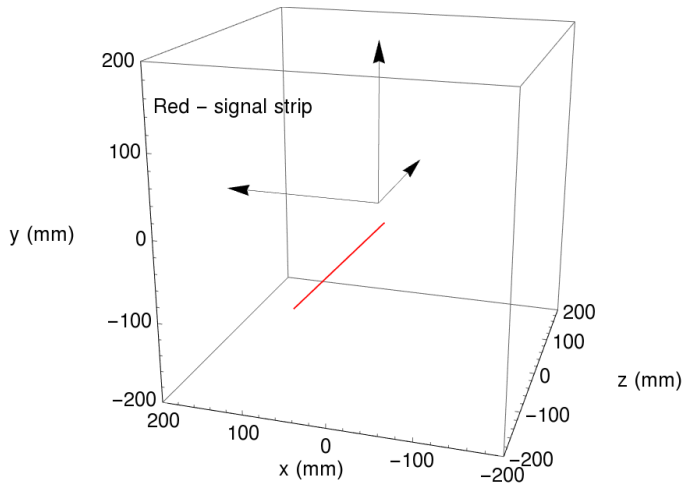Take the cross product of $\hat{d}$ with $\hat{s}$ (see Equation 14).

$$\hat{m} = \hat{d} \times \hat{s} \tag{17}$$

which is a vector perpendicular to the sensor (since $\hat{d}$ and $\hat{s}$ should both lie in the plane). Then take the dot product with $\hat{n}$
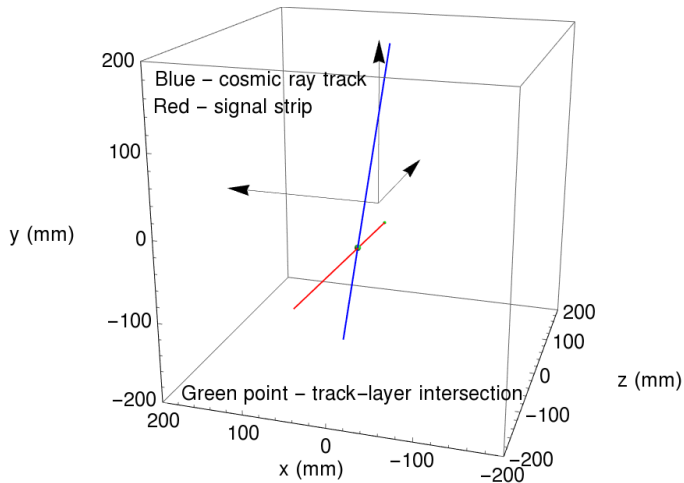
$$\hat{m} \cdot \hat{n} = \pm 1 \tag{18}$$

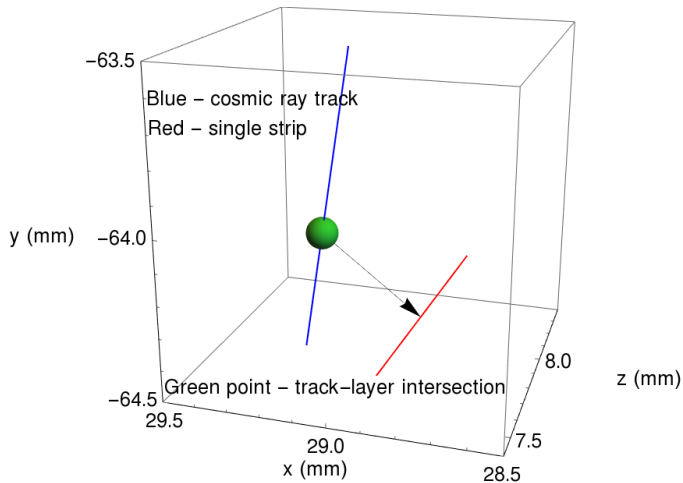and use this result to assign a sign.

Visualization/Validation

Visualization

Blue – cosmic ray track
Red – signal strip

Green point – track–layer intersection

y (mm)

x (mm)

z (mm)

References

1. 'Distance from a point to a line', Wikipedia, `https://en.wikipedia.org/wiki/Distance_from_a_point_to_a_line`
2. Weisstein, Eric W. 'Plane.' From MathWorld–A Wolfram Web Resource. `http://mathworld.wolfram.com/Plane.html`

Back-Up Slides

$$\frac{dDOCAQ}{dmyz}$$

DOCA - distance of closest approach, myz - slope in $y-z$ plane.

Back-Up Slides

The equation of a line can be given in vector form:

$$\mathbf{x} = \mathbf{a} + t\mathbf{n}$$

Here $\mathbf{a}$ is the position of a point on the line, and $\mathbf{n}$ is a unit vector in the direction of the line. Then as scalar t varies, $\mathbf{x}$ gives the locus of the line.

The distance of an arbitrary point $\mathbf{p}$ to this line is given by

$$\text{distance}(\mathbf{x} = \mathbf{a} + t\mathbf{n}, \mathbf{p}) = \|(\mathbf{a} - \mathbf{p}) - ((\mathbf{a} - \mathbf{p}) \cdot \mathbf{n})\mathbf{n}\|.$$

This formula can be derived as follows: $\mathbf{a} - \mathbf{p}$ is a vector from $\mathbf{p}$ to the point $\mathbf{a}$ on the line. Then $(\mathbf{a} - \mathbf{p}) \cdot \mathbf{n}$ is the projected length onto the line and so

$$((\mathbf{a} - \mathbf{p}) \cdot \mathbf{n})\mathbf{n}$$

is a vector that is the projection of $\mathbf{a} - \mathbf{p}$ onto the line. Thus

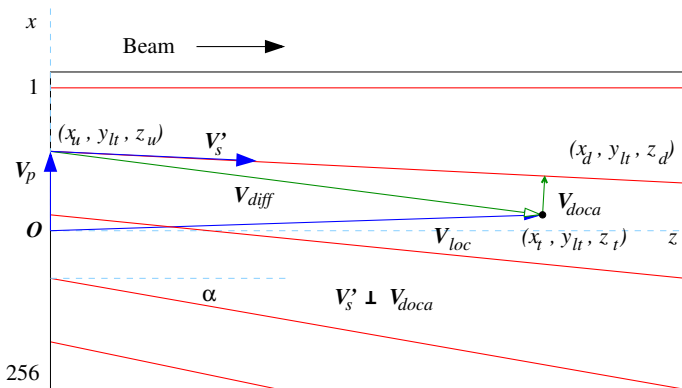$$(\mathbf{a} - \mathbf{p}) - ((\mathbf{a} - \mathbf{p}) \cdot \mathbf{n})\mathbf{n}$$

is the component of $\mathbf{a} - \mathbf{p}$ perpendicular to the line. The distance from the point to the line is then just the norm of that vector.[10] This more general formula is not restricted to two dimensions.

Illustration of the vector formulation.

Relating equation for $\vec{DOCA}$ on wiki to this problem.

$$\vec{a} = \vec{s}_u \qquad t = d_{st}$$
$$\hat{n} = \hat{s} \qquad \vec{p} = \vec{V}_{trk}(d_{tp})$$
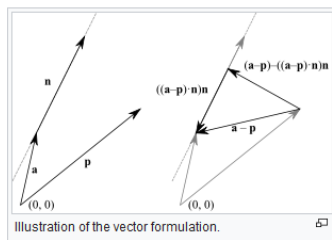
Distance of Closest Approach

$\vec{V}_p$ - origin $\rightarrow$ upstream endpoint

$\vec{V}_{loc}$ - origin $\rightarrow$ track point on module

$\vec{V}_{doca}$ - track point $\rightarrow$ strip ($\vec{V}_{doca} \perp \vec{V}_s'$)

$\vec{V}_s'$ - unit vector along strip

$\vec{V}_{diff} = \vec{V}_{loc} - \vec{V}_p$