# Connecting the Schwinger Parameter $\Delta E$ to $v_{cut}$ in EXCLURAD

In the Schwinger method one calculates the radiative correction for the scattering of an electron in a Coulomb field. This corresponds to inclusive electron scattering. An essential step in the calculation is to integrate over the radiative tail of the energy of a scattered electron to arrive at a correction factor for the yield lost to the emission of photons. The parameters of that integration are defined in Figure 1.[1] The parameter $\Delta E$ is the energy range

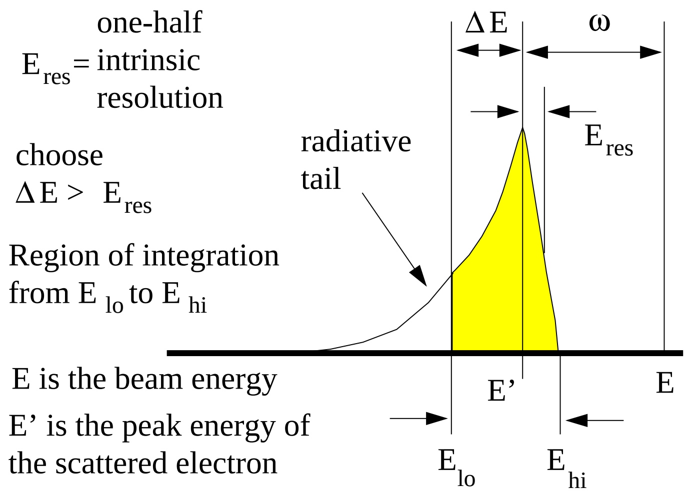


Figure 1: Energy spectrum of scattered electron showing definitions of quantities used in Schwinger radiative correction calculation.

over which the integral is performed (starting at the unradiated energy of the electron) to estimate the yield lost to radiated photons.

Afanasev, *at al.* follow an analogous procedure in their more sophisticated approach.[2] They integrate over the radiative tail of the scattered electron, but they perform the integration in terms of the covariant 'inelasticity' $v$ defined as

$$v = \Lambda^2 - m_u^2 \tag{1}$$

where $m_u$ is the mass of the undetected hadron and $\Lambda$ is the four-momentum of the missing or undetected particles. The quantity $v$ can be rewritten as

$$v = W^2 + m_h^2 - m_u^2 - 2WE_h \tag{2}$$

where $W$ is the mass of the system recoiling against the proton, $m_h$ is the mass of the detected hadron, and $E_h$ is the center-of-mass energy of the detected hadron. To determine the relationship between $\Delta E$ and $v$ consider the usual expression for $W^2$

$$W^2 = M^2 + 2M(E - E') - Q^2 \tag{3}$$

where

$$Q^2 \approx 4EE' \sin^2 \frac{\theta}{2} \tag{4}$$

$M$ is the target mass, and $\theta$ is the electron scattering angle. However, for an event with a radiated photon, the measured energy of the scattered electron is not $E'$, but some lower energy

$$E_{lo} = E' - \Delta E \tag{5}$$

so $W$ for this event will not be 'correct'. The new value of $W$ is

$$W_{rad}^2 = M^2 + 2M(E - E_{lo}) - 4EE_{lo} \sin^2 \frac{\theta}{2} \tag{6}$$

Using Equations 5 and 6 in the expression for $v$ in Equation 2 one obtains the following function of $\Delta E$.

$$v = M^2 + 2M(E - E' + \Delta E) - 4E(E' + \Delta E)\sin^2\frac{\theta}{2} + m_h^2 - m_u^2 - 2E_h\sqrt{M^2 + 2M(E - E' + \Delta E) - 4E(E' + \Delta E)\sin^2\frac{\theta}{2}} \quad (7)$$

This expression can be re-arranged so

$$v = W_0^2 + m_h^2 - m_u^2 + 2\Delta E(M + 2E\sin^2\frac{\theta}{2}) - 2E_h\sqrt{W_0^2 + 2\Delta E(M + 2E\sin^2\frac{\theta}{2})} \quad (8)$$

where

$$W_0^2 = M^2 + 2M(E - E') - 4EE'\sin^2\frac{\theta}{2} \quad (9)$$

and the quantities $E$, $E'$, and $\theta$ are determined by the electron kinematics. The hadron energy $E_h$ is determined by the choice of the angle of the outgoing hadron relative to $\vec{q}$, the three-vector of the momentum transfer. The masses $M$, $m_h$, and $m_u$ are all known.

As an example of applying Equation 7 consider the following kinematics. The results of the calculation are

| | | |
|---|---|---|
| $E = 2.558\ GeV$ | $E' = 2.345\ GeV$ | $\theta = 14.84°$ |
| $m_h = 0.938\ GeV$ | $m_u = 0.940\ GeV$ | $\theta_h^{cm} = 45°$ |
| $M = 1.876\ GeV$ | $Q^2 = 0.52\ (GeV/c)^2$ | $W = 1.93\ GeV$ |

Table 1: Kinematics for calculating $v(\Delta E)$.

shown below.

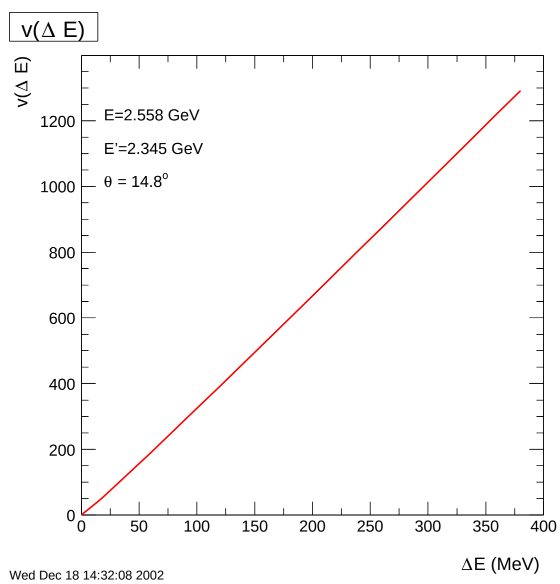


Figure 2: Dependence of $v$ on $\Delta E$ for the kinematics listed in Table 1.

## References

[1] Juan Cornejo, *radiative corrections, external and internal bremsstrahlung factors*, Retrieved on January 30, 2003 from http://www.calstatela.edu/academic/nuclear_physics/schwin12_extbrems.html.

[2] A.Afanasev, I.Akushevich, V.Burkert, and K.Joo, Phys.Rev., **D66**, 074004, 2002.