2.2 The Einstein Model of a Solid

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Physics 3P41

The 2-state paramagnet

- Where is the 2-state model used?
- An example: 2-state paramagnet
- If you apply a magnetic field to a solid, the unpaired electrons will tend to align in that field (nuclei have very small magnetic moments too, but we'll ignore them for now).
- These electronic moments, in the absence of a field, interact only weakly with themselves in most cases (exception: ferromagnets, where the moments can align even without a field. These are the magnets in compasses and on your fridge).
- Now, these dipoles can point in any direction in a solid. For simplicity, let's assume they can only align with a field, or against a field (sometimes called the <u>Ising model</u>, see Chap. 8). So we have:

B $\big\uparrow$ $\quad \uparrow\ \uparrow\ \downarrow\ \uparrow\ \downarrow\ \uparrow\ \uparrow$ (etc.)

What is the multiplicity of the no. of up states ($N_\uparrow$) and the no. of down states ($N_\downarrow$)?

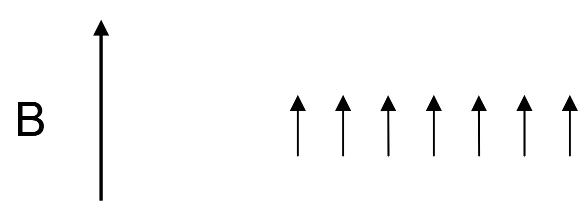
The 2-state paramagnet

- $N = N_{\uparrow} + N_{\downarrow}$

No of up states   No. of down states

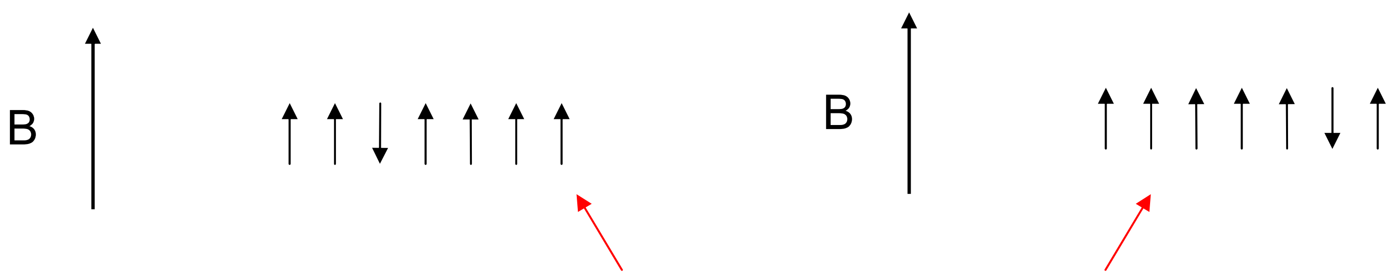
$$\Omega(N_{\uparrow}) = \binom{N}{N_{\uparrow}} = \frac{N!}{N_{\uparrow}!(N - N_{\uparrow})!} = \frac{N!}{N_{\uparrow}!\,N_{\downarrow}!}$$

- The total energy of this system is defined as the total no. of up and down dipoles, so saying what the <u>macrostate</u> the system is in is the same as specifying its <u>total energy</u>.
- Eg. The lowest energy state – all the spins align in a field



The 2-state paramagnet

- Since it costs energy to flip a spin, higher energy states are defined by the no. of spins pointing in the opposite direction. But it doesn't matter <u>which</u> spin you flip.

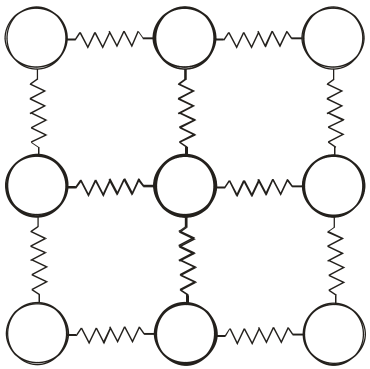


These 2 states have the same energy – order doesn't count

- So, knowing the multiplicity is important, and knowing the macrostate (eg. how many spins are flipped) is important. Macrostates typically <u>characterize</u> a system (and define properties such as the energy)

2.2 The Einstein Model of a Solid

- Einstein made many contributions to statistical physics
- Problem (1907): In a solid, there are atoms held together by spring-like forces:



- Each atom is a harmonic oscillator (for now, it doesn't matter which direction it vibrates in)
- Potential energy: $\frac{1}{2} k_s x^2$

Spring constant

Displacement

The Einstein Model of a Solid

- Quantum Mechanics tells us that the levels are quantized:

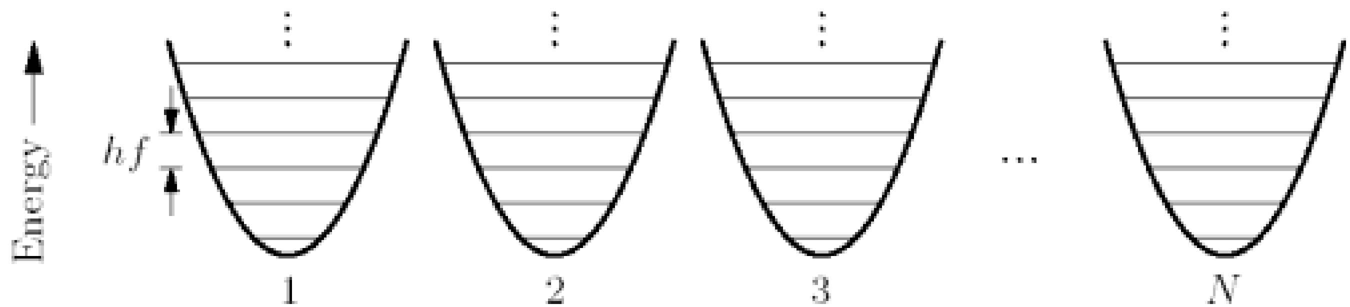


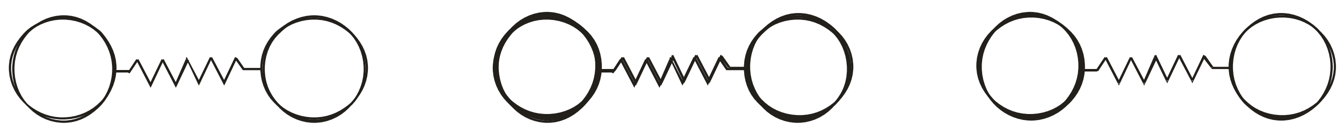
Figure 2.2. In quantum mechanics, any system with a quadratic potential energy function has evenly spaced energy levels separated in energy by hf, where f is the classical oscillation frequency. An Einstein solid is a collection of N such oscillators, all with the same frequency.

Planck's constant

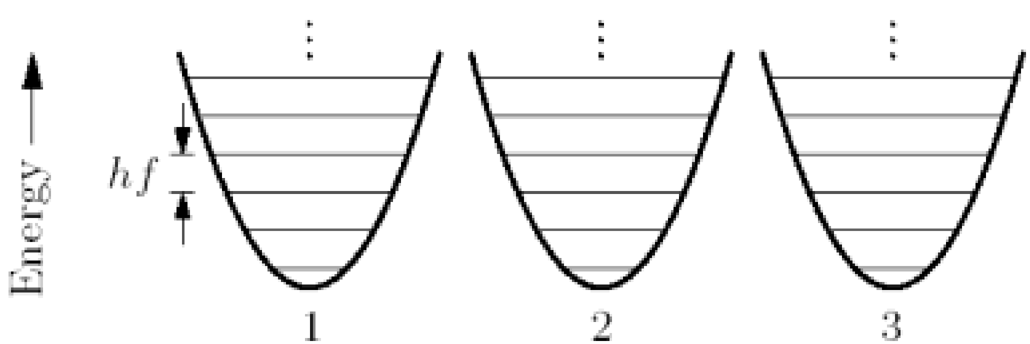
- The energy levels are quantized in units of E = hf
- This is the picture we have for N oscillators in a solid of N atoms. Other examples of this: diatomic gases such as N_2 or H_2 (which each act like little oscillators)

The Einstein Model of a Solid

- This system is called (as you may have guessed) an Einstein solid.
- It is easiest to think about this as a system of 1D oscillators



- Let’s look at a simple example: N = 3 oscillators



$E_n = (n + 1/2)\,hf$
$(n = 0, 1, 2, \ldots)$

The Einstein Model of a Solid

- Let's look at the total no. of states per total energy:

Oscillator	1	2	3	Total Energy	
Energy State	0	0	0	0	$\Omega(0) = 1$
	1	0	0	1	
	0	1	0	1	$\Omega(1) = 3$
	0	0	1	1	
	2	0	0	2	
	0	2	0	2	
	0	0	2	2	$\Omega(2) = 6$
	1	1	0	2	
	1	0	1	2	
	0	1	1	2	

The Einstein Model of a Solid

Oscillator	1	2	3	Total Energy
Energy State	3	0	0	3
	0	3	0	3
	0	0	3	3
	2	1	0	3
	2	0	1	3
	1	2	0	3
	0	2	1	3
	1	0	2	3
	0	1	2	3
	1	1	1	3

$\Omega(3) = 10$

The Einstein Model of a Solid

- Notice a pattern:

$$\Omega(3) = \frac{(3+3-1)!}{3!2!} = \frac{5!}{3 \bullet 2 \bullet 2} = 10$$

$$\Omega(2) = \frac{(2+3-1)!}{2!2!} = \frac{4!}{2 \bullet 2} = 6$$

$$\Omega(1) = \frac{(1+3-1)!}{1!2!} = \frac{3!}{2} = 3$$

$$\Omega(0) = \frac{(1+3-1)!}{1!1!} = 1$$

- So, for N oscillators, with total energy q:

$$\Omega(N,q) = \binom{q+N-1}{q} = \frac{(N+q-1)!}{q!(N-1)!}$$

(Homework: Prove this!)
See page 55 of text.