Investigative Physics
Module 1: Activity Units for Physics 131

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Abstract

The exercises in this manual have been developed to support an investigative physics course that emphasizes active learning. Some of these units have been taken from the Workshop Physics project at Dickinson College and the Tools for Scientific Thinking project at Tufts University and modified for use at the University of Richmond. Others have been developed locally.

The units are made up of activities designed to guide your investigations in the laboratory. The written work will consist primarily of documenting your class activities by filling in the entries in the spaces provided in the units. The entries consist of observations, derivations, calculations, and answers to questions. Although you may use the same data and graphs as your partner(s) and discuss concepts with your classmates, all entries should reflect your own understanding of the concepts and the meaning of the data and graphs you are presenting. Thus, each entry should be written in your own words. Indeed, it is very important to your success in this course that your entries reflect a sound understanding of the phenomena you are observing and analyzing.

We wish to acknowledge the support we have received for this project from the University of Richmond and the Instrumentation and Laboratory Improvement program of the National Science Foundation. Also, we would like to thank our laboratory directors for their invaluable technical assistance.

Contents

1 Measuring and Graphing Horizontal Motion[1]

Name ______________________ Section ______________ Date ______________

Objectives

- To explore the nature of horizontal motion.
- To learn to use Excel for graphing and fitting data.

Measuring the Horizontal Motion of a Bowling Ball

A key to understanding how to describe motion near the surface of the earth is to observe horizontal motions and vertical motions separately. Eventually, situations in which an object undergoes both horizontal and vertical motion can be analyzed and understood as a combination of these two kinds of basic motions.

Lets start with horizontal motion. How do you think the horizontal position of a bowling ball changes over time as it rolls along on a smooth surface? For example, suppose you were to roll the ball a distance of 6.0 meters on a fairly level smooth floor. Do you expect that the ball would: (1) move at a steady speed, (2) speed up, or (3) slow down? To observe the horizontal motion of a "bowling ball" you can use a bocce ball which is slightly smaller but quite similar to a regulation bowling ball.

Apparatus

- A bocce ball
- 3 stop watches
- A 2-meter stick
- Masking tape for marking distances
- A smooth level surface ($>$ 7 meters in length)

Activity 1: Horizontal Motion

(a) What do you predict will happen to the position of the ball as a function of time? Will the ball move at a steady speed, speed up, or slow down after it leaves the bowler's hand? Why?

(b) Find a 7 meter length of smooth floor and use masking tape to mark off a starting line and distances of 2.00 m, 4.00 m, and 6.00 m. from the starting line. Then: (1) Bowl the ball along the surface. (2) Measure the time it takes to travel 2.0 m, 4.0 m & 6.0 m. (3) Record the results in the table below.

Note: This is a cooperative project. You will need a bowler, three people to measure the times, and someone to stop the ball. Practice several times before recording data in the table below.

t (s)	x (m)
0.00	0.00

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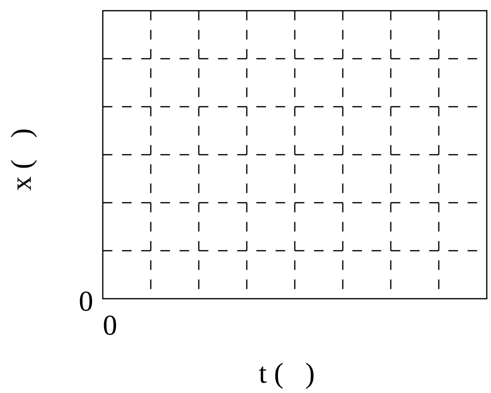
(c) Calculate the average speed, v, in m/s of the bowling ball as it travels the 6.00 meters.

Graphing the Horizontal Motion

In this activity, you should graph your data for the position of the ball as a function of the rolling-time of the ball. This graphing should be done both by hand and on the computer.

Activity 2: Drawing a Graph of Position vs. Time

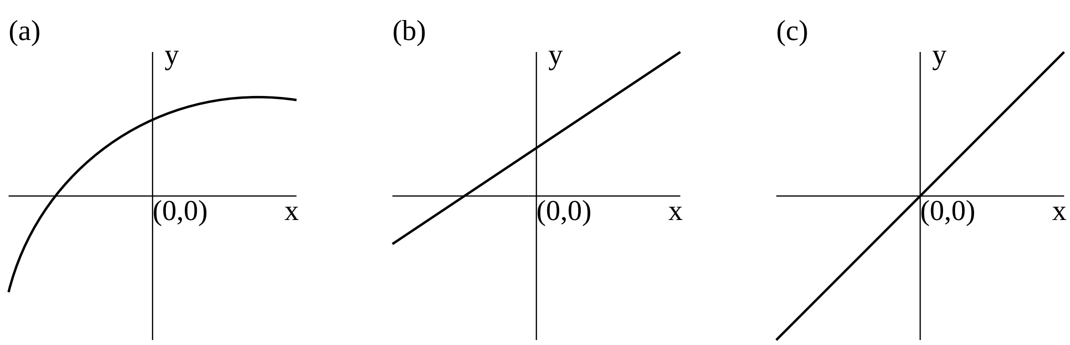
(a) Fill in the units and the scale numbers in the graph below and plot the data you collected. You should plot a "fourth" data point on your graph by reasoning out what rolling-time you would measure for your ball if you were to roll it a distance of zero meters.



(b) Now use Excel on the computer to create the same graph. See Appendix C for instructions. Print the graph and insert a copy in your notebook at the end of this unit.

How does the Position of the Ball Vary with Time?

We are interested in the mathematical nature of the relationship between position and time for rolling on a level surface. Some definitions of mathematical relationships are shown in the sketches below. Figure (a) shows a function y that increases with x so that $y = f(x)$. In sketch (b) y is a linear function that increases with x so that $y = mx + b$, where m is the slope and b is the y intercept. In figure (c) y is proportional to x ($y = mx$ where $b = 0$).



Activity 3: The Mathematical Relationships

(a) By comparing the shape of the graph you have just produced with the sketches shown above, would you say that the position, x, increases with time, t? Decreases with time? Is it a linear function of t? Is it proportional to t? Explain.

(b) How do the results compare with the prediction you made in Activity 1? Are you surprised?

(c) What do you think would happen to the slope, m, of the graph, if the ball had been rolled faster? Would it increase? Decrease? Stay the same?

Activity 4: Mathematical Modeling

(a) Create a mathematical model of the bowling ball motion data you collected in Activity 1. This can be done by using Excel to fit the data with a line. See Appendix C for instructions. Be sure to label the graph. Then print it and insert a copy into your notebook. Does the line provide a good description of the data?

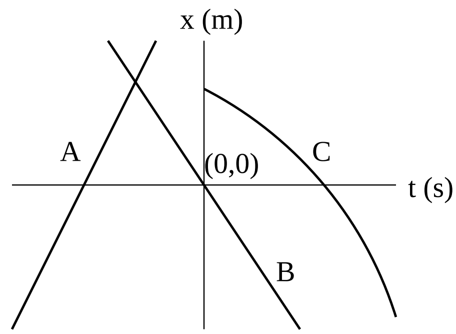
(b) Write the equation describing the motion in the form Position (m) = (? m/s) Time (s) + (? m).

(c) Compare the slope of the line with the average speed you calculated in Activity 1. A good way to do this is by calculating the % difference between the two (See Appendix A).

(d) Give a brief discussion of the meaning of the slope of a graph of Position vs. Time. What does it tell you?

Homework

The diagram below shows the graphs of three possible relationships between the time, t, in seconds and the position, x, in centimeters that the object has traveled.



(a) Which graphs represent position as a linear function of time? A, B, and/or C?

(b) Which graphs, if any, show position as proportional to time?

2 Walking Speed

Name ______________________ Section ____________ Date ____________

Objectives

- Introduction to measuring length and time.
- Introduction to metric units of length.

Apparatus

- stop watch
- 2-meter stick

Activity

(a) Use the stop watch and meter stick to determine your walking speed in m/s.

walking speed ______________________

(b) Describe the technique you used to perform the measurement and show the calculation of walking speed.

(c) Use the stop watch and meter stick to determine your partner's walking speed in m/s.

walking speed ______________________ How does this compare with his/her result?

Question

What sort of unit is speed (fundamental or derived)?

3 Converting Units

Name ______________________ Section ___________ Date ___________

Objectives:

- Introduction to units
- Exploration of relations between different systems of measurement

Apparatus:

- string
- scissors
- meter stick

Activity:

1. Cut the string into three pieces, 1-meter, 1-foot, and 30-cm, respectively.
2. Compare them. Enter into the table below how many lengths of one piece it requires to make each of the others.
3. Cut a length of string equivalent to your anatomical foot; compare this piece to the others.

		equal one of these?			
	pieces	1-meter	1-foot	30-cm	anatomical foot
How many of these	1-meter				
	1-foot				
	30-cm				
	anatomical foot				

4. Devise your own measurement system; name it; select length, mass, and time standards and choose names and abbreviations for them; and define these.

system name ______________________________

length: name ___________ abbreviation _____ definition ________________
mass: name ___________ abbreviation _____ definition ________________
time: name ___________ abbreviation _____ definition ________________

5. Compare your measurement system to the Standard International (MKSA) system.

1 meter = ___ ___________s
1 kilogram = ___ ___________s
1 second = ___ ___________s
1 ___________ = ___ meters
1 ___________ = ___ kilograms
1 ___________ = ___ seconds

Questions:

1. Are anatomical units practical for measurement? Explain.

2. What criteria should be satisfied by a good clock?

3. Can you explain the nearly universal use of the metric system in scientific work?

4 Measurement and Uncertainty[2]

Name ______________________ Section ____________ Date ____________

Objectives

- To learn to measure length using a meter stick and a vernier caliper.
- To learn to express the results of measurements with the appropriate number of significant figures.
- To learn how to compensate for systematic error in measurements so that accuracy can be improved.

Measuring Lengths and Significant Figures

We are interested in determining the number of significant figures in length measurements you might make. How is the number of significant figures determined? Suppose God could tell us that the "true" width of a certain car key in centimeters was:

2.435789345646754456540123544332975774281245623... etc.

If we were to measure the key width with a ruler that is lying around the lab, the precision of our measurement would be limited by the fact that the ruler only has lines marked on it every 0.1 cm. We could estimate to the nearest 1/100th of a centimeter how far the key edge is from the last mark. Thus, we might agree that the best estimate for the width of the key is 2.44 cm. This means we have estimated the key width to three significant figures.

If God announces that the width of a pair of sun glasses is 13.27655457787654267787... cm, then upon direct measurement we might estimate the width to be 13.28 or 13.27 or 13.26 cm. In this case the estimated width is four significant figures. Obviously, there is uncertainty about the "true" value of the right-most digit.

The number of significant figures in a measurement is given by the number of digits from the most certain digit on the left of the number up to and including the first uncertain digit on the right. In reporting a number, all digits except the significant digits should be dropped. (See the discussion of significant figures in Appendix A.)

Let's do some length measurements to find out what factors might influence the number of significant figures in a measurement.

Apparatus

- A meter stick
- A vernier caliper
- A rectangular board

Activity 1: Length Measurements with the Meter Stick

(a) What factors might make a determination of the "true" length of an object measured with the meter stick uncertain?

(b) Measure the width of the board with the meter stick at least seven times and create a table in the space below to list the measurements. For best results, you should use different regions of the meter stick so that, when an average of these measurements is made, non-uniformities in the scale will tend to cancel. Also, you

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should avoid using the end of the meter stick which might be worn and, therefore, would not be a true zero. The apparent change in the reading on the scale due to the position of the eye is called parallax, an effect which can introduce error into the reading. To reduce the uncertainty due to parallax, you should place the meter stick on edge so that the scale is close to the object being measured. (See Appendix E)

(c) In general, when a series of measurements is made, the best estimate is the average of those measurements. In the space below list the minimum measurement, the maximum measurement, and the best estimate for the width of your board.

(d) Based on these measurements, write the width of your board as a value plus or minus an "uncertainty".

(e) How many significant figures should you report in your best estimate? Why?

(f) For your board, what limits the number of significant figures most - variation in the actual width of the board or limitations in the accuracy of the meter stick? How do you know?

Activity 2: Length Measurements with the Vernier Caliper

(a) Measure the thickness of the board at least seven times and record the results in a table in the space below. Make the measurements at different places along each of the two edges. If you have questions about how to read the vernier, see Appendix E. If you still have questions, consult your instructor.

(b) Record the minimum, maximum, and average of your measurements below.

(c) Based on these measurements, write the thickness of your board as a value plus or minus an uncertainty.

(d) How many significant figures should you report in your average? Why?

Activity 3: Calculation of Cross-Sectional Area

(a) Calculate the cross-sectional area of the board in the space below.

(b) Calculate the uncertainty in the area, ΔA, as follows:

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta w}{w}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

(c) Write the cross-sectional area as a value plus or minus an uncertainty, rounding off as appropriate.

(d) How many significant figures should you report in your result? Why?

The Inevitability of Uncertainty

In common terminology there are three kinds of "errors": (1) mistakes or human errors, (2) systematic errors due to measurement or equipment problems and (3) inherent uncertainties.

Activity 4: Error Types

(a) Give an example of how a person could make a "mistake" or "human error" while taking a length measurement.

(b) Give an example of how a systematic error could occur because of the condition of the meter stick when a set of length measurements are being made.

(c) What might cause inherent uncertainties in a length measurement?

5 Measurement of Length, Mass, Volume, and Density

Name ______________________ Section __________ Date __________

Objectives:

- Learn to measure length with the vernier caliper and mass with the platform balance
- Apply knowledge of units and significant figures
- Understand the dependence of mass and density on dimensions

Apparatus:

- vernier caliper
- platform balance
- set of wooden disks

Activity:

1. Find the dimensions in centimeters of each of the disks using the vernier caliper (see Appendix E). Use the averages of three trials for each dimension (diameter, D; width, W) in the calculations of the volumes ($V = \pi r^2 W$).

2. Find the mass, M, of each disk using the laboratory balance.

3. Calculate the density, ρ.

disk	D_1 (cm)	D_2 (cm)	D_3 (cm)	W_1 (cm)	W_2 (cm)	W_3 (cm)	D (cm)	W (cm)	V (cc)	M (g)	ρ (g/cc)
1											
2											
3											
4											
5											

4. Graph mass versus radius and mass versus radius squared.

Questions:

1. How does the density depend on the size of a disk?

2. What is the nature of the relationship between mass and radius? What is the dependency?

3. What is the smallest part of a centimeter that can be read or estimated with a meter stick? With a vernier caliper? Which reading is more reliable? Explain.

4. When determining the volume of a disk, which dimension, diameter or width, should be measured more carefully? Explain.

5. What is the volume of the largest disk in cubic millimeters? In liters? What is its mass in kilograms?

6 Determining π

Name ______________________ Section ____________ Date ____________

Objectives:

- Determine the relationship between circumference and diameter
- Understand the meaning of π

Apparatus:

- Wooden disks
- Meter stick
- String

Activity:

1. With a meter stick, measure the diameter of one of the disks. Enter the value (or an average of several such measurements) in the table below.
2. With a string and meter stick, measure the circumference of the same disk, and enter the value (or average of several measurements) in the table below.
3. Repeat the previous steps 1. and 2. for each of the disks.

disk	diameter (cm)	circumference (cm)
1		
2		
3		
4		
5		

4. Graph circumference versus diameter using your measurement data.
5. Fit data and determine the slope of the resulting line.

slope ______________________

Questions:

1. What quantity is identified with the slope of the graph?

2. Must the line go through the origin? Explain.

3.If the diameter of the largest disk increased by a factor of 2.7, by how much would its circumference change?

4. If you formed a circle with a string 15 mm shorter than the circumference of the smallest disk, its diameter would be how much smaller than the disk's?

7 Measurement Uncertainty and Variation[3]

Name ______________________________ Section _______________ Date _______________

Objectives

- To explore the mathematical meaning of the standard deviation and standard error associated with a set of measurements.
- To investigate random and systematic variations associated with a set of measurements.

Statistics - The Inevitability of Uncertainty

With care and attention, it is commonly believed that both mistakes and systematic errors can be eliminated completely. However, inherent uncertainties do not result from mistakes or errors. Instead, they can be attributed in part to the impossibility of building measuring equipment that is precise to an infinite number of significant figures. The ruler provides us with an example of this. It can be made better and better, but it always has an ultimate limit of precision.

Another cause of inherent uncertainties is the large number of random variations affecting any phenomenon being studied. For instance, if you repeatedly drop a baseball from the level of the lab table and measure the time of each fall, the measurements will most probably not all be the same. Even if the stop watch was gated electronically so as to be as precise as possible, there would be small fluctuations in the flow of currents through the circuits as a result of random thermal motion of atoms and molecules that make up the wires and circuit elements. This could change the stop watch reading from measurement to measurement. The sweaty palm of the experimenter could cause the ball to stick to the hand for an extra fraction of a second, slight air currents in the room could change the ball's time of fall, vibrations could cause the floor to oscillate up and down an imperceptible distance, and so on.

Repeated Time-of-Fall Data

In the first two activities, you and your partners will take repeated data on the time of fall of a ball and study how the data varies from some average value for the time-of-fall.

Apparatus

- A ball
- A stop watch
- A 2-meter stick

Activity 1: Timing a Falling Ball

(a) Drop the ball so it falls through a height of exactly 2.0 m at least 20 times in rapid succession and measure the time of fall. Be as exact as possible about the height from which you drop the ball. Record the data in a table below.

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(b) Enter your data as a single column in Excel and find the average (mean) of the data. See Appendix C for instructions. Report the mean value in the space below.

The Standard Deviation as a Measure of Uncertainty

How certain are we that the average fall-time determined in the previous activity is accurate? The average of a number of measurements does not tell the whole story. If all the times you measured were the same, the average would seem to be very precise. If each of the measurements varied from the others by a large amount, we would be less certain of the meaning of the average time. We need criteria for determining the certainty of our data. Statisticians often use a quantity called the standard deviation as a measure of the level of uncertainty in data. The standard deviation is usually represented by the Greek letter σ (sigma). A customary way of expressing an experimentally determined value is: Mean $\pm\ \sigma$. The formal mathematical definition of σ can be found in Appendix A.

In the next activity you will use Excel (see Appendix C) to calculate the value of the standard deviation for the repeated fall-time data you just obtained and explore how the standard deviation is related to variation in your data. In particular, you will try to answer this question: What percentage of your data lies within one standard deviation of the average you calculated?

Activity 2: Standard Deviation

(a) Report the value for the standard deviation of your data in the space below.

(b) Calculate the average plus the standard deviation, $\langle t \rangle + \sigma$, and the average minus the standard deviation, $\langle t \rangle - \sigma$, and record the results in the space below.

(c) Determine the number of your data points that lie within $\pm\sigma$ of the average and write the result in the space below. Also, calculate the percentage of data points lying within a standard deviation of the average and report that result.

(d) Combine your results with those obtained by the other groups in the class and create a table in the space below with the following column headings: Lab Station, $\langle t \rangle$ (s), σ (s), %Data $\pm\sigma$.

(e) Study the last column, which represents the percentage of data points lying within one standard deviation of the average. What does the standard deviation, σ, tell you about the approximate probability that another measurement will lie within $\pm\sigma$ of the average?

Systematic Error - How About the Accuracy of Your Timing Device and Timing Methods?

As the result of problems with your measuring instrument or the procedures you are using, each of your measurements may tend to be consistently too high or too low. If this is the case, you probably have a source of systematic error. There are several types of systematic error.

Most of us have set a watch or clock only to see it gain or lose a certain amount of time each day or week. In ordinary language we would say that such a time keeping device is inaccurate. In scientific terms, we would say that it is subject to systematic error. In the case of a stopwatch or digital timer that doesn't run continuously like a clock, we have to ask an additional set of questions. Does it start up immediately? Does it stop exactly when the event is over? Is there some delay in the start and stop time? A delay in starting or stopping a timer could also cause systematic error.

Finally, systematic error can be present as a result of the methods you and your partner are using for making the measurement. For example, are you starting the timer exactly at the beginning of the event being measured and stopping it exactly at the end? Are you dropping the ball from a little above the exact starting point each time? A little below?

It is possible to correct for systematic error if you can quantify it. Suppose that God, who is a theoretical physicist, said that the distance in meters, y, that a ball falls after a time of t seconds near the earth's surface in most places is given by the equation

$$y = \frac{1}{2}gt^2$$

where g is the gravitational constant [equal to 9.8 (m/s)/s]. (In this idealized equation the effects of air resistance have been neglected.)

In the activity that follows, you should compare your average time-of-fall with that expected by theory to determine if a systematic error exists.

Activity 3: Is There Systematic Error in the Data?

(a) Calculate the theoretical, God given, time-of-fall in the space below.

(b) Does the theoretical value lie in the range of your own average value with its associated uncertainty? If not, you probably have a source of systematic error.

(c) If you seem to have systematic error, explain whether the measured times tend to be too short or too long and list some of the possible causes of it in the space below.

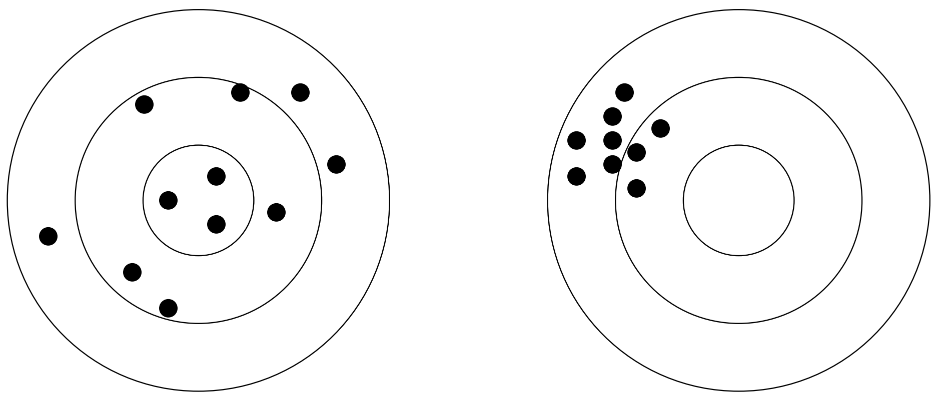
Homework

1. Suppose you made the following five length measurements of the width of a piece of $8\frac{1}{2}'' \times 11''$ paper which has been cut carefully by a manufacturer using an unfamiliar centimeter rule: 21.33 cm, 21.52 cm, 21.47 cm, 21.21 cm, 21.45 cm. (a) Find the mean and standard deviation of the measurements. The formal mathematical definition of standard deviation is given in Appendix A.

(b) Is there any evidence of uncertainty in the measurements or are they precise? Explain.

(c) Is there any evidence of systematic error in the measurements? If so, what might cause this? Explain.

2. Suppose Ashley and Ryan each throw darts at targets as shown below. Each of them is trying very hard to hit the bulls eye each time. Discuss in essay form which of the two students has the least amount of random error associated with his or her throws and is thus more precise. Is one of the students less accurate is the sense of having a systematic error associated with his or her throws? What factors like eyesight and coordination might cause one to be more precise and another more accurate?



Ashley Ryan

8 Position vs. Time Graphs[4]

Name ______________________ Section ____________ Date ____________

Objectives

- To learn about two of the ways that physicists can describe motion in one dimension-words and graphs.
- To learn how to relate graphs of position vs. time to the motions they represent.

Introduction

The focus of this unit on kinematics is to be able to describe your position as a function of time using words and graphs. You will use a motion detector attached to a computer in the laboratory to learn to describe one-dimensional motion.

The ultrasonic motion detector sends out a series of sound pulses that are of too high a frequency to hear. These pulses reflect from objects in the vicinity of the motion detector and some of the sound energy returns to the detector. The computer is able to record the time it takes for reflected sound waves to return to the detector and then, by knowing the speed of sound in air, figure out how far away the reflecting object is. There are several things to watch out for when using a motion detector. (1) Do not get closer than 0.15 meters from the detector because it cannot record reflected pulses which come back too soon. (2) The ultrasonic waves come out in a cone of about 15°. It will see the closest object. Be sure there is a clear path between the object whose motion you want to track and the motion detector. (3) The motion detector is very sensitive and will detect slight motions. You can try to glide smoothly along the floor, but don't be surprised to see small bumps in velocity graphs. (4) Some objects like bulky sweaters are good sound absorbers and may not be "seen" well by a motion detector. You may want to hold a book or a board in front of you if you have loose clothing on.

Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (Position Graphs application)
- Wooden board
- Masking tape for marking distances

Position vs. Time Graphs of Your Motion

The purpose of this unit is to learn how to relate graphs of position as a function of time to the motions they represent. How does a position vs. time graph look when you move slowly? Quickly? What happens when you move toward the motion detector? Away? After completing the next few activities, you should be able to look at a position vs. time graph and describe the motion of the object. You should also be able to look at the motion of an object and sketch a graph representing that motion.

Note that the motion detector measures the distance of an object from the detector, and that the motion detector is located at the origin of each graph. It is common to refer to the distance of an object from some origin as the position of the object. Therefore, it is better to refer to these graphs as position vs. time graphs than distance vs. time graphs.

You will use the *DataStudio* software to do the following activities. An application has been created for this purpose. Launch the **Position Graphs** application from the **131 Workshop** submenu, which is in the **Programs** section of the **Start** menu. To start a data run, click the **Start** button. To stop a data run, click the

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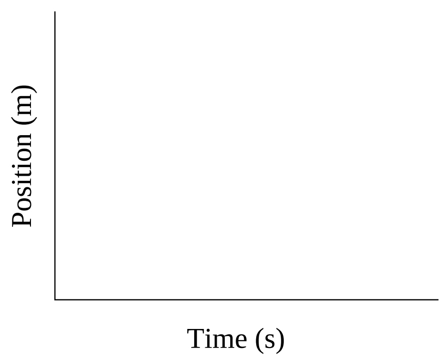
Stop button. After a data run, the graph can be expanded by clicking on the **Scale to fit** button in the upper left corner of the Graph window. Multiple data sets can be displayed on the same graph. Data can be removed from the graph by selecting **Delete Last Data Run** or **Delete All Data Runs** from the **Experiment** menu. When you are finished with the activities, choose **Quit** from the **File** menu and do not save this activity.

Before you begin the activities, you should mark a position scale on the floor. To do this, position one person at approximately 1 meter in front of the motion detector and take data for 1 second. The computer will display a horizontal line showing the position measured by the detector. The person standing in front of the detector should then adjust his/her position and the procedure repeated until the 1 meter position is established. Mark the 1 meter position on the floor with a piece of masking tape and then mark the 2, 3 and 4 meter positions using the 2 meter stick.

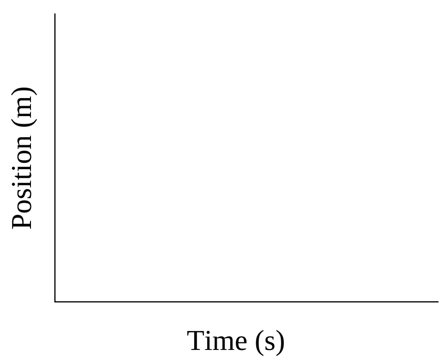
Activity 1: Making Position vs. Time Graphs

Make position-time graphs for the following motions and sketch the graph you observe in each case:

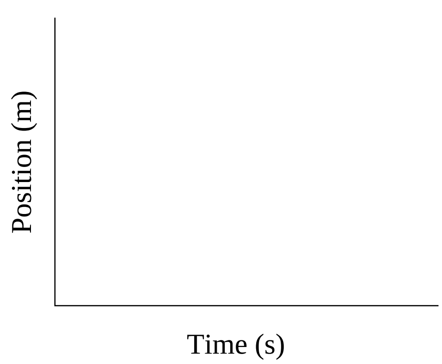
(a) Starting at 0.5 m, walk away from the origin (i.e., the detector) slowly and steadily.



(b) Walk away from the origin medium-fast and steadily.



(c) Walk toward the detector (origin) slowly and steadily.

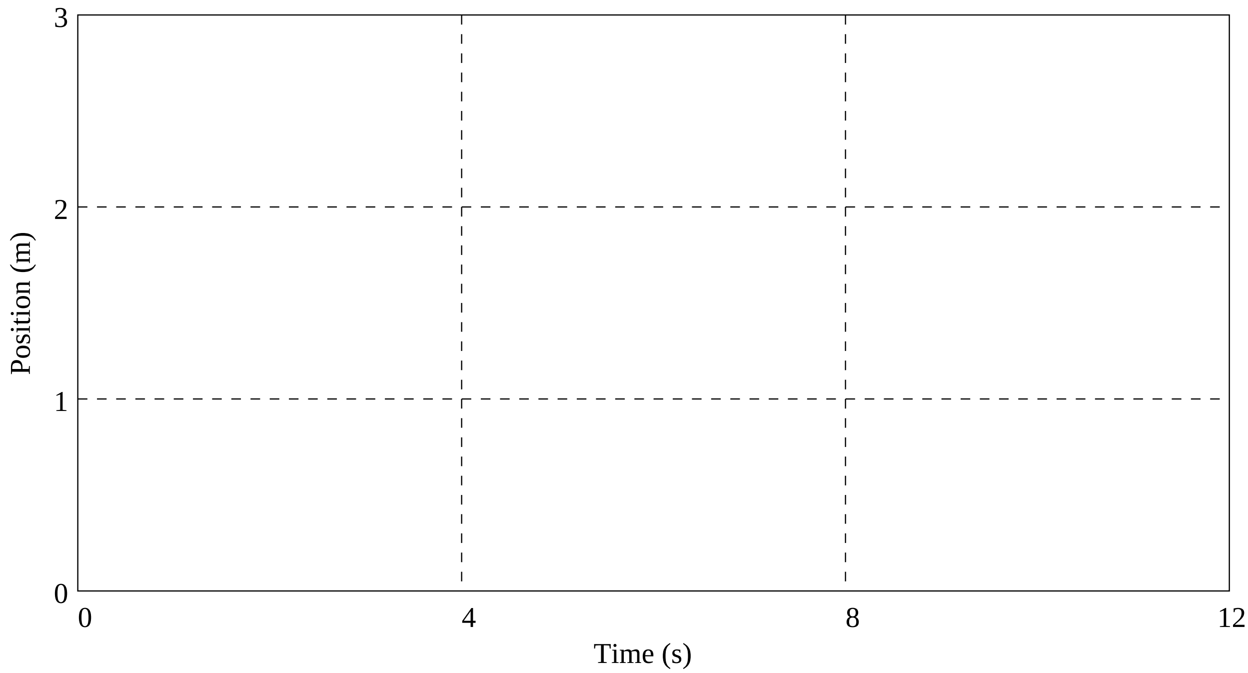


(d) Describe the difference between the graph you made by walking away slowly and the one made by walking away more quickly.

(e) Describe the difference between the graph made by walking toward and the one made walking away from the motion detector.

Activity 2: Predicting a Position vs. Time Graph

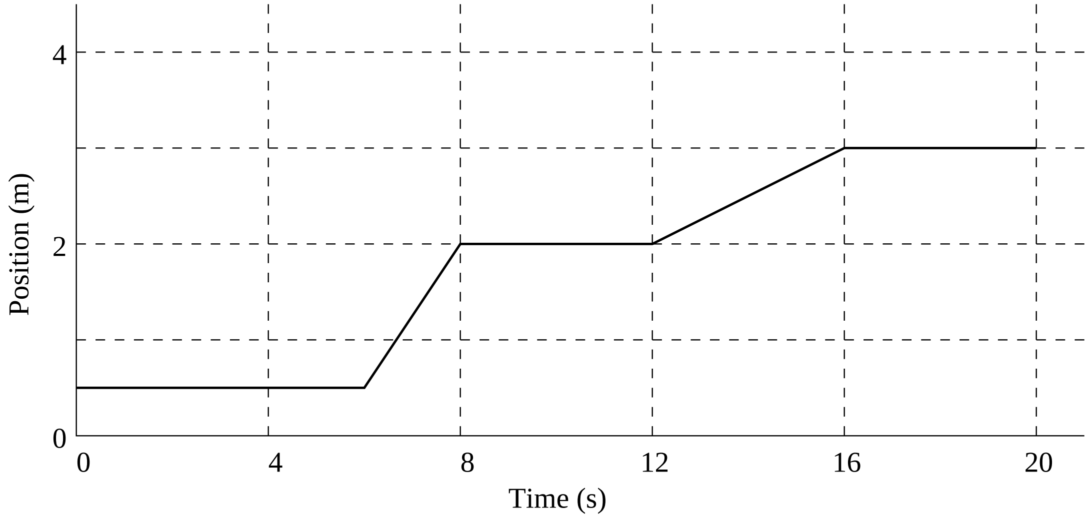
(a) Suppose your were to start 1.0 m in front of the detector and walk away slowly and steadily for 4 seconds, stop for 4 seconds, and then walk toward the detector quickly. Sketch your prediction on the axes below using a dashed line.



(b) Test your prediction by moving in the way described and making a graph of your motion with the motion detector. Sketch the trace of your actual motion on the above graph with a solid line.

(c) Is your prediction the same as the final result? If not, describe how you would move to make a graph that looks like your prediction.

Activity 3: Matching Position vs. Time Graphs

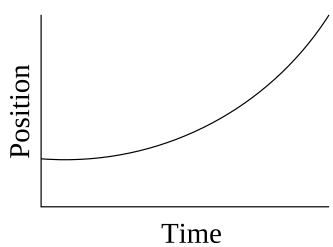


(a) Describe in your own words how you would move in order to match the graph shown above.

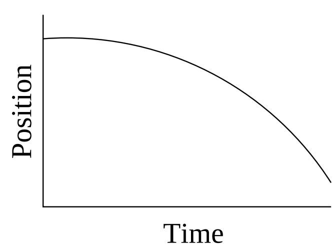
(b) Move to match the above graph on the computer screen. You may try a number of times. It helps to work as a team. Get the times right. Get the positions right. Do this for yourself. (Each person in your group should do his or her own match.) You will not learn very much by just watching!

(c) What was the difference in the way you moved to produce the two differently sloped parts of the graph you just matched?

(d) Make curved position vs. time graphs like those shown below.



Graph 1



Graph 2

(e) Describe how you must move to produce a position vs. time graph with each of the shapes shown.

Graph 1 answer:

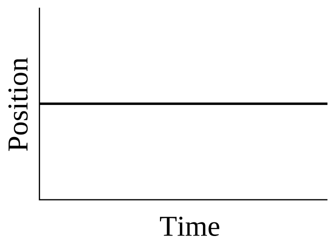
Graph 2 answer:

(f) What is the general difference between motions which result in a straight-line position vs. time graph and those that result in a curved-line position vs. time graph?

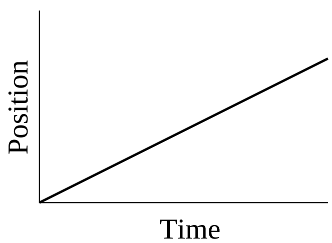
Homework

Answer the following questions in the spaces provided.

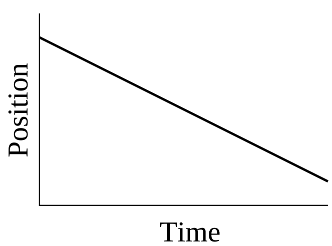
1.What do you do to create a horizontal line on a position-time graph?



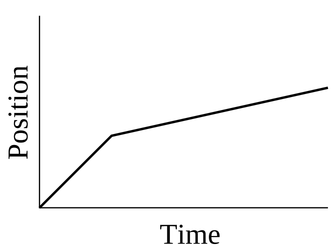
2. How do you walk to create a straight line that slopes up?



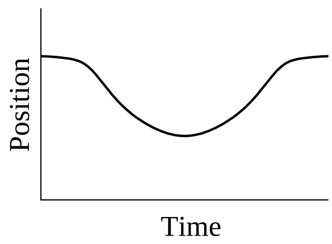
3. How do you walk to create a straight line that slopes down?



4. How do you move so the graph goes up steeply at first, then continues up gradually?



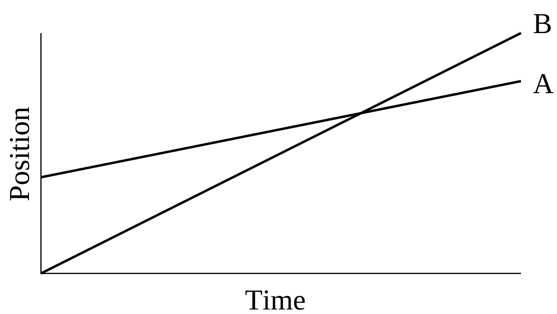
5. How do you walk to create a U-shaped graph?



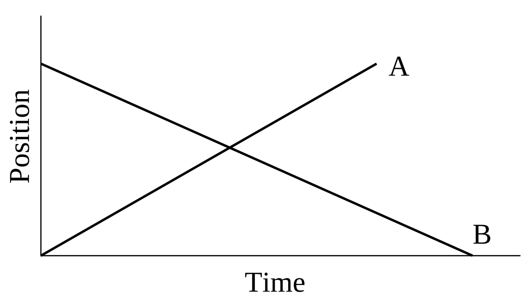
Answer the following about the two objects , A and B, whose motion produced the following position-time graphs.

6. (a) Which object is moving faster? (b) Which starts ahead? Define what you mean by "ahead."

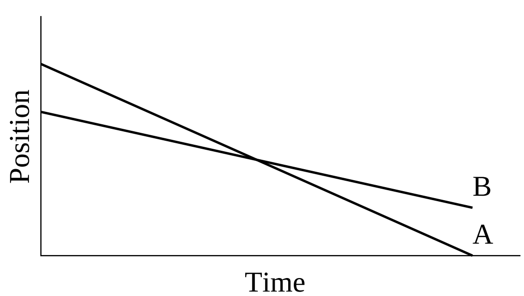
(c) What does the intersection mean?



7. (a) Which object is moving faster? (b) Which object has a negative velocity according to the convention we have established?

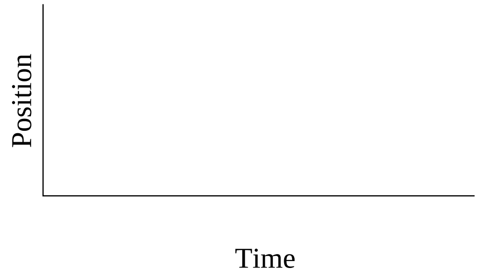


8. (a) Which object is moving faster? (b) Which starts ahead? Define what you mean by "ahead."

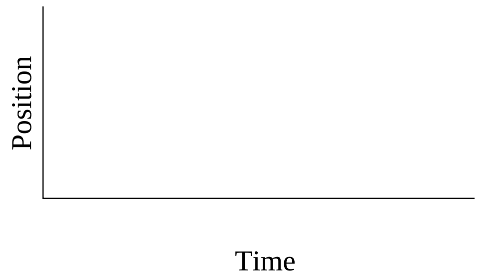


Sketch the position-time graph corresponding to each of the following descriptions of the motion of an object.

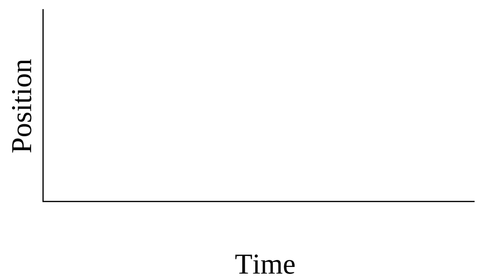
9. The object moves with a steady (constant) velocity away from the origin.



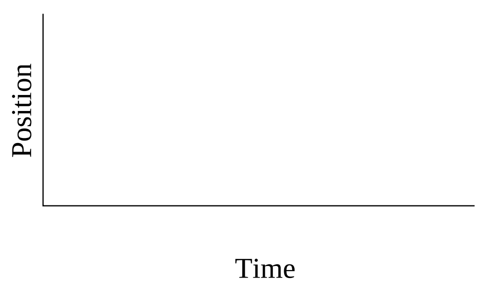
10. The object is standing still.



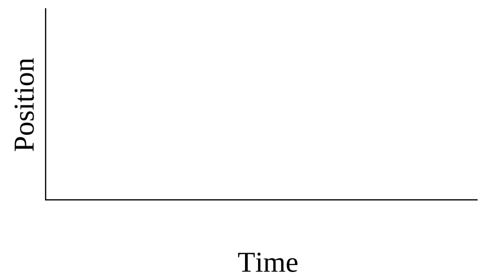
11. The object moves with a steady (constant) velocity toward the origin for 5 seconds and stands still for 5 seconds.



12. The object moves with a steady velocity away from the origin for 5 seconds, then reverses direction and moves at the same speed toward the origin for 5 seconds.



13. The object moves away from the origin, starting slowly and speeding up.


Position
Time

9 Velocity vs. Time Graphs[5]

Name ______________________________ Section _______________ Date _______________

Objectives

- To acquire an intuitive understanding of speed and velocity in one dimension.
- To learn how to relate graphs of velocity vs. time to the motions they represent.

Introduction

You have already plotted your position as a function of time. Another way to represent your motion during an interval of time is with a graph which describes how fast and in what direction you are moving from moment to moment. How fast you move is known as your speed. It is the rate of change of position with respect to time. Velocity is a quantity which takes into account your speed and the direction you are moving. Thus, when you examine the motion of an object moving along a line, its velocity can be positive or negative depending on whether the object is moving in the positive or negative direction.

Graphs of velocity over time are more challenging to create and interpret than those for position. A good way to learn to interpret them is to create and examine velocity vs. time graphs of your own body motions, as you will do in the next few activities.

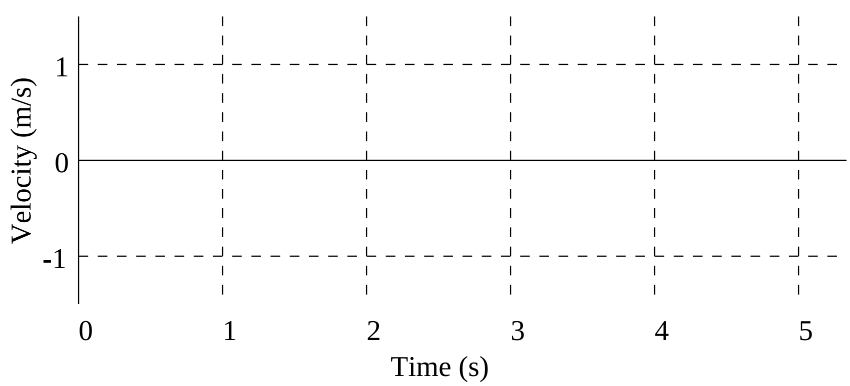
Use the **Velocity Graphs** application in the **131 Workshop** submenu (in the **Programs** section of the **Start** menu) to make the graphs in the following activities.

Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (Velocity Graphs application)
- Wooden board

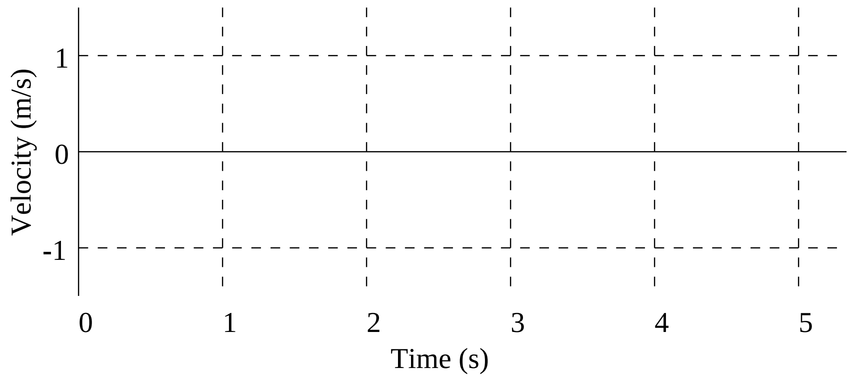
Activity 1: Making Velocity vs. Time Graphs

(a) Make a velocity graph by walking away from the detector slowly and steadily. Try again until you get a graph you're satisfied with and then sketch your result on the graph that follows. (We suggest you draw smooth patterns by ignoring smaller bumps that are mostly due to your steps.)

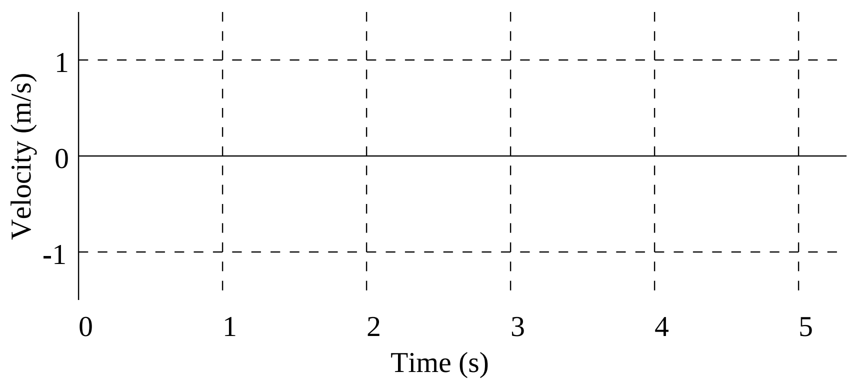


[5] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) Make a velocity graph, walking away from the detector steadily at a medium speed. Sketch your graph below.



(c) Make a velocity graph, walking toward the detector slowly and steadily. Sketch your graph below.



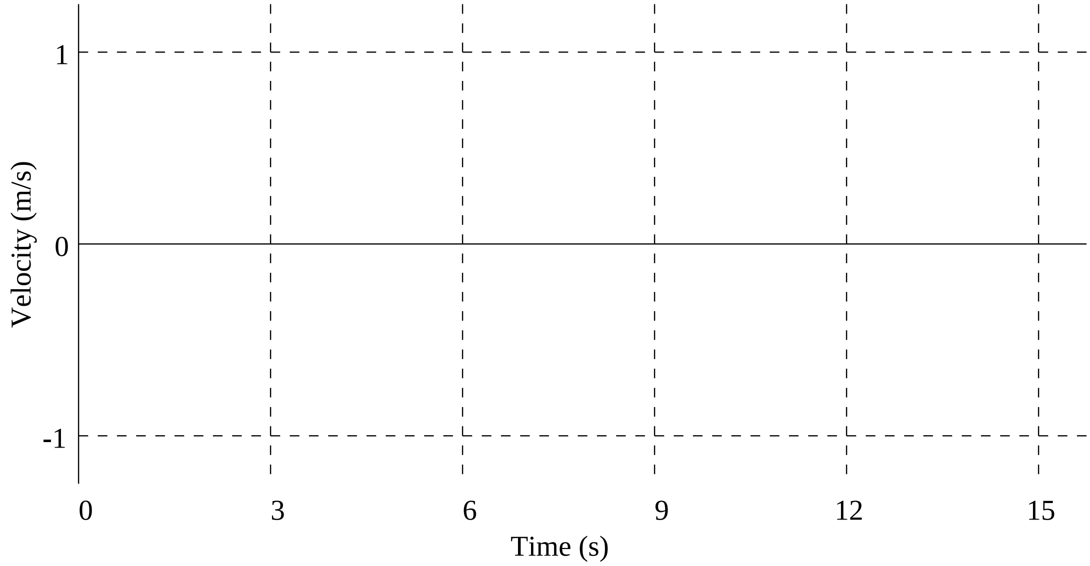
(d) What is the most important difference between the graph made by slowly walking away from the detector and the one made by walking away more quickly?

(e) How are the velocity vs. time graphs different for motion away and motion toward the detector?

Activity 2: Predicting a Velocity vs. Time Graph

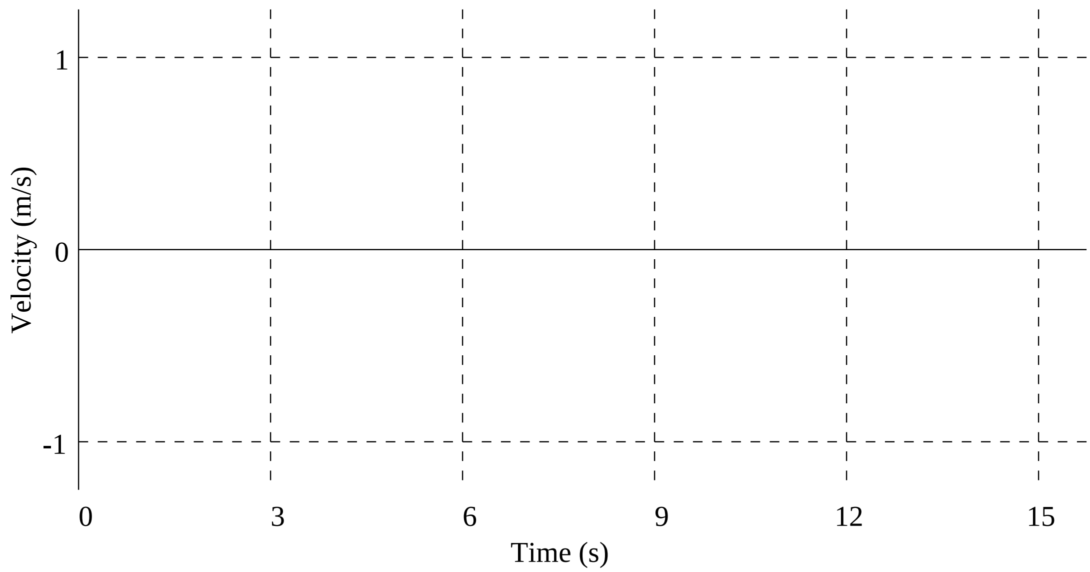
Suppose you were to undergo the following sequence of motions: (1) walk away from the detector slowly and steadily for 6 seconds, (2) stand still for 6 seconds, (3) walk toward the detector steadily about twice as fast as before.

(a) Use a dashed line in the graph that follows to record your prediction of the shape of the velocity graph that will result from the motion described above.



(b) Compare predictions with your partner(s) and see if you can all agree. Use a solid line to sketch your group prediction in the graph above.

(c) Adjust the sampling time to 15 s and then test your prediction. Repeat your motion until you are confident that it matches the description in words and then draw the actual graph on the axes below. Be sure the 6-second stop shows clearly.



(d) Did your prediction match your real motion? If not, what misunderstanding of what elements of the graph did you have?

Velocity Vectors

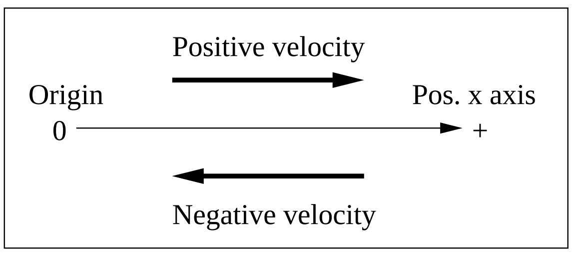
The two ideas of speed and direction can be combined and represented by vectors. A velocity vector is represented by an arrow pointing in the direction of motion. The length of the arrow is drawn proportional to the speed;

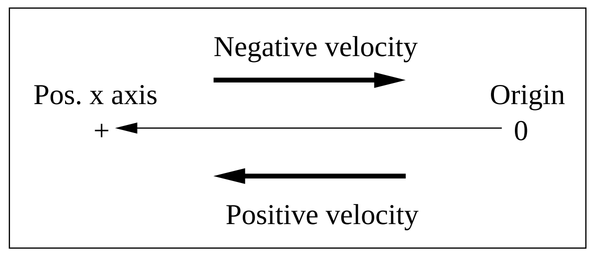
the longer the arrow, the larger the speed. If you are moving toward the right, your velocity vector can be represented by the arrow shown below.

If you were moving twice as fast toward the right, the arrow representing your velocity vector would look like:

while moving twice as fast toward the left would be represented by the following arrow:

What is the relationship between a one-dimensional velocity vector and the sign of velocity? This depends on the way you choose to set the positive x-axis.





In both diagrams the top vectors represent velocity toward the right. In the left diagram, the x-axis has been drawn so that the positive x-direction is toward the right. Thus the top arrow represents positive velocity. However, in the right diagram, the positive x-direction is toward the left. Thus the top arrow represents negative velocity. Likewise, in both diagrams the bottom arrows represent velocity toward the left. In the left diagram this is negative velocity, and in the right diagram it is positive velocity.

Activity 3: Sketching Velocity Vectors

Sketch below velocity vectors representing the three parts of the motion described in the prediction you made in Activity 2.

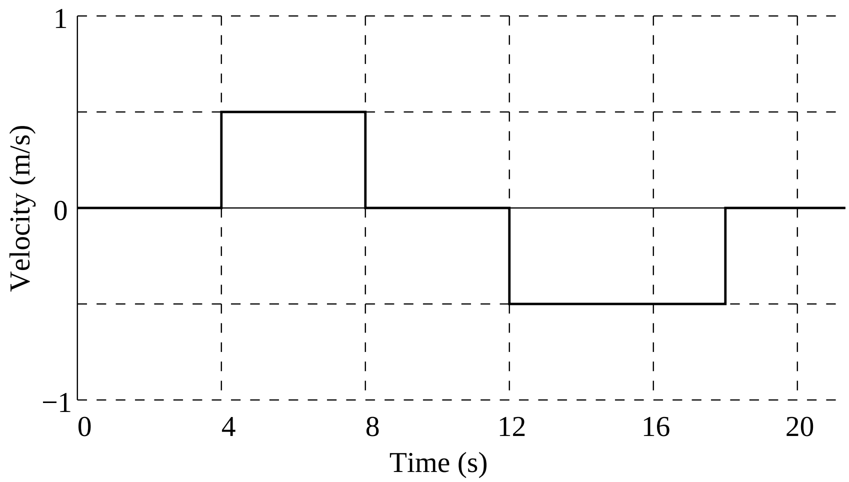
(a)Walking slowly away from the detector:

(b) Standing still:

(c) Walking rapidly toward the detector:

Activity 4: Matching a Velocity Graph

(a) Describe how you think you will have to move in order to match the velocity graph shown below.



(b) Move in such a way that you can reproduce the graph shown. You may have to practice a number of times to get the movements right. Work as a team and plan your movements. Get the times right. Get the velocities right. You and each person in your group should take a turn. Then sketch your group's best match on the above graph.

(c) Describe how you moved to match each part of the graph.

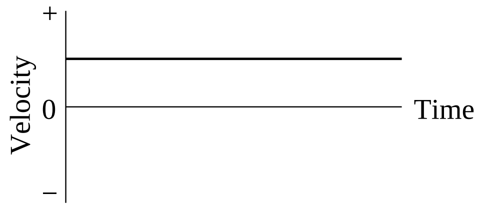
(d) Is it possible for an object to move so that it produces an absolutely vertical line on a velocity-time graph? Explain.

(e) Did you run into the motion detector on your return trip? If so, why did this happen? How did you solve the problem? Does a velocity graph tell you where to start? Explain.

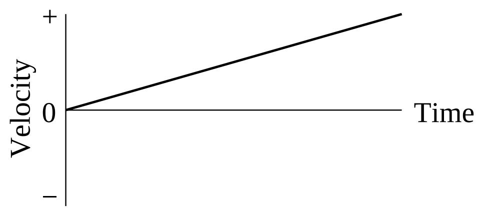
Homework

Answer the following questions in the spaces provided.

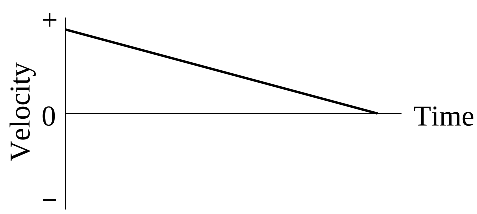
1. How do you move to create a horizontal line in the positive part of a velocity-time graph, as shown below?



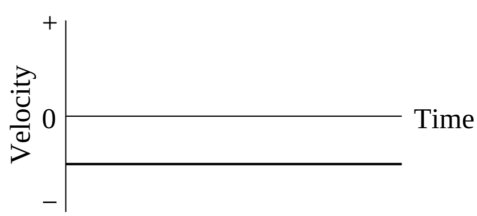
2. How do you move to create a straight-line velocity-time graph that slopes up from zero, as shown below?



3. How do you move to create a straight-line velocity-time graph that slopes down, as shown below?

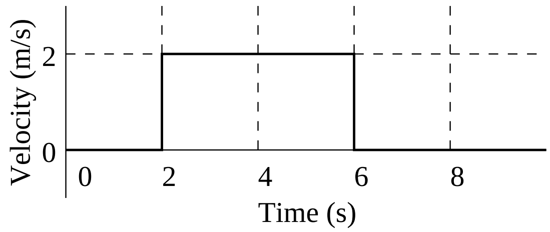


4. How do you move to make a horizontal line in the negative part of a velocity-time graph, as shown below?

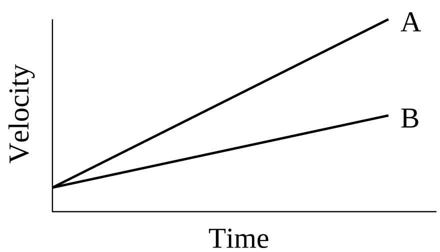


5. The velocity-time graph of an object is shown below. Figure out the total change in position (displacement) of the object. Show your work.

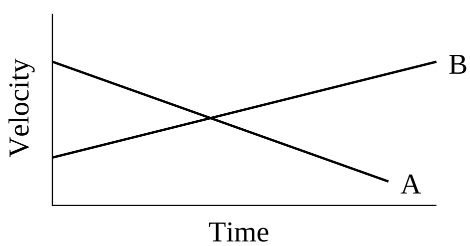
Displacement = _____________ meters.



6. The velocity graph below shows the motion of two objects, A and B. Answer the following questions separately. Explain your answers when necessary. (a) Is one faster than the other? If so,which one is faster? (A or B) (b) What does the intersection mean? (c) Can one tell which object is “ahead”? (define “ahead”) (d) Does either object A or B reverse direction? Explain.



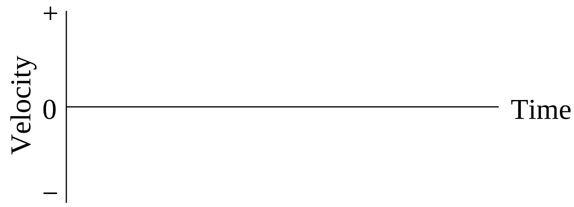
7. The velocity graph below shows the motion of two objects, A and B. Answer the following questions separately. Explain your answers when necessary. (a) Is one faster than the other? If so,which one is faster? (A or B) (b) What does the intersection mean? (c) Can one tell which object is “ahead”? (define “ahead”) (d) Does either object A or B reverse direction? Explain.



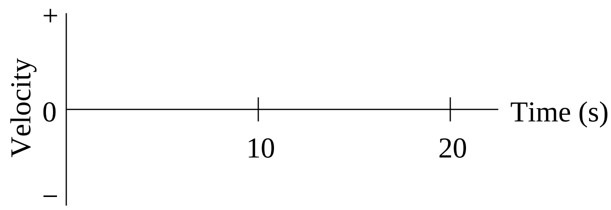
Sketch the velocity-time graph corresponding to each of the following descriptions of the motion of an object.

8. The object is moving away from the origin at a constant velocity.

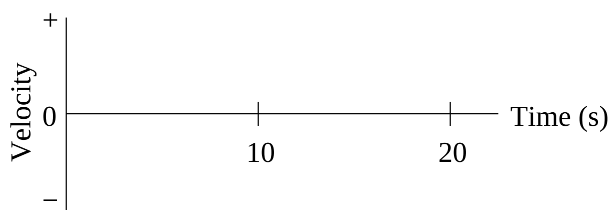
9. The object is standing still.



10. The object moves toward the origin at a steady (constant) velocity for 10 s and then stands still for 10 s.



11. The object moves away from the origin at a steady (constant) velocity for 10 s, reverses direction and moves back toward the origin at the same speed for 10 s.



10 Relating Position and Velocity Graphs[6]

Name ______________________________ Section _______________ Date _______________

Objective

To understand the relationship between position vs. time and velocity vs. time graphs.

Introduction

You have looked at position and velocity vs. time graphs separately. Since position vs. time and velocity vs. time graphs are different ways to represent the same motion, it ought to be possible to figure out the velocity at which someone is moving by examining her/his position vs. time graph. Conversely, you ought to be able to figure out how far someone has traveled (change in position) from a velocity vs. time graph.

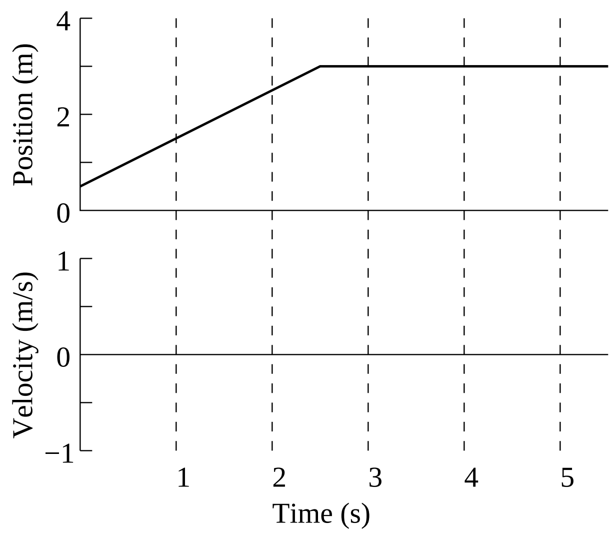
To explore how position vs. time and velocity vs. time graphs are related, you will use the **Position & Velocity Graphs** application (in the **131 Workshop** submenu of the **Programs** section under the **Start** menu). For some of the runs it may be necessary to adjust the time axis for one of the graphs so that the time scales are the same for the position and velocity graphs.

Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (Position & Velocity Graphs application)
- Wooden board

Activity 1: Predicting Velocity Graphs from Position Graphs

(a) Carefully study the position graph shown below and predict the velocity vs. time graph that would result from the motion. Using a dashed line, sketch your prediction of the corresponding velocity vs. time graph on the velocity axes.



[6] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) After each person in your group has sketched a prediction, test your prediction by matching the position vs. time graph shown. When you have made a good duplicate of the position graph, sketch your actual graph over the existing position vs. time graph. Use a solid line to draw the actual velocity graph on the same graph with your prediction. (Do not erase your prediction).

(c) How would the position graph be different if you moved faster? Slower?

(d) How would the velocity graph be different if you moved faster? Slower?

Activity 2: Average Velocity Calculations

(a) Find your average velocity from your velocity graph in the previous activity. To do this, use the Smart Tool on the graph menu bar (sixth box along the top of the graph window) to determine the velocity at ten points on the graph. Record these ten values, then average them together to get an estimate of the average velocity.

Velocity values (m/s) ________ ________ ________ ________ ________ ________ ________ ________ ________ ________

Average value of the velocity: ________________ m/s

Standard deviation: ________________ m/s

Average velocity with uncertainty: ________________________ m/s

(b) Average velocity during a particular time interval can also be calculated as the change in position divided by the change in time. (The change in position is often called the displacement.) By definition, this is also the slope of the position vs. time graph for that time period. Use the cursors on the position vs. time graph to read the position and time coordinates for two typical points while you were moving. (For a more accurate answer, use two points as far apart as possible but still typical of the motion, and within the time interval over which you took velocity readings in Activity 1.) Record the coordinates of the points below.

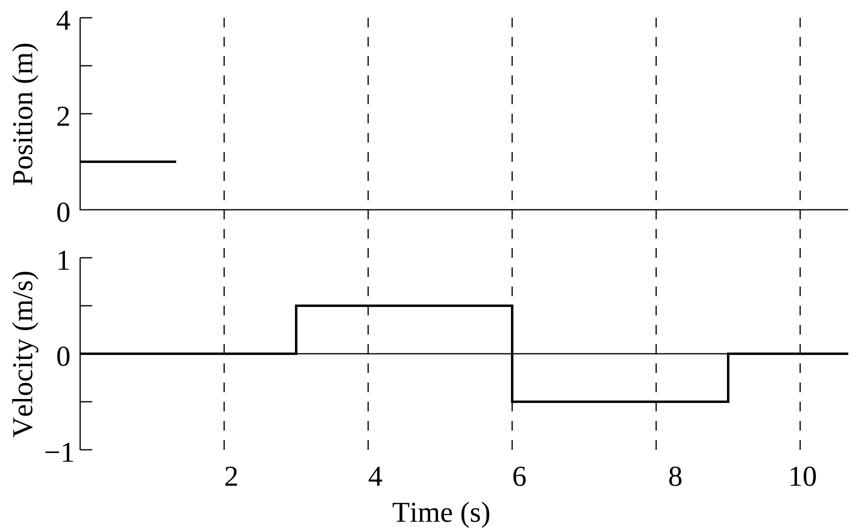
(c) Calculate the change in position (displacement) between the two points in part (b). Also calculate the corresponding change in time (time interval). Divide the change in position by the change in time to calculate the average velocity. Show your calculations below.

(d) Is the average velocity positive or negative? Is this what you expected?

(e) Does the average velocity you just calculated from the position graph agree with the average velocity you estimated from the velocity graph? Do you expect them to agree? How would you account for any differences?

Activity 3: Finding Position from a Velocity Graph

(a) Carefully study the velocity graph that follows. Using a dashed line, sketch your prediction of the corresponding position graph on the bottom set of axes. (Assume that you started at the 1-meter mark.)



(b) After each person has sketched a prediction, do your group's best to duplicate the bottom (velocity vs. time) graph by walking. When you have made a good duplicate of the velocity vs. time graph, draw your actual result over the existing velocity vs. time graph. (Use a solid line to draw the actual position vs. time graph on the same axes with your prediction. Do not erase your prediction.)

(c) How can you tell from a velocity vs. time graph that the moving object has changed direction?

(d) What is the velocity at the moment the direction changes?

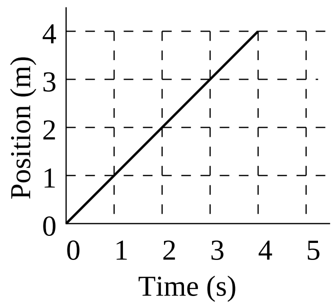
(e) Is it possible to actually move your body (or an object) to make vertical lines on a position vs. time graph? Why or why not? What would the velocity be for a vertical section of a position vs. time graph?

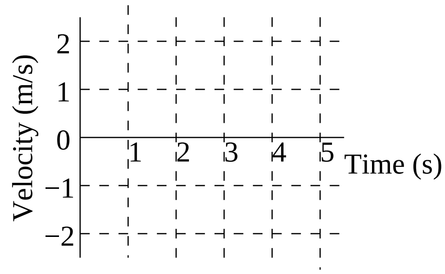
(f) How can you tell from a position vs. time graph that your motion is steady (motion at a constant velocity)?

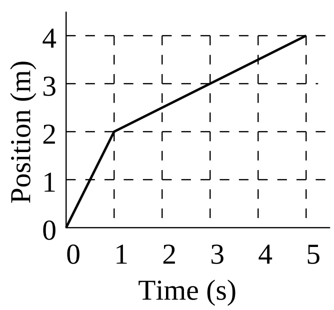
(g) How can you tell from a velocity vs. time graph that your motion is steady (constant velocity)?

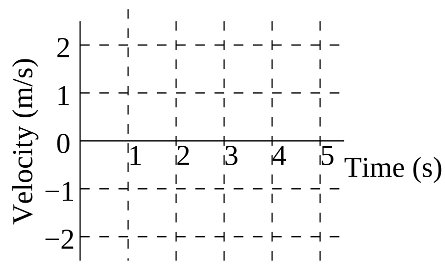
Homework

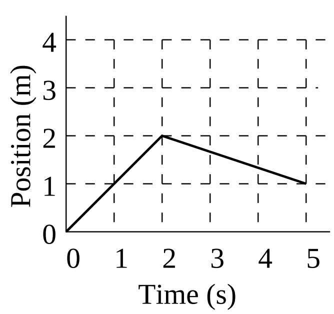
1. Draw the velocity graphs for an object whose motion produced the position-time graphs shown below on the left. Position is in meters and velocity in meters per second. Note: Unlike most real objects, you can assume these objects can change velocity so quickly that it looks instantaneous with this time scale.

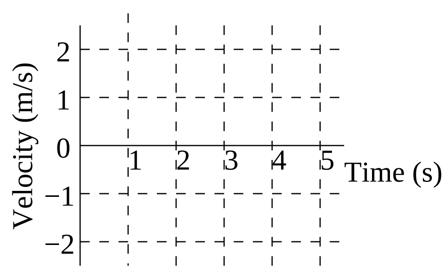




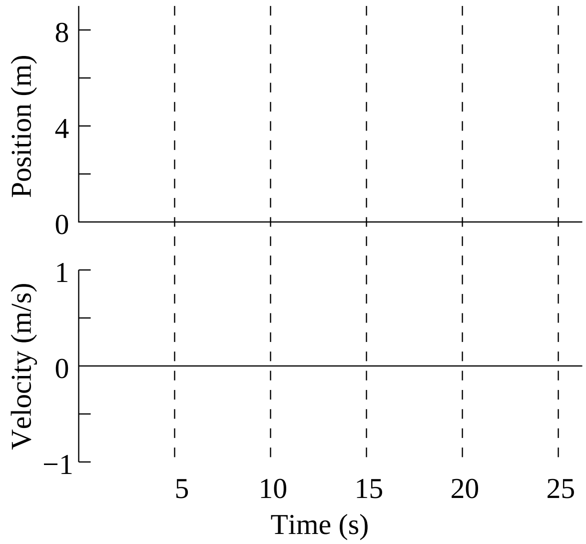








2. Draw careful graphs below of position and velocity for a cart that (a) moves away from the origin at a slow and steady (constant) velocity for the first 5 seconds; (b) moves away at a medium-fast, steady (constant) velocity for the next 5 seconds; (c) stands still for the next 5 seconds; (d) moves toward the origin at a slow and steady (constant) velocity for the next 5 seconds; (e) stands still for the last 5 seconds.



8
4
0
Position (m)
1
0
−1
Velocity (m/s)
5
10
15
20
25
Time (s)

11 Instantaneous Velocity

Name ______________________ Section __________ Date __________

Objectives:

- To make plausible the idea of velocity at an instant.
- To demonstrate that nonuniform motion appears increasingly uniform as the time interval becomes successively shorter.
- To understand the notion of "taking a limit".

Apparatus:

- Track and cart
- *Science Workshop 750 Interface*
- *DataStudio* software (Instantaneous Velocity application)
- Motion detector
- Lab stand

Activity 1: An experiment

1. Open Instantaneous Velocity application (under **Start** → **Programs** → **131 Workshop**).
2. You will release a cart to roll down the track toward the motion detector. *Stop the cart before it hits the motion detector*. Also, be sure that you aim the motion detector at the cart and that there are no people or extraneous objects within a 20 degree cone of the track (with vertex at the motion detector). Spurious motions within the sensitivity of the motion detector give false signals.
3. Place the cart near the 2-meter mark at the raised end of the track, opposite that on which the motion detector is mounted.
4. Release the cart from rest and start recording data.
5. A graph with dots at regular *time* intervals will be created on the screen. Stop the cart before it hits the motion detector. You may need to repeat the data taking until you get a reasonable looking chart (see Question 1).
6. Print out a copy of the graph for each member of the lab group.

Questions:

1. Describe the distribution of the dots. Are they as you would have expected? Is the motion it represents uniform or not? Explain.

2. Using the cursor select ten data points on your graph. Click on the Zoom In button on the graph menu bar until the highlighted data points span the graph. Notice that the Zoom In button has the effect of viewing the data over successively shorter time intervals. What does examining the data over successively shorter time intervals demonstrate? Explain.

3. Use the Smart Tool on the graph menu bar to record the position and time for the first and last points highlighted on the graph. Calculate $\Delta x / \Delta t$ and explain the significance of this quantity.

Discussion You just approximated the instantaneous velocity for the central point of your section of dots. The instantaneous velocity is the velocity the object would have at a particular instant if it were moving at uniform velocity during the entire interval. You also approximated a derivative: the ratio of the displacement to the time interval as the time interval approaches zero seconds.

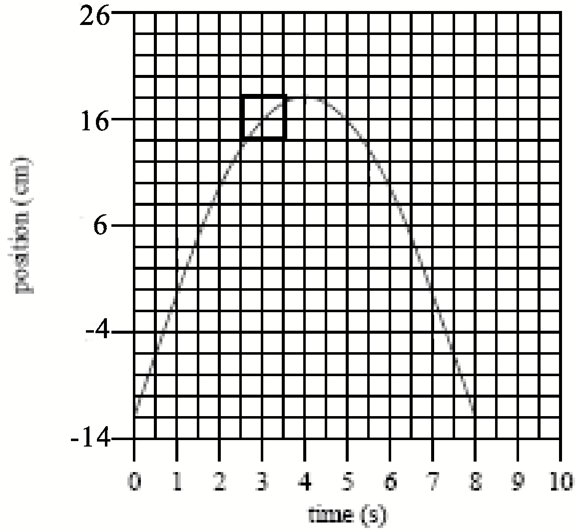
Activity 2: A Graphing Exercise

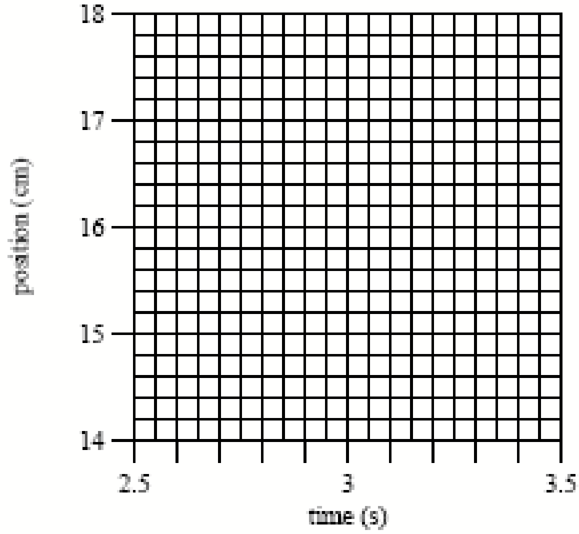
1. On the next page, you will find three position versus clock reading graphs. The first one depicts motion over an eight second time interval. We want to know the instantaneous velocity at clock reading 3 sec.
2. On the second graph, replot the outlined segment of the first graph, this time magnifying both position and time by a factor of ten.
3. On the third graph, magnify again by a factor of ten the equivalent section of the second graph around clock reading 3 sec and position 16 cm.
4. If you are satisfied with the straightness of the line in the third graph, calculate $\frac{\Delta s}{\Delta t}$, the average velocity for this interval. Since the line is sufficiently straight, the result of your calculation may be identified also as the instantaneous velocity at the midpoint of the interval.
5. The curve in the first graph depicts the motion of the object. Determine the equation of motion (the equation of the curve).
6. Take the derivative of this equation of motion and evaluate it at the central clock reading of your final interval. This is also a determination of the instantaneous velocity at that clock reading.

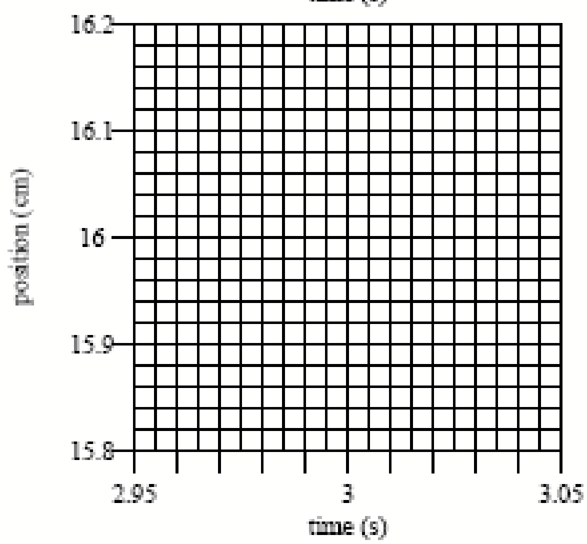
Questions:

1. Compare your two calculations of the instantaneous velocity. Are they the same or do they differ? Should they be the same or different? Explain.

2. Plot a line on the first graph through the point (3s,16cm) which has the slope of the line in the last graph. What is this line in relation to the motion curve?


26
16
6
-4
-14
0 1 2 3 4 5 6 7 8 9 10
time (s)
position (cm)


18
17
16
15
14
2.5 3 3.5
time (s)
position (cm)


16.2
16.1
16
15.9
15.8
2.95 3 3.05
time (s)
position (cm)

12 Changing Motion[7]

Name ______________________________ Section _______________ Date _______________

Objectives

- To learn how to relate graphs of acceleration vs. time to the motions they represent.
- To understand the relationship between position vs. time, velocity vs. time, and acceleration vs. time graphs.

Velocity and Acceleration Graphs

We are interested in having you learn to describe simple motions in which the velocity of an object is changing. In order to learn to describe motion in more detail for some simple situations, you will be asked to observe and describe the motion of a dynamics cart on a track. Although graphs and words are still important representations of these motions, you will also be asked to draw velocity vectors, arrows that indicate both the direction and speed of a moving object. Thus, you will also learn how to represent simple motions with velocity diagrams .

In the last session, you looked at position vs. time and velocity vs. time graphs of the motion of your body as you moved at a "constant" velocity. The data for the graphs were collected using a motion detector. Your goal in this session is to learn how to describe various kinds of motion in more detail. It is not enough when studying motion in physics to simply say that "the object is moving toward the right" or "it is standing still." You have probably realized that a velocity vs. time graph is better than a position vs. time graph when you want to know how fast and in what direction you are moving at each instant in time as you walk. When the velocity of an object is changing, it is also important to know how it is changing. The rate of change of velocity is known as the acceleration.

In order to get a feeling for acceleration, it is helpful to create and learn to interpret velocity vs. time and acceleration vs. time graphs for some relatively simple motions of a cart on a track. You will be observing the cart with the motion detector as it moves at a constant velocity and as it changes its velocity at a constant rate. Use the **P, V & A Graphs** application (under **Start → Programs → 131 Workshop**) for all of the activities in this unit.

Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (P, V & A Graphs application)
- Dynamics cart and track
- Lab stand to incline the track

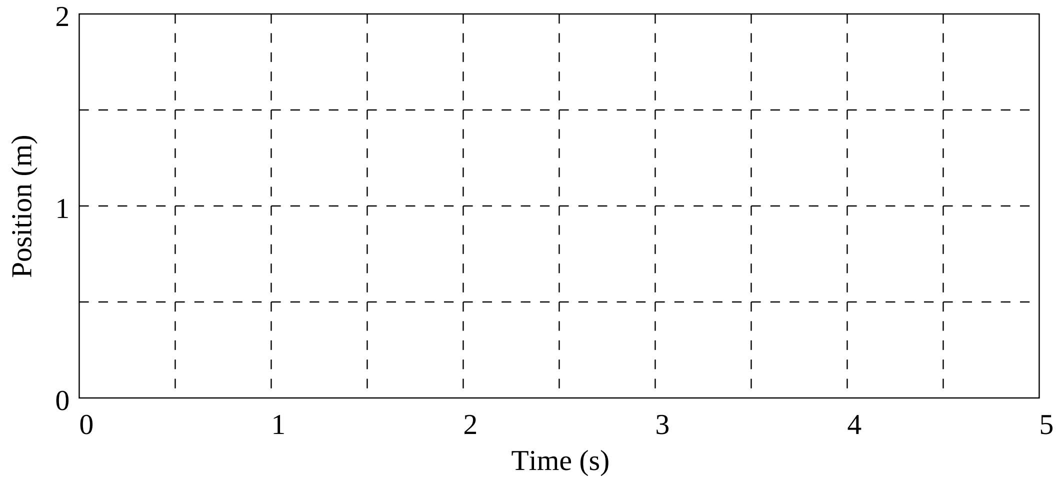
Graphing a Constant Velocity Cart Motion

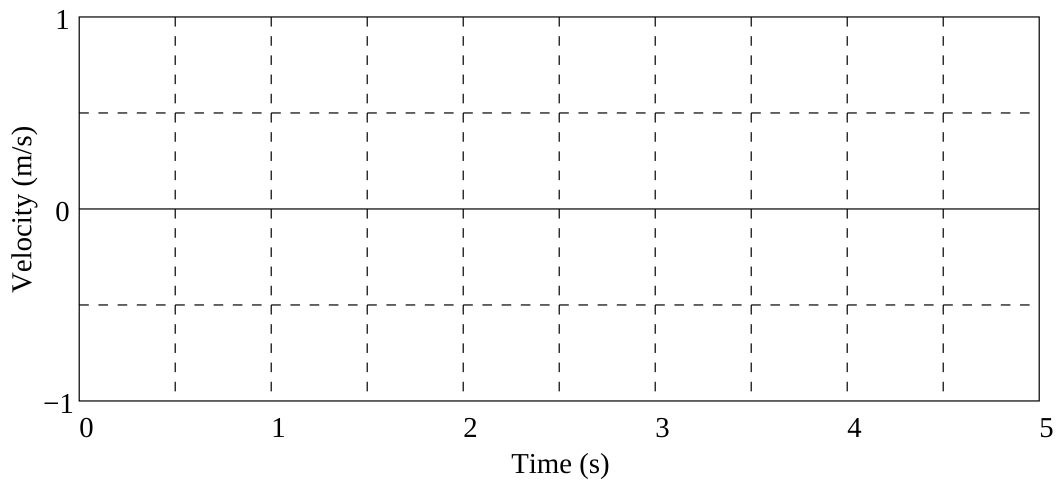
Let's start by giving the cart a push along the level track and graphing its motion.

Activity 1: Position, Velocity and Acceleration Graphs of Constant Velocity

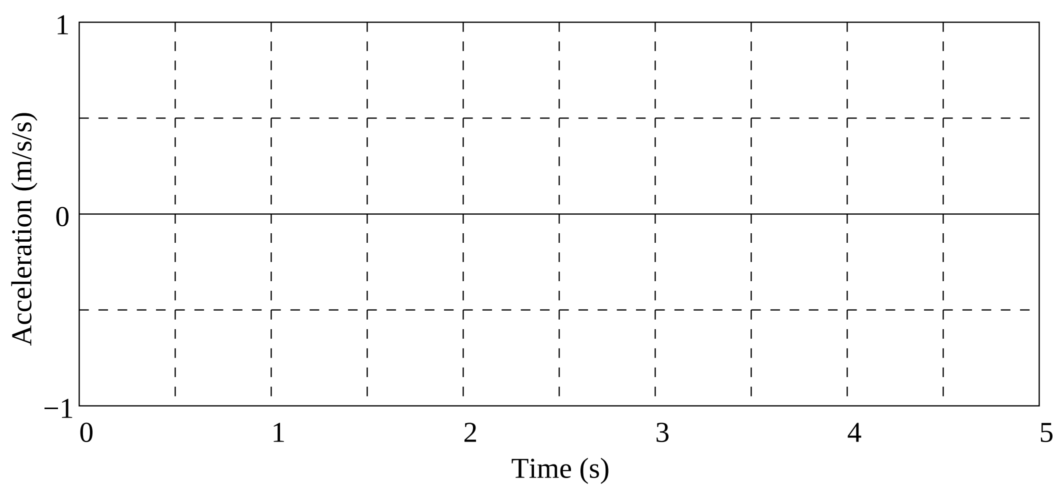
(a) Based on your observations of the motions of your body in the last session, how should the position and velocity graphs look if you move the cart at a constant velocity away from the motion detector starting at the 0.5 meter mark? Sketch your predictions with dashed lines on the axes that follow.

[7] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.





(b) Acceleration is defined as the time rate of change of velocity. Sketch your prediction of the cart acceleration on the axes that follow using a dashed line.



(c) Test your prediction. Be sure that the cart is never closer than 0.15 meter from the motion detector. Try several times until you get a fairly constant velocity. Sketch your results with solid lines on the axes shown above. The acceleration vs. time graphs will exhibit small fluctuations due to irregularities in the motion of the cart. You should ignore these fluctuations and draw smooth patterns.

(d) Did your graphs agree with your predictions? What characterizes constant velocity motion on a position vs. time graph?

(e) What characterizes constant velocity motion on a velocity vs. time graph?

(f) What characterizes constant velocity motion on an acceleration vs. time graph?

Finding Accelerations

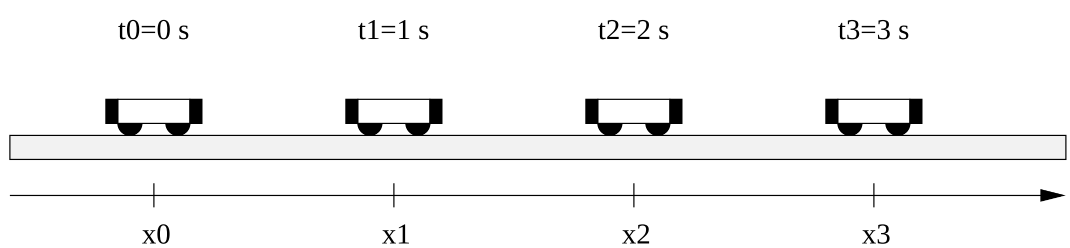
To find the average acceleration of the cart during some time interval (the average time rate of change of its velocity), you must measure its velocity at two different times, calculate the difference between the final value and the initial value and divide by the time interval.

To find the acceleration vector from two velocity vectors, you must first find the vector representing the change in velocity by subtracting the initial velocity vector from the final one. Then you divide this vector by the time interval.

Activity 2: Representing Acceleration

(a) Calculate the average acceleration during some time interval from your velocity graph in Activity 1. Does the result agree with your acceleration graph in Activity 1?

(b) The diagram below shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving at a constant velocity away from the motion detector.



(c) Explain how you would find the vector representing the change in velocity between the times 1.0 s and 2.0 s in the diagram above. From this vector, what value would you calculate for the acceleration? Explain. Is this value in agreement with the acceleration graph you obtained in Activity 1?

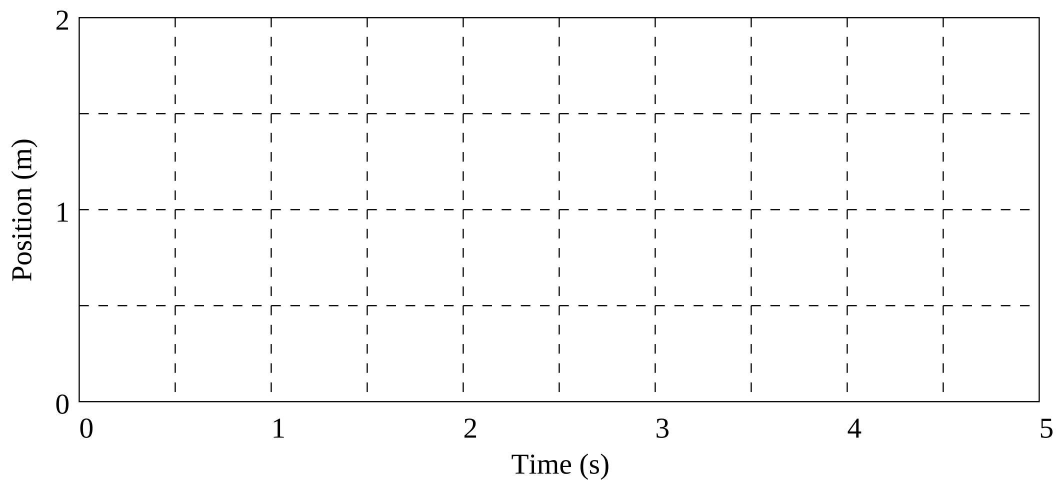
Speeding Up at a Moderate Rate

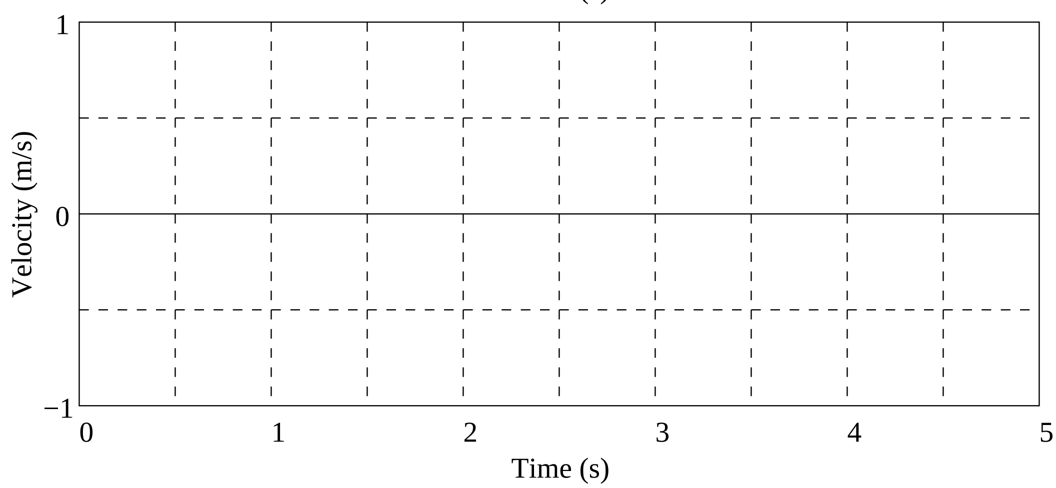
In the next activity you will look at velocity and acceleration graphs of the motion of a cart when its velocity is changing. You will be able to see how these two representations of the motion are related to each other when the cart is speeding up.

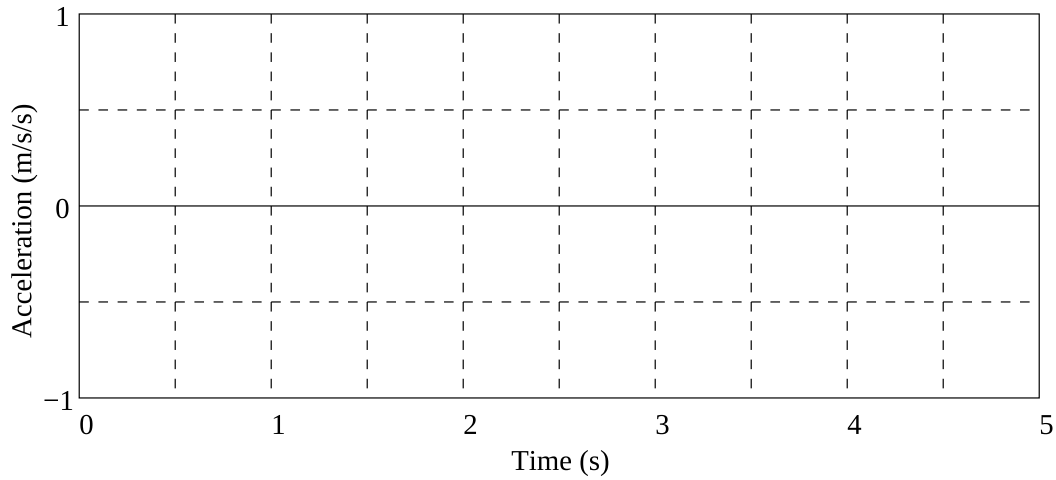
In order to get your cart speeding up smoothly use the lab stand to raise the track several centimeters at the end where the motion detector is mounted.

Activity 3: Graphs Depicting Speeding Up

(a) Predict the shape of the position, velocity, and acceleration vs. time graphs for the cart moving away from the sensor and speeding up. Sketch your predictions on the following axes using dashed lines.







(b) Create graphs of the motion of your cart as it moves away from the detector and speeds up. Sketch the graphs neatly on the above axes using solid lines.

(c) How does your position graph differ from the position graphs for steady (constant velocity) motion?

(d) What feature of your velocity graph signifies that the motion was away from the detector?

(e) What feature of your velocity graph signifies that the cart was speeding up? How would a graph of motion with a constant velocity differ?

(f) During the time that the cart is speeding up, is the acceleration positive or negative? How does speeding up while moving away from the detector result in this sign of acceleration? Hint: Remember that acceleration is the rate of change of velocity. Look at how the velocity is changing.

(g) How does the velocity vary in time as the cart speeds up? Does it increase at a steady rate or in some other way?

(h) How does the acceleration vary in time as the cart speeds up? Is this what you expect based on the velocity graph? Explain.

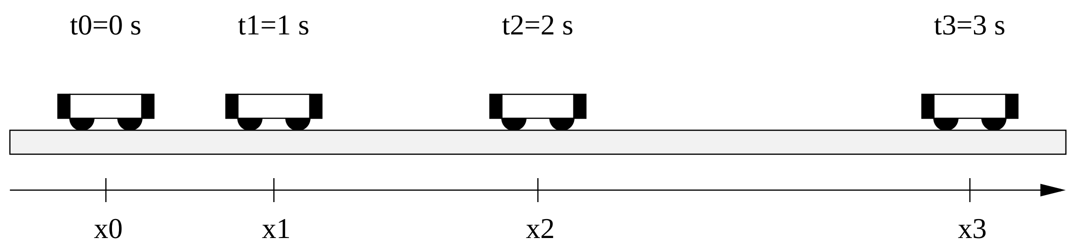
(i) Do not delete the graphs from the computer screen. They will be used in Activity 5.

Using Vectors to Describe the Acceleration

Let's return to the Vector Diagram representation and use it to describe the acceleration.

Activity 4: Acceleration Vectors

(a) The diagram that follows shows the positions of the cart at equal time intervals. At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving away from the motion detector and speeding up.



(b) Show below how you would find the approximate length and direction of the vector representing the change in velocity between the times 1.0 s and 2.0 s using the diagram above. No quantitative calculations are needed. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Activity 3 (f)?

Measuring Acceleration

In this investigation you will analyze the motion of your accelerated cart quantitatively. This analysis will be quantitative in the sense that your results will consist of numbers. You will determine the cart's acceleration from the slope of your velocity vs. time graph and compare it to the average acceleration read from the acceleration vs. time graph. You can display actual values for your acceleration and velocity data using the Smart Tool on the graph window menu.

Activity 5: Calculating Accelerations

(a) List 10 of the typical accelerations of the cart measured in Activity 3. Use the Smart Tool on the acceleration vs. time graph to get these values. (Only use values from the portion of the graph after the cart was released and before you stopped it.)

(b) Calculate the average value of the acceleration and record it below. Also calculate the standard deviation and write the acceleration with its uncertainty.

(c) The average acceleration during a particular time period is defined as the change in velocity divided by the change in time. This is the average rate of change of velocity. By definition, the rate of change of a quantity graphed with respect to time is also the slope of the curve. Thus the (average) slope of an object's velocity vs. time graph is the (average) acceleration of the object.

Use the Smart Tool to read the velocity and time coordinates for two typical points on the velocity vs. time graph. For a more accurate answer, use two points as far apart in time as possible but still during the time the cart was speeding up. Record the points in the space below.

(d) Calculate the change in velocity between points 1 and 2. Also calculate the corresponding change in time (time interval). Divide the change in velocity by the change in time. This is the average acceleration. Show and then summarize your calculations below.

(e) Is the acceleration positive or negative? Is this what you expected?

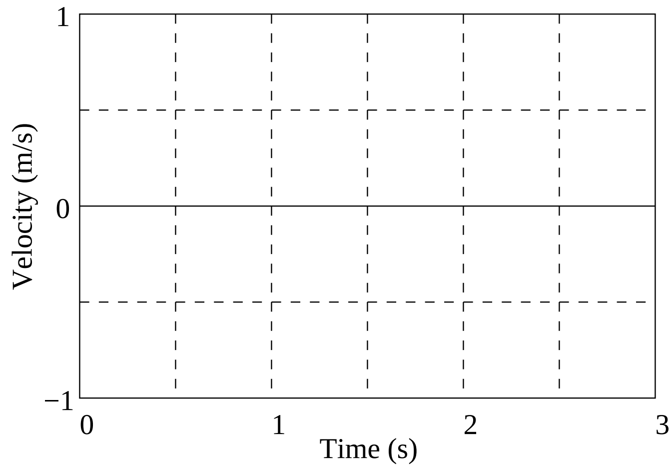
(f) Does the average acceleration you just calculated agree with the average acceleration you calculated from the acceleration vs. time graph? Do you expect them to agree? How would you account for any differences?

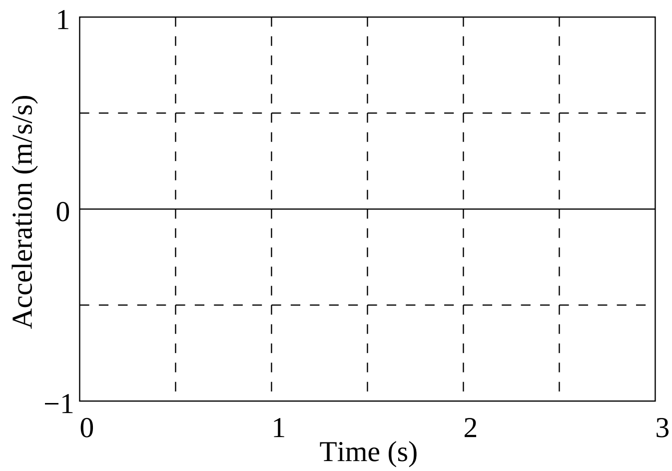
Speeding Up at a Faster Rate

Suppose that you accelerate your cart at a faster rate by raising the end of the track several more centimeters. How would your velocity and acceleration graphs change?

Activity 6: Graphs for a Greater Acceleration

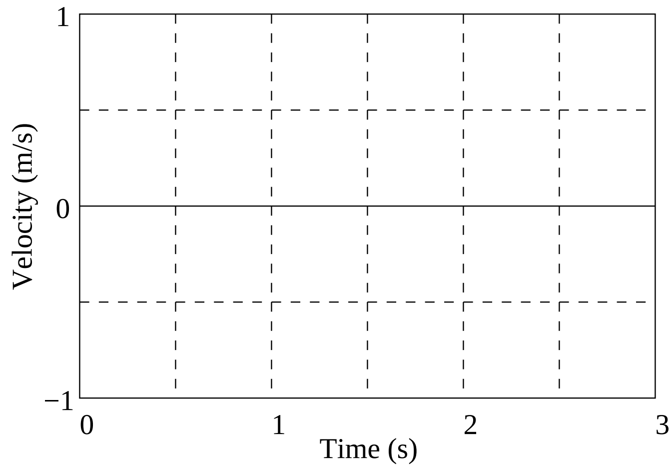
(a) Resketch the velocity and acceleration graphs you found in Activity 3 using the axes that follow.

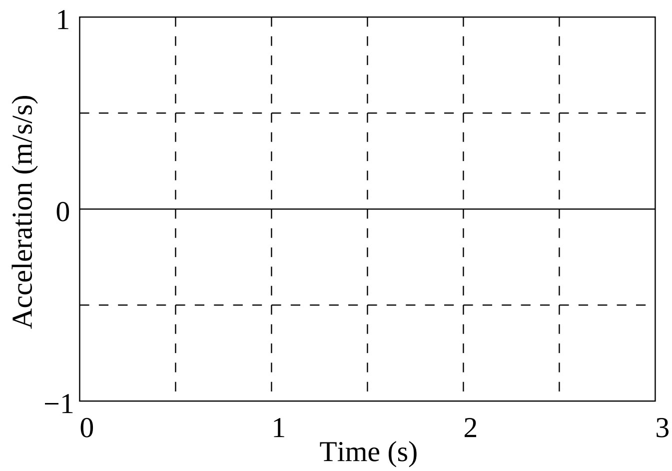




(b) In the previous set of axes, use a dashed line or another color to sketch your predictions for the general graphs that depict a cart speeding up at a faster rate. Exact predictions are not expected. We just want to know how you think the general shapes of the graphs will change.

(c) Test your predictions by accelerating the cart with the end of the track raised several centimeters more than in Activity 3. Repeat if necessary to get nice graphs and then sketch the results using the axes that follow.



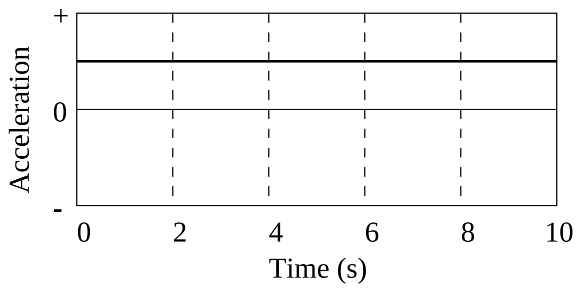


(d) Did the general shapes of your velocity and acceleration graphs agree with your predictions? How is the greater magnitude (size) of acceleration represented on a velocity vs. time graph?

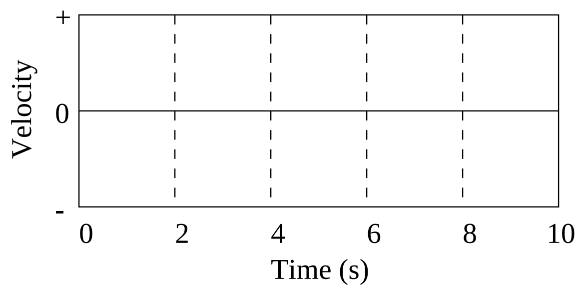
(e) How is the greater magnitude (size) of acceleration represented on an acceleration vs. time graph?

Homework

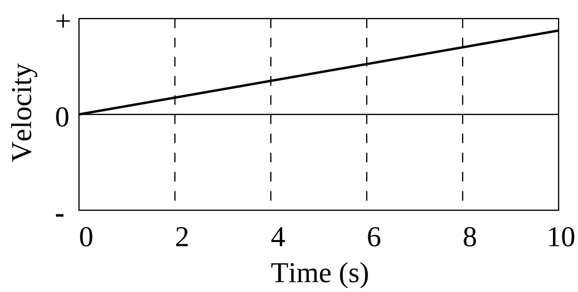
1. An object moving along a line (the + position axis) has the acceleration-time graph shown below. Describe how might the object move to create this graph if it is moving away from the origin?

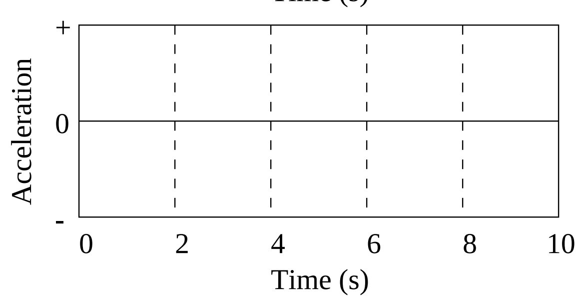


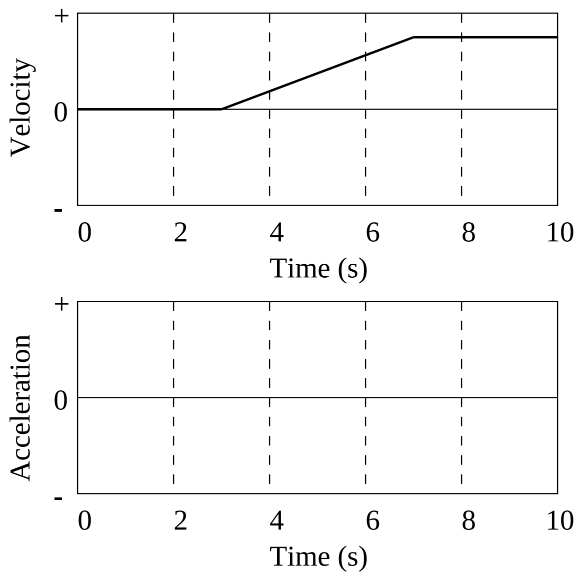
2. Sketch on the axes below a velocity-time graph that goes with the above acceleration-time graph.



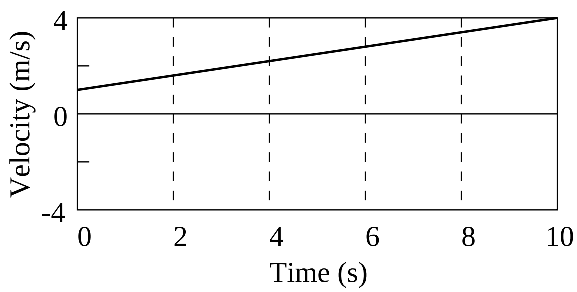
3. For each of the velocity-time graphs below, sketch the shape of the acceleration-time graph that goes with it.





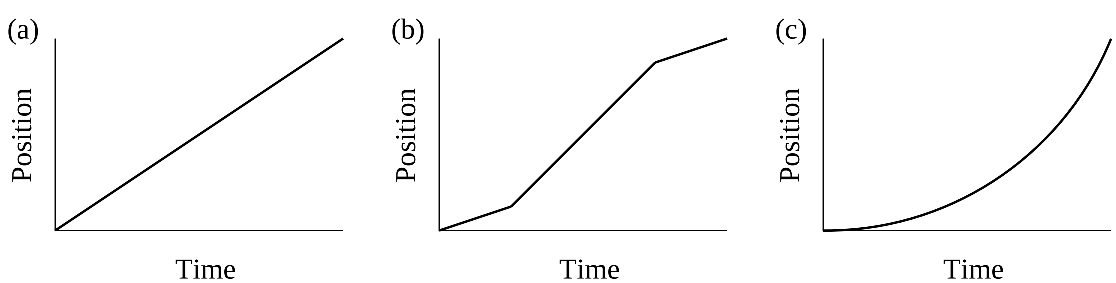


4. The following is a velocity-time graph for a car.



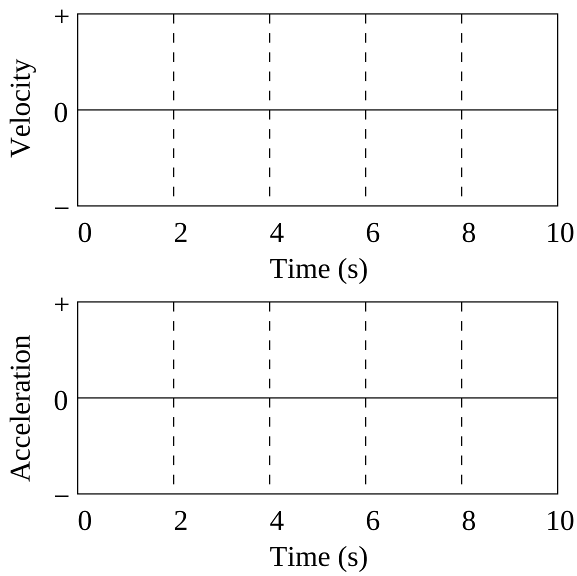
What is the average acceleration of the car? Show your work below.

5. Which position-time graph below could be that for a cart that is steadily accelerating away from the origin?

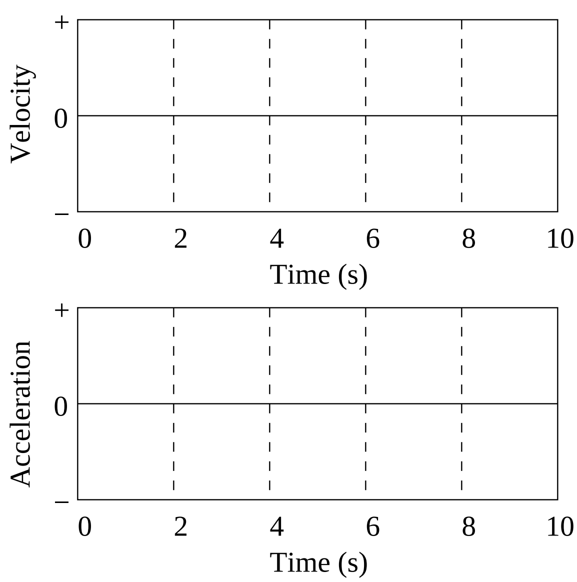


A car can move along a line (the + position axis). Sketch velocity-time and acceleration-time graphs which correspond to each of the following descriptions of the car's motion.

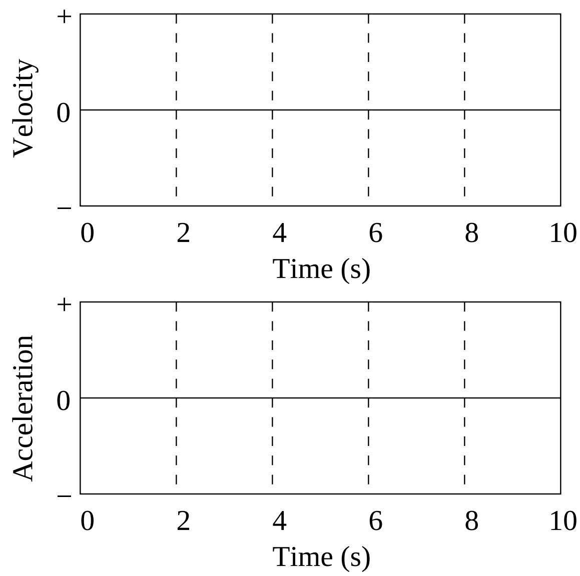
6. The car starts from rest and moves away from the origin increasing its speed at a steady rate.



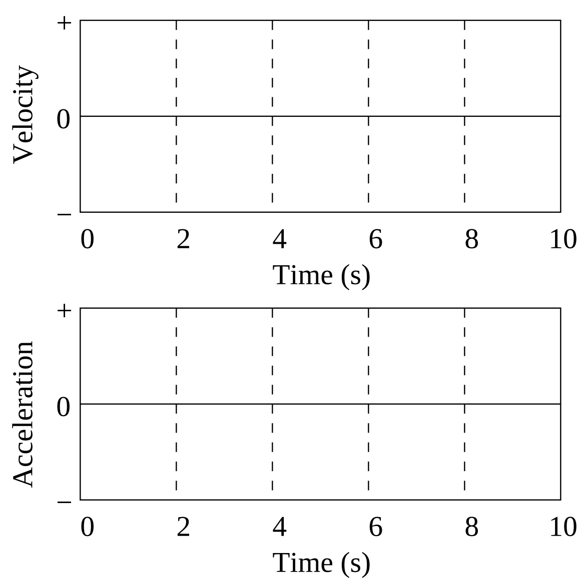
7. The car is moving away from the origin at a constant velocity.



8. The car starts from rest and moves away from the origin increasing its speed at a steady rate twice as large as in (6) above.



9. The car is moving away from the origin at a constant velocity twice as large as in (7) above.



13 Slowing Down, Speeding Up, and Turning[8]

Name ______________________ Section ____________ Date ____________

Objectives

- To learn how to relate graphs of acceleration vs. time to the motions they represent.
- To understand the relationship between velocity vs. time and acceleration vs. time graphs.

About Slowing Down, Speeding Up and Turning

In the previous session, you explored the characteristics of position vs. time, velocity vs. time and acceleration vs. time graphs of the motion of a dynamics cart. In the cases examined, the cart was always moving away from a motion detector, either at a constant velocity or with a constant acceleration. Under these conditions, the velocity and acceleration are both positive. You also learned how to find the magnitude of the acceleration from velocity vs. time and acceleration vs. time graphs, and how to represent the velocity and acceleration using vectors.

In the motions you studied in the last session, the velocity and acceleration vectors representing the motion of the cart both pointed in the same direction. In order to get a better feeling for acceleration, it will be helpful to examine velocity vs. time and acceleration vs. time graphs for some slightly more complicated motions of a cart on an inclined track. Again you will use the motion detector to observe the cart as it changes its velocity at a constant rate. Only this time the motion may be toward the detector, and the cart may be speeding up or slowing down.

Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (P, V & A Graphs application)
- Dynamics cart and track
- Lab stand to incline the track

Slowing Down and Speeding Up

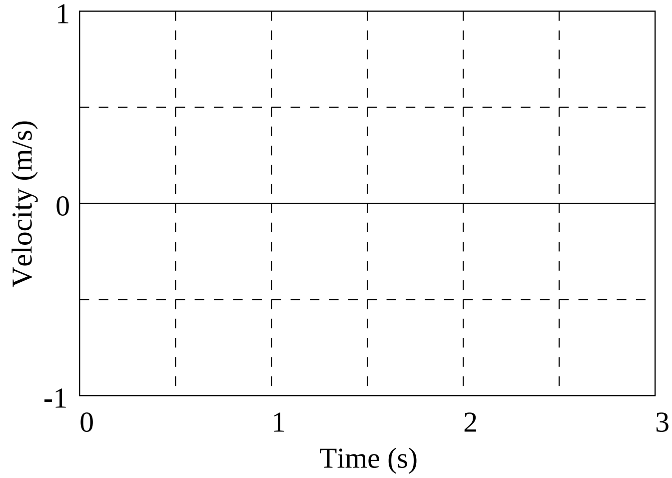
In this activity you will look at a cart moving along an inclined track and slowing down. A car being brought to rest by the steady action of brakes is a good example of this type of motion. Later you will examine the motion of the cart toward the motion detector and speeding up. In both cases, we are interested in the shapes of the velocity vs. time and acceleration vs. time graphs, as well as the vectors representing velocity and acceleration.

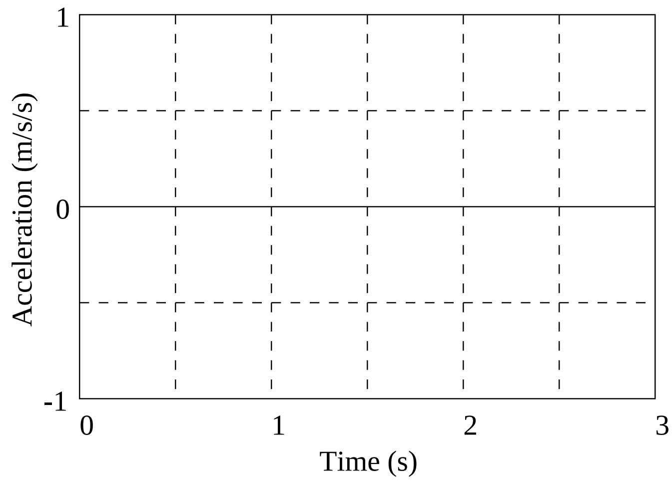
Let's start with the creation of velocity and acceleration graphs of when it is moving away from the motion detector and slowing down. To do this activity, the track should be inclined with a lab stand at one end and the motion detector set up at the lower end of the track. Adjust the lab stand so that the track is raised a few centimeters at the opposite end from where the motion detector is located. Now when you give the cart a push away from the motion detector, it will slow down after it is released. In this activity you will examine the velocity and acceleration of this motion.

Activity 1: Graphs Depicting Slowing Down

(a) If you give the cart a push away from the motion detector and release it, will the acceleration be positive, negative or zero (after it is released)? Sketch your predictions for the velocity vs. time and acceleration vs. time graphs on the axes below using dashed lines.

[8]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.





(b) To test your predictions, locate the cart 0.15 m from the motion detector and gently push the cart away from the motion detector once it starts clicking. Catch the cart before it turns around or hits the end of the track.

Draw the results on the axes above using solid lines for the part of the motion after the cart is released. You may have to try a few times to get a good run. The acceleration vs. time graphs will exhibit small fluctuations due to irregularities in the motion of the cart. You should ignore these fluctuations and draw smooth patterns.

(c) Did the shapes of your velocity and acceleration graphs agree with your predictions? How is the sign of the acceleration represented on a velocity vs. time graph?

(d) How is the sign of the acceleration represented on an acceleration vs. time graph?

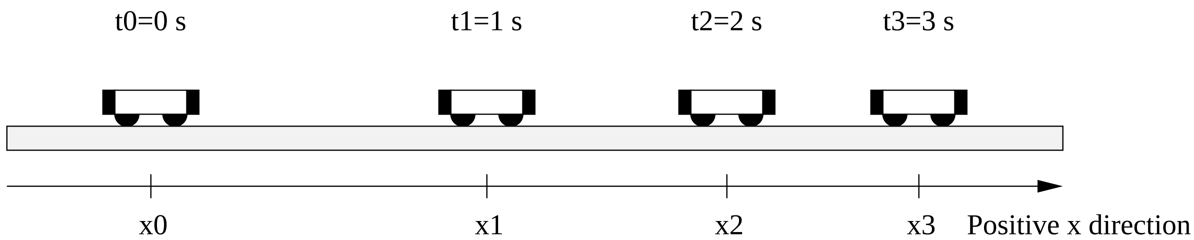
(e) Is the sign of the acceleration what you predicted? How does slowing down while moving away from the detector result in this sign of acceleration? Hint: Remember that acceleration is the rate of change of velocity. Look at how the velocity is changing.

Constructing Acceleration Vectors for Slowing Down

Let's consider a diagrammatic representation of a cart which is slowing down and use vector techniques to figure out the direction of the acceleration.

Activity 2: Vector Diagrams for Slowing Down

(a) The diagram that follows shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving away from the motion detector and slowing down.



(b) Show below how you would find the vector representing the change in velocity between the times 1s and 2 s in the diagram above. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Question (e) in Activity 1?

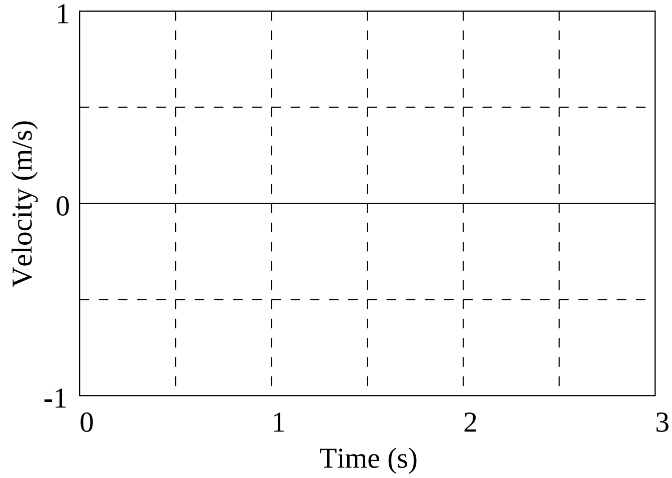
(c) Based on your observations in this activity and in the last session, state the general rules to predict the sign and direction of the acceleration if you know the sign of the velocity (i.e., the direction of motion) and whether the object is speeding up or slowing down.

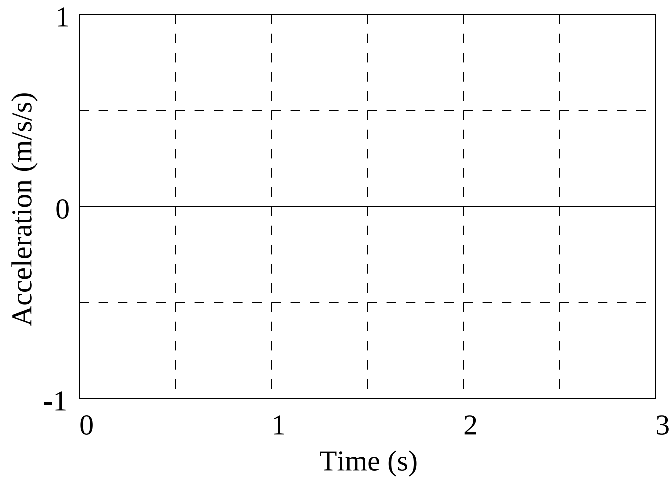
Speeding Up Toward the Motion Detector

Let's investigate another common situation. Suppose the cart is allowed to speed up when traveling toward the motion detector. What will be the direction of the acceleration? Positive or negative?

Activity 3: Graphs Depicting Speeding Up

(a) Use the general rules that you stated in Activity 2 to predict the shapes of the velocity and acceleration graphs. Sketch your predictions using dashed lines on the axes that follow.





(b) Test your predictions by releasing the cart from rest at the raised end of the track after the motion detector starts clicking. Catch the cart before it gets too close to the detector.

Draw the results using solid lines on the axes above. You may have to try a few times to get a good run.

(c) How does your velocity graph show that the cart was moving toward the detector?

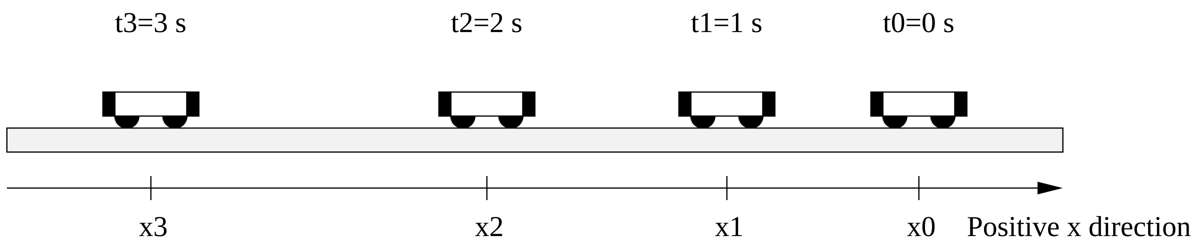
(d) During the time that the cart was speeding up, is the acceleration positive or negative? Does this agree with your prediction? Explain how speeding up while moving toward the detector results in this sign of acceleration. Hint: Think about how the velocity is changing.

Constructing Acceleration Vectors for Speeding Up

Let's consider a diagrammatic representation of a cart which is speeding up and use vector techniques to figure out the direction of the acceleration.

Activity 4: Vector Diagrams for Speeding Up

(a) The diagram that follows shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving toward the motion detector and speeding up.



(b) Show below how you would find the vector representing the change in velocity between the times 1s and 2 s in the diagram above. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Question (d) in Activity 3?

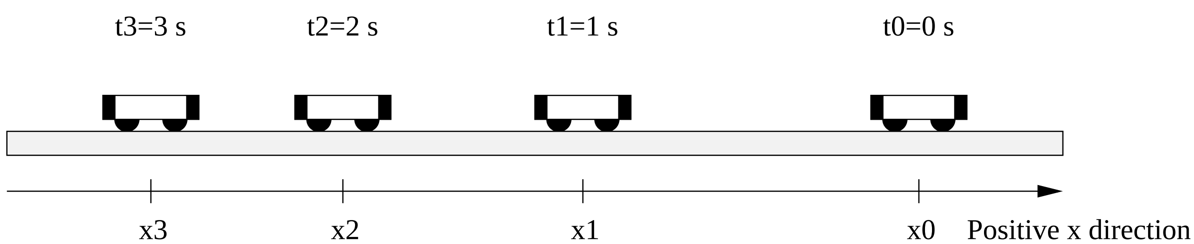
Moving Toward the Detector and Slowing Down

There is one more possible combination of velocity and acceleration for the cart, that of moving toward the detector while slowing down.

Activity 5: Slowing Down Toward the Detector

(a) Use your general rules to predict the direction and sign of the acceleration when the cart is slowing down as it moves toward the detector. Explain why the acceleration should have this direction and this sign in terms of the velocity and how the velocity is changing.

(b) The diagram below shows the positions of the cart at equal time intervals for slowing down while moving toward the detector. At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving toward the motion detector and slowing down.



(c) Show below how you would find the vector representing the change in velocity between the times 1s and 2 s in the diagram above. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with the prediction you made in part (a)?

Acceleration and Turning Around

In the last session and in the first activity in this session, you looked at velocity vs. time and acceleration vs. time graphs for a cart moving in one direction with a changing velocity. In this investigation you will look at what happens when the cart slows down, turns around and then speeds up.

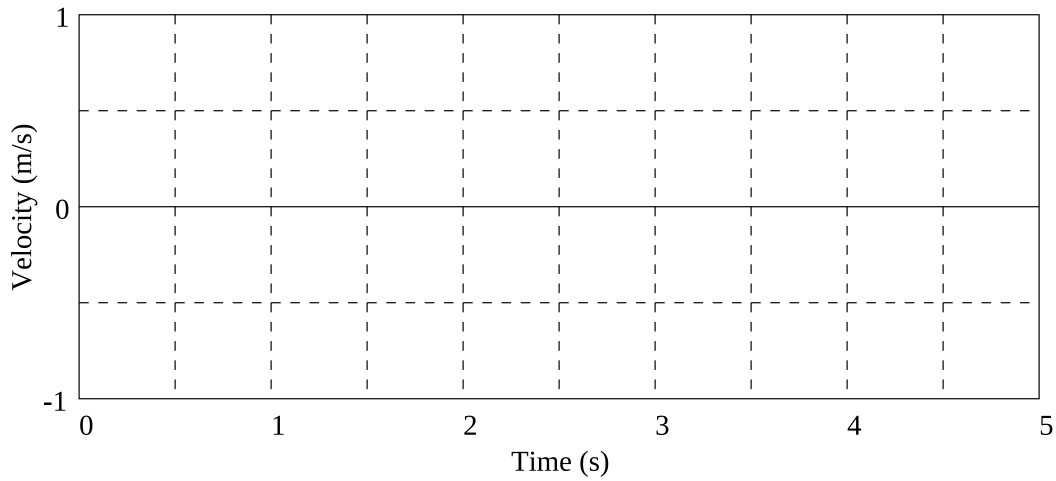
To practice this motion you should position the cart 0.15 m from the detector and give the cart a gentle push away from the motion detector. It should move up the track, slow down, reverse direction and then move back down toward the detector. Try it without activating the motion detector! Be sure that the cart does not hit the end of the track before it turns around.

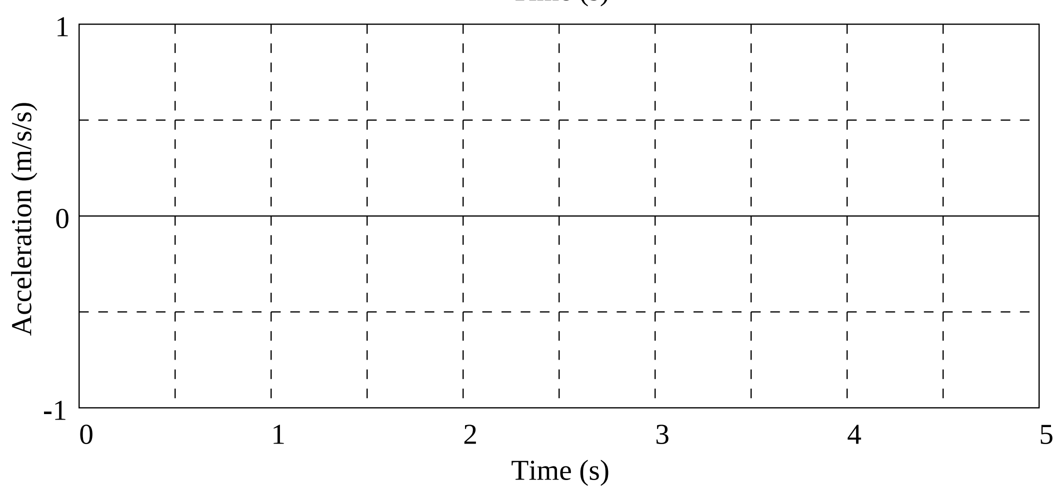
Activity 6: Reversing Direction

(a) For each part of the motion-away from the detector, at the turning point, and toward the detector, predict in the table that follows whether the velocity will be positive, zero or negative. Also indicate whether the acceleration is positive, zero or negative.

	Moving Away	Turning Around	Moving Toward
Velocity			
Acceleration			

(b) Sketch the predicted shapes of the velocity vs. time and acceleration vs. time graphs of this entire motion on the axes that follow using dashed lines.





(c) Test your predictions by making graphs of the motion. Use the procedures you used in the slowing down and speeding up activities. You may have to try a few times to get a good run. When you get a good run, sketch both graphs on the axes above using solid lines.

(d) Did the cart have a zero velocity at any point in the motion? (Hint: Look at the velocity graph. What was the velocity of the cart at the end of the ramp?) Does this agree with your prediction? How much time did it spend at zero velocity before it started back toward the detector?

(e) According to your acceleration graph, what is the acceleration at the instant the cart comes to rest? Is it positive, negative or zero? Does this agree with your prediction?

(f) Explain the observed sign of the acceleration at the end of the ramp. (Hint: Remember that acceleration is the rate of change of velocity.)

(g) Print a copy of the velocity and acceleration graphs for each person in your group and add the graphs to this unit.

(h) Notice that the slope of the velocity graph is not quite the same for positive velocities as it is for negative velocities. (This difference can also be seen on the acceleration graph.) What accounts for this difference?

Tossing a Ball

Suppose you throw a ball up into the air. It moves upward, reaches its highest point and then moves back down toward your hand. We will now consider what can be said about the directions of its velocity and acceleration vectors at various points.

Activity 7: The Rise and Fall of a Ball

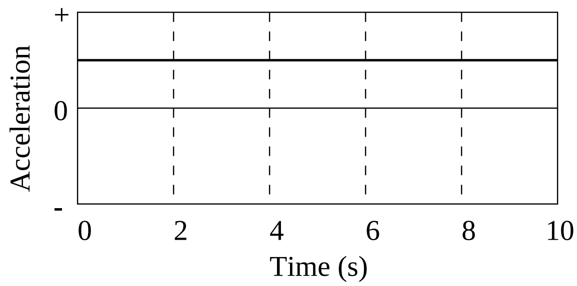
(a) Consider the ball toss carefully. Assume that upward is the positive direction. Indicate in the table that follows whether the velocity is positive, zero or negative during each of the three parts of the motion. Also indicate if the acceleration is positive, zero or negative. Hint: Remember, to find the acceleration you must look at the change in velocity.

	Moving Up (After Release)	At Highest Point	Moving Down
Velocity			
Acceleration			

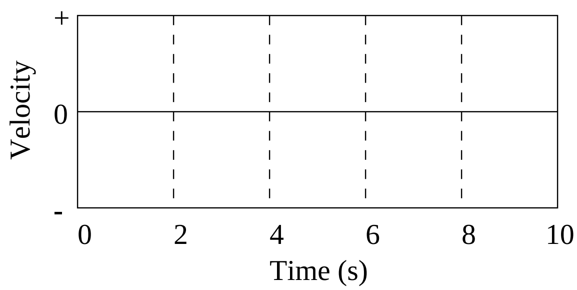
(b) In what ways is the motion of the ball similar to the motion of the cart which you just observed?

Homework

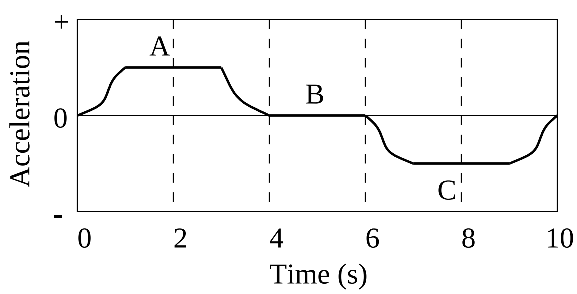
1. An object moving along a line (the + position axis) has the acceleration-time graph below. How might the object move to create this graph if it is moving toward the origin?



2. Sketch on the axes below the velocity-time graph that goes with the above acceleration- time graph.



3. How would an object move to create each of the three labeled parts of the acceleration- time graph shown below?

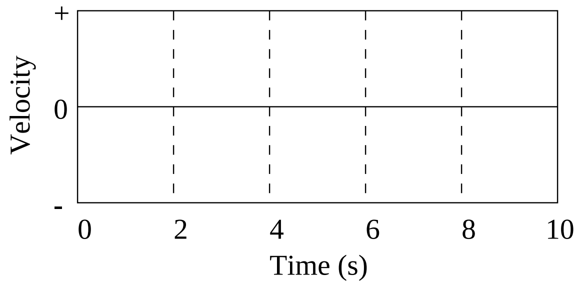


A:

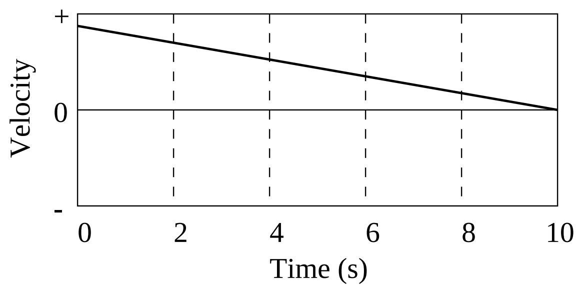
B:

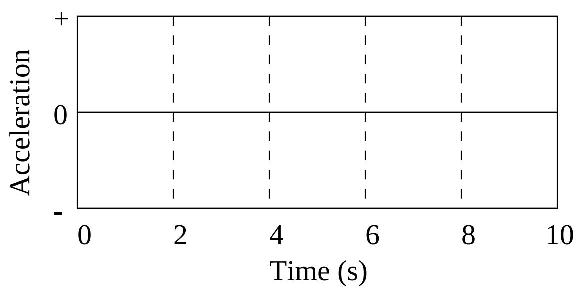
C:

4. Sketch below a velocity-time graph which might go with the acceleration-time graph in question (3).



5. Sketch the shape of the acceleration-time graph that goes with the velocity-time graph shown below.

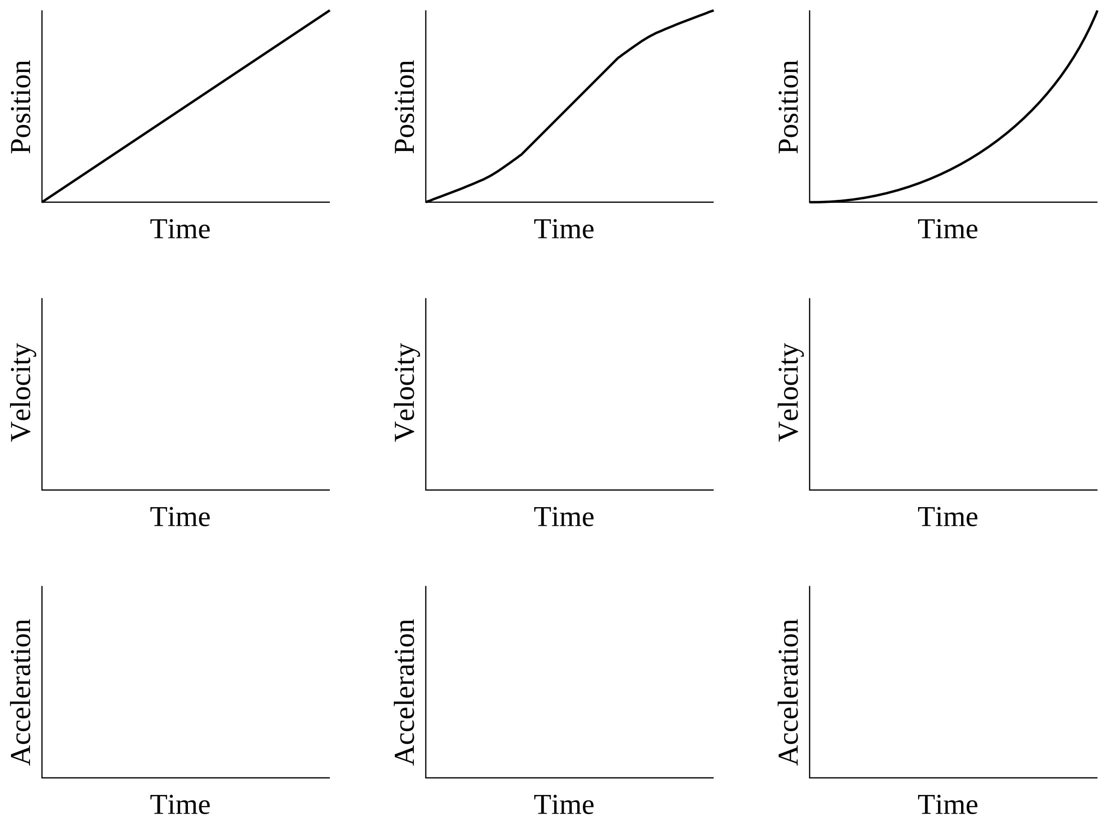




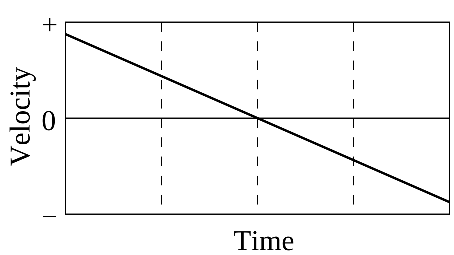
6. A car moves along a line [the + position axis]. Fill in the table below with the sign (+ or -) of the velocity and acceleration of the car for each of the motions described.

	Position	Velocity	Acceleration Speeding Up	Acceleration Slowing Down
Car Moves Away from the Origin	+			
Car Moves Toward the Origin	+			

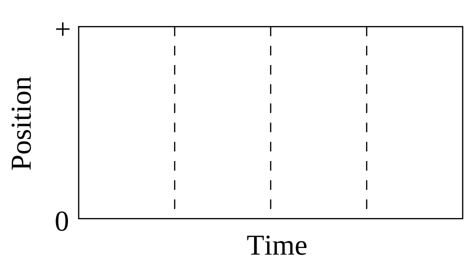
7. For each of the position-time graphs shown, sketch below it the corresponding velocity- time and acceleration-time graphs.



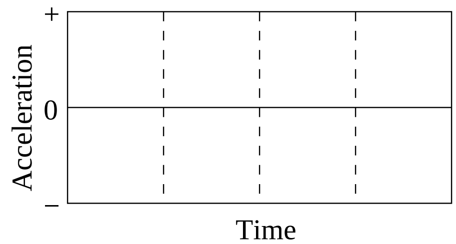
8. Describe how you would move to produce the velocity-time graph shown below.



9. Sketch a position-time graph corresponding to the velocity-time graph above.

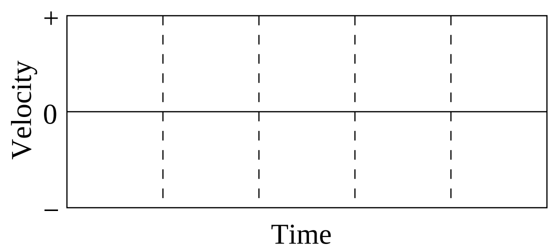


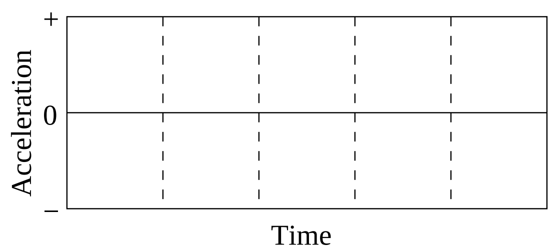
10. Sketch an acceleration-time graph corresponding to the velocity-time graph above.



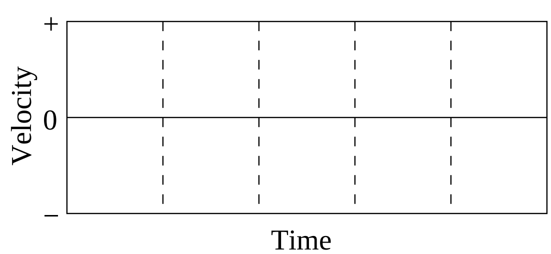
A car can move in either direction along a line (the + position axis). Sketch velocity-time and acceleration-time graphs that correspond to each of the following descriptions of the car's motion.

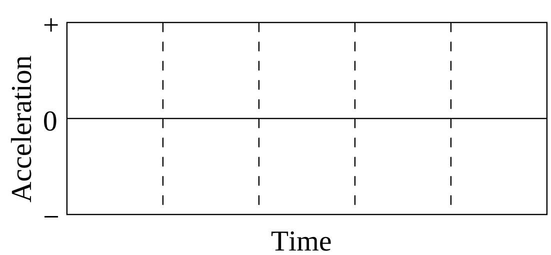
11. The car is moving toward the origin at a constant velocity.



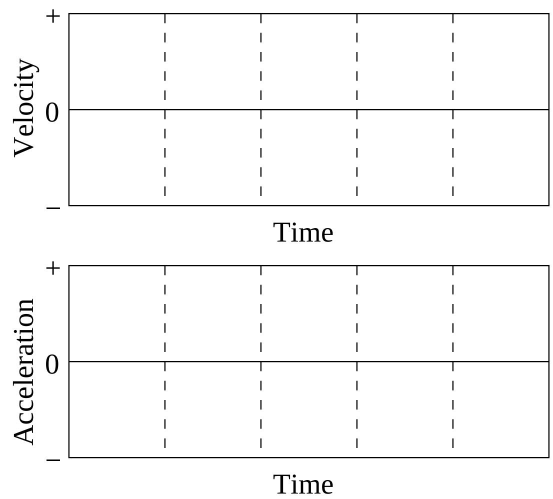


12. The car starts from rest and moves toward the origin, speeding up at a steady rate.



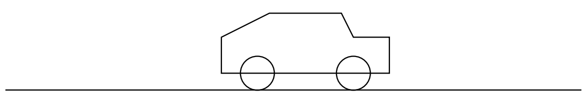


13. A ball is tossed in the air. It moves upward, reaches its highest point and falls back downward. Sketch a velocity-time and an acceleration-time graph for the ball from the moment it leaves the thrower's hand until the moment just before it reaches her hand again. Consider the positive direction to be upward.

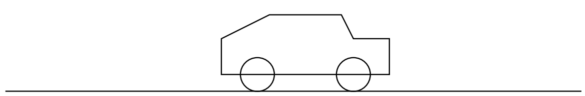


14. Each of the pictures below represents a car driving down a road. The motion of the car is described. In each case, draw velocity and acceleration vectors above the car which might represent the described motion. Also specify the sign of the velocity and the sign of the acceleration. (The positive direction is toward the right.)

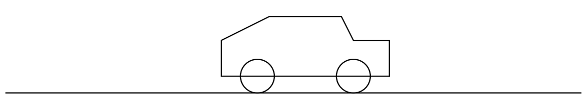
(a) The driver has stepped on the accelerator and the car is just starting to move forward.



(b) The car is moving forward. The brakes have been applied. The car is slowing down, but has not yet come to rest.

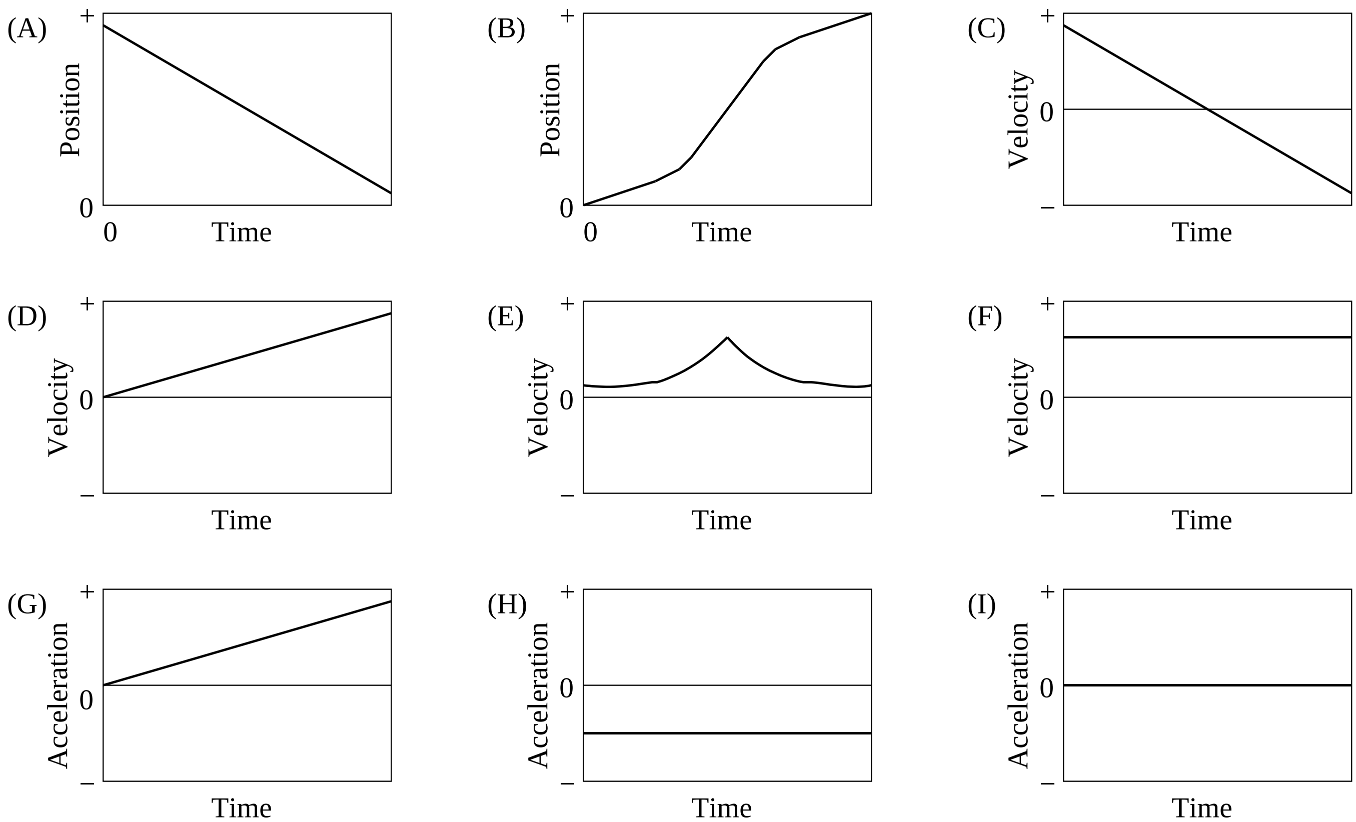


(c) The car is moving backward. The brakes have been applied. The car is slowing down, but has not yet come to rest.



The following graphs represent the motions of objects along the positive position axis. Notice that the motion of the objects is represented by position, velocity, or acceleration graphs.

Answer the following questions. You may use a graph more than once or not at all, and there may be more correct choices than blanks. If none of the graphs is correct, answer none.



15. Pick one graph that gives enough information to indicate that the velocity is always negative. ________

Pick three graphs that represent the motion of an object whose velocity is constant.

16. ________ 17. ________ 18. ________

19. Pick one graph that definitely indicates an object has reversed direction. ________

20. Pick one graph that might possibly be that of an object standing still. ________

Pick 3 graphs that represent the motion of objects whose acceleration is changing.

21. ________ 22. ________ 23. ________

Pick a velocity graph and an acceleration graph that could describe the motion of the same object during the time shown.

24. Velocity graph. ________ 25. Acceleration graph. ________

14 Equations to Define Velocity and Acceleration[9]

Name ______________________ Section ____________ Date ____________

Objectives

To learn how physicists use mathematical equations to describe simple one-dimensional motions by:

1. Understanding the mathematical definitions of both average and instantaneous velocity and acceleration as well as the meaning of the slope of a position vs. time graph and of a velocity vs. time graph.
2. Learning to use different techniques for measuring length and time and to use mathematical definitions of average velocity and acceleration in one dimension to determine these quantities from fundamental measurements.

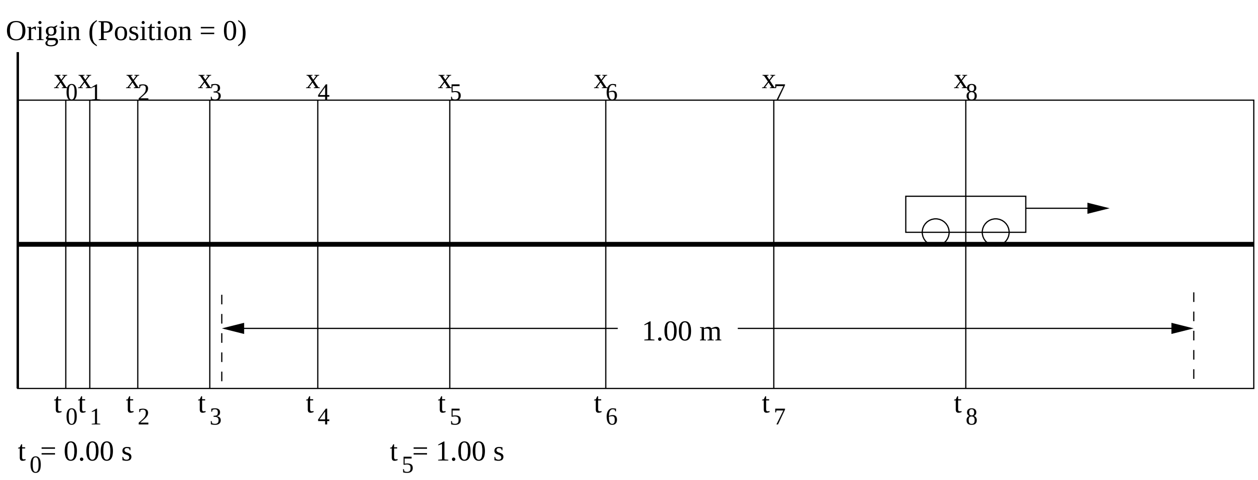
Apparatus

- Ruler with centimeter scale

Measuring Position as a Function of Time

When you used the motion detector, the computer took care of all the length and time measurements needed to track motion automatically. In order to understand more about how the motion software actually translates measurements into one dimensional velocities and accelerations it is helpful to make your own length and time measurements for a cart system.

Consider the type of uniformly accelerating cart motion that you studied in the previous two units. Suppose that instead of a motion detector you have a video camera off to one side so you can film the location of the cart 30 times each second. (This is the rate at which a standard video camera records frames.) By displaying frames at regular time intervals it is possible to view the position of the cart on each frame as shown in the figure below. This figure shows a scale diagram of the position of an accelerating cart at 8 equally spaced time intervals. The cart actually moved a distance of just less than 1 meter. Every 6th frame was displayed in the cart movie, so that 5 frames were recorded each second. At each time the center of the cart is located in the upper left corner of the rectangle with a number 1 in it.



Activity 1: Position vs. Time from a Cart Video

[9]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(a) Let's start the measuring process by recording the key scaling factors that will be used to calibrate the time and distance measurements. How much time, t, has elapsed between frame 0 and frame 1, between frame 1 and frame 2, etc. What is the calibration factor (i.e., how many real meters are represented by each centimeter in the diagram)?

t = ______________ s m/cm = ______________

(b) Use a ruler to measure the cart's distance from the origin (i.e., its position) in cm at each of the times (0.00 s, 0.20 s, etc.) and fill in columns 1 and 2 in table the below.

(c) Use the scaling factor between "diagram centimeters" and real meters to calculate the position in meters of the cart. Place the results in column 3.

Position	Distance from origin in diagram 1 x (cm)	Elapsed time 2 t (s)	Actual distance from origin 3 x (m)	Average velocity 4 $\langle v \rangle$ (m/s)	Average acceleration 5 $\langle a \rangle$ [(m/s)/s]
x_0		0.000		-	-
	-	0.100	-		-
x_1		0.200		-	
	-	0.300	-		-
x_2		0.400		-	
	-	0.500	-		-
x_3		0.600		-	
	-	0.700	-		-
x_4		0.800		-	
	-	0.900	-		-
x_5		1.000		-	
	-	1.100	-		-
x_6		1.200		-	
	-	1.300	-		-
x_7		1.400		-	
	-	1.500	-		-
x_8		1.600		-	-

How Do You Define Average Velocity Mathematically?

By considering the work you did with the motion detector and with the measurements you just performed in Activity 1, you should be able to define average velocity along a line in words or even mathematically. Remember that velocity is the rate of change of position divided by the time interval over which the change occurred.

Note: Mathematically, change is defined as the difference between the final value of something minus the initial value of something.

$$\text{Change} = (\text{Final Value}) - (\text{Initial Value})$$

Activity 2: Defining Velocity in One Dimension

(a) Describe in words as accurately as possible what the word "velocity" means by drawing on your experience with studying velocity graphs of motion. Hint: How can you tell from the graph the direction an object moves? How can you tell how fast it is moving?

(b) Suppose that you have a long tape measure and a timer to keep track of a cart or your partner who is moving irregularly along a line. For the purposes of this analysis, assume that the object of interest is a mere point mass.

Describe what you would need to measure and how you would use these measurements to calculate velocity at a given moment in time.

(c) Can you put this description in mathematical terms? Denote the average velocity with the symbol $\langle v \rangle$. Suppose the distance from the origin (where the motion detector was when it was being used) to your partner is x_1 at a time t_1 just before the moment of interest and that the distance changes to x_2 at a later time t_2 which is just after the moment of interest. Write the equation you would use to calculate the average velocity, $\langle v \rangle$, as a function of x_1, x_2, t_1, and t_2. What happens to the sign of $\langle v \rangle$ when x_1 is greater than x_2?

(d) Use the mathematical definition in part (c) to calculate the average velocity for each time interval of the cart motion described in Activity 1 and fill in column 4 of the table in Activity 1. Show at least one sample calculation in the space below. Important note: $t_2 - t_1$ represents a time interval, Δt, between two measurements of position and is not necessarily the total time that has elapsed since a clock started.

Defining Average Acceleration Mathematically

By considering the work you did with the motion detector, you should be able to define average acceleration in one dimension mathematically. It is similar to the mathematical definition of average velocity which you developed in Activity 2. All of the circumstances in which accelerations are positive and negative are described by the equation that defines them.

Activity 3: Defining Average Acceleration

(a) Describe in words as accurately as possible what the word "acceleration" means by drawing on your experience with studying velocity and acceleration graphs of motion.

(b) Suppose the cart's average velocity is $\langle v_1 \rangle$ at a time t_1 and that the average velocity changes to $\langle v_2 \rangle$ at a later time t_2. Write an equation for the average acceleration in the space below.

(c) Use the mathematical definition in part (b) to calculate the average acceleration of the cart motion depicted in Activity 1 and fill in column 5 of the table in Activity 1 for each time interval (i.e., 0.00 to 0.20 s, 0.20 s to 0.40 s, etc.). Show at least one sample calculation in the space below.

(d) Suppose you are walking away from a motion detector. How does the rate of your walking change if $\langle v_1 \rangle$ is greater than $\langle v_2 \rangle$? Is your acceleration positive or negative? Use the mathematical equation in part (b) to explain your answer.

(e) Suppose you are walking toward a motion detector. How is your speed (i.e., magnitude of velocity) changing for $\langle v_1 \rangle > \langle v_2 \rangle$? Is your acceleration positive or negative? Be very careful with your mathematics on this one. It's tricky!

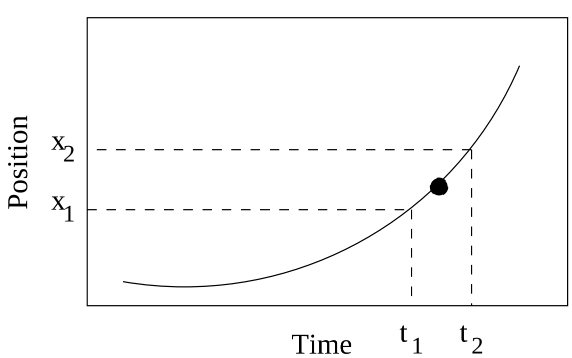
(f) Is the sign of $\langle a \rangle$ always the same as the sign of the velocities? Why or why not?

Instantaneous Velocity and the Slope of a Position vs. Time Graph

What happens if we want to know the velocity of an object at a single instant? That is, we'd like to estimate the velocity of an object during a time interval which is too small to measure directly. Since velocity is a measure of the change in position over time, it is possible to use techniques developed in calculus to estimate how a continuously varying function of position vs. time changes during a very short time interval. Let's start by considering how we might determine the slope of a continuous function and proceed from there.

Activity 4: Defining the Slope or Tangent

(a) In the figure below, what is the equation for the average slope of the curve at the highlighted point in terms of x_1, x_2, t_1, and t_2?



(b) How is the value of the slope related to the average velocity in the time interval $[t_1, t_2]$?

(c) Since the rate of change of position is increasing as time goes on (so that the position "curve" is not linear), how can you calculate a more accurate value of the slope? Hint: Feel free to use different x and t values in your "calculation" to correspond to a different time interval.

(d) How would you find the "exact" value of the slope at the point in time of interest?

(e) Look up the formal definition of a derivative in your calculus book and list it below. Usually it has to do with $f(x)$ and how it changes with x.

(f) Notice that in our position vs. time graph we are interested in how x changes with t. Thus, we would use the notation $x(t)$ to indicate that x is a function of t. By letting x play the role of f and t play the role of x, rewrite the definition of the derivative.

(g) How might the instantaneous value of velocity at the highlighted point be related to the derivative of x with respect to t at that same point?

(h) Suppose that $x(t) = t^2 + 1$, where x is in centimeters and t is in seconds. What is the derivative of this function with respect to time? What is its instantaneous velocity?

(i) What is the instantaneous velocity, v, in cm/s at $t = 1$ s? At $t = 2$ s?

Acceleration as the Slope of a Velocity Graph

Just as velocity is the rate of change of position, acceleration is the rate of change of velocity. How do we find the acceleration of an object at a single instant (i.e., during a time interval which is too small to measure directly). Since acceleration is the rate of change of velocity, the acceleration of an object is given by the slope of a smooth curve drawn through its velocity vs. time graph.

Let's apply this graphical analysis approach to the task of finding the instantaneous acceleration for the cart motion described in Activity 1.

Activity 5: Accelerations from the Cart Data

(a) Refer to the data that you analyzed and recorded in the table in Activity 1. Use Excel to fit the velocity (y axis) vs. time (x axis) data with a line. Label the resulting graph and put a copy in your notebook. The slope of the line represents the acceleration of the cart. Report its value in the space below.

(b) How does the value for acceleration which you determined from the slope compare to those you determined earlier from the average accelerations at the mid-point of each time interval between average velocity values. In other words, how does the slope compare with the averages reported in column 5 of the table in Activity 1? Find the average and standard deviation of the elements in column 5 of the table and report them in the space below. (You can use Excel for this.)

(c) Does the acceleration determined by the slope lie within one standard deviation of the average of the average accelerations?

15 Gravity and Free Fall[10]

Name ______________________ Section ____________ Date ____________

Objectives

To explore the phenomenon of gravity and study the nature of motion along a vertical line near the earth's surface.

Overview

When an object falls close to the surface of the earth, there is no obvious force being applied to it. Whatever is causing it to move is invisible. Most people casually refer to the cause of falling motions as the action of "gravity." What is gravity? Can we describe its effects mathematically? Can Newton's laws be interpreted in such a way that they can be used for the mathematical prediction of motions that are influenced by gravity? In this investigation we will study the phenomenon of gravity for vertical motion. You will need:

Apparatus

- A tennis ball.
- A video analysis system (*VideoPoint*).
- Graphing software (*Excel*).

Vertical Motion: Describing How Objects Rise and Fall

Let's begin the study of the phenomenon of gravity by predicting the nature of the motion of an object, such as a tennis ball, when it is tossed up and then allowed to fall vertically near the surface of the earth. This is not easy since motion happens pretty fast! To help you with this prediction you should toss a ball in the laboratory several times and see what you think is going on.

Activity 1: Predicting the Motion of a Tossed Ball

(a) Toss a ball straight up a couple of times and then describe how you think it might be moving when it is moving upward. Some possibilities include: (1) rising at a constant velocity; (2) rising with an increasing acceleration; (3) rising with a decreasing acceleration; or (4) rising at a constant acceleration. What do you think? (You may want to review Activity 7 of Experiment 13.)

(b) Explain the basis for your prediction.

(c) Now describe how you think the ball might be moving when it is moving downward. Some possibilities include: (1) falling at a constant velocity; (2) falling with an increasing acceleration; (3) falling with a decreasing acceleration; or (4) falling at a constant acceleration. What do you think?

[10] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

(d) Explain the basis for your prediction.

(e) Do you expect the acceleration when the ball is rising to be different in some way from the acceleration when the ball is falling? Why or why not?

(f) What do you think that the acceleration will be at the moment when the ball is at its highest point? Why?

The motion of a tossed ball is too fast to observe carefully by eye without the aid of special instruments. In the next activity we will use a video analysis system to study the motion of a freely falling object. You will use a sequence of these video frames and mathematical modeling techniques to find an equation that describes the fall.

Activity 2: Analyzing the Motion of a Tennis Ball

(a) Make a movie of a tennis ball in flight by following these steps.

1. Turn on the video camera and center the field of view on the region where you will toss the ball. This region should be about 2 meters from the camera to get a large enough area for the flight of the ball. Place a ruler or meter stick somewhere in the field of view where it won't interfere with the motion. This ruler will be used later to determine the scale.
2. Make a movie of the tennis ball being dropped from rest. Make sure most of the trajectory is visible to the camera. See **Appendix D: Video Analysis** for details on making the movie.

IMPORTANT: Do NOT save the movie to your netfiles space. Save it to the DESKTOP as indicated in **Appendix D** number 2. (Before logging out later you can save the movie file to your own space on Saturn.)

(b) Determine the vertical position, y, of the tennis ball at different times during the motion. To do this follow the instructions in the second section of **Appendix D: Video Analysis - Analyzing the Movie**.

(c) Use *Excel* to plot a graph of y vs. t. See **Appendix C: Introduction to Excel** for details. Affix a copy of the graph to this unit.

(d) What is the initial value of y (usually denoted y_0)?

(e) By examining your data table, calculate the approximate value of the initial velocity of the ball in the y-direction. Include the sign of the velocity and its units. (Use the convention that on the y-axis up is positive and down is negative.)

(f) Examine the graph of your data. What does the nature of this motion look like? Constant velocity, constant acceleration, an increasing or decreasing acceleration? How does your observation compare with the prediction you made earlier in this unit for the ball on its way down?

(g) Using the convention that on the y-axis up is positive and down is negative, is the acceleration positive or negative (i.e., in what direction is the magnitude of the velocity increasing)?

(h) If you think the object is undergoing a constant acceleration, use the fitting capability of *Excel* (see **Appendix C: Introduction to Excel** for details) to find an equation that describes y as a function of t as the ball drops. Hints: (1) You might try to model the system with a second order equation like the kinematic equation for uniformly accelerated motion. (2) Write the equation of motion in the space below. Then use coefficients of the best-fit equation to find the values of a, v_0 and y_0 with the appropriate units. Note: Since the acceleration is caused by gravity, our notation for it will be g rather than just a. Also, when you have found a good fit to the data, print it and attach a copy to this unit.

1. The equation of motion with proper units is: $y =$

2. The acceleration with proper sign and units is: $g =$

3. The initial velocity with proper sign and units is: $v_0 =$

4. The initial position with proper sign and units is: $y_0 =$

16 Acceleration of Gravity

Name ______________________ Section ____________ Date ____________

Objective

To determine the acceleration of gravity near the surface of the earth.

Overview

In a previous activity, you performed a video analysis of an object in free fall to determine the equation of motion and to extract a value for the acceleration of gravity near the surface of the earth. Now we will make a more accurate determination of the acceleration of gravity by timing the motion of a freely falling object. The "picket fence" has evenly spaced black bars on a piece of clear plastic. When dropped through the photogate, the bars interrupt the light beam. By measuring the distance between the bars, and using the time measurements of the photogate, the acceleration of the freely falling picket fence can be calculated.

Apparatus

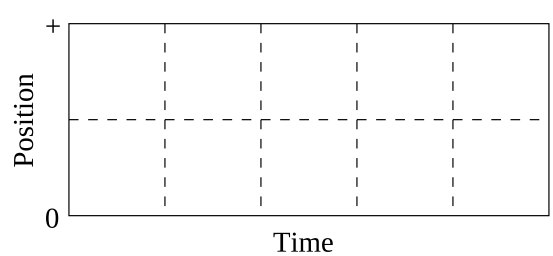
- *Science Workshop 750 Interface*
- Photogate
- Picket fence
- *DataStudio* software (Free Fall application)

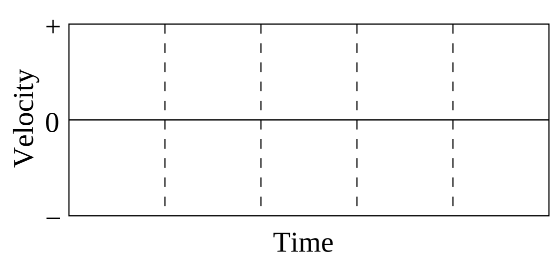
Note: On using the "Picket Fence"

1. When performing free-fall experiments, place a box with packing material under the experiment to cushion the fall of the picket fence.
2. For accurate results drop the picket fence through the photogate vertically.
3. To achieve vertical alignment of the picket fence hold it between your thumb and forefinger, centered at the top of the bar, before releasing.

Activity 1: Position and Velocity Graphs

(a) Consider an object in free fall near the surface of the earth. Sketch your predictions for the position vs. time and velocity vs. time graphs of the motion of the object using dashed lines on the axes below. Assume that the positive position axis is down.





(b) Test your predictions by measuring the free fall of the picket fence. Open the Free Fall application. Have one person hold the picket fence in a vertical position above the gap between the arms of the photogate. Have another person start recording data and then have the first person drop the picket fence through the photogate. **Stop** the recording once the picket fence has passed completely through the photogate. The computer will display graphs of position versus time and velocity versus time. Sketch the results on the above axes using solid lines.

(c) How do your predictions compare with the results of the measurements?

(d) What can you say about the magnitude and sign of the acceleration in this case?

Activity 2: Determining the Acceleration of Gravity

(a) Fit the velocity vs. time graph to determine the acceleration. Repeat this procedure to get a total of 20 good runs and record your results in a data table below. For one of the runs, print the graph window and put a copy in your notebook at the end of this unit.

(b) Use Excel to find the mean and standard deviation for the acceleration measurements and record the results below in the form $g = \text{Mean} \pm \sigma$.

(c) Compare your average acceleration with the standard value of 9.80 m/s^2. This is done by calculating the % difference. The % difference is calculated by subtracting the accepted value from your value, dividing by the accepted value and multiplying by 100.

(d) Does your mean acceleration fall within one standard deviation of the accepted value? Is there any indication of a systematic error? Explain.

17 Human Reaction Time

Name ________________________ Section ____________ Date ____________

Objectives:

- Application of the free-fall concept
- Appreciation of the magnitude of reaction time

Introduction

Since you now know the magnitude of gravitational acceleration, you can determine your reaction time from the distance a meter stick drops before you catch it.

Apparatus:

- meter stick
- stop watch

Activity: [Note: everyone should determine their individual reaction time.]

1. With a lab partner holding a meter stick vertically from the 0.0 end, hold your thumb and index finger close to, but not touching, opposite sides of the stick at the 50.0-cm mark.
2. The holder should release the stick without warning and you should try to grasp it as quickly as possible. Record the position at which you caught the stick.
3. Repeat the exercise five times. Then calculate the average position and the characteristic displacement, s, of the stick before you can catch it.

P_1	P_2	P_3	P_4	P_5	Ave. P	$s = 50.0 - P$

4. Calculate your reaction time, t. (show calculations)

Reaction time $\equiv t =$ ________________________

Questions:

1. How many feet would a car traveling 55 mi./hr travel during your reaction time? (show calculations)
2. Could you catch a dropped dollar bill (15.5 cm long) if your fingers were initially at the a) center, b) bottom? Use the experimental data just taken to justify your answers.

18 The Tossed Ball

Name ______________________ Section ____________ Date ____________

Objectives:

- Application of the free-fall concept
- Investigation of linear motion in two directions with constant acceleration

Introduction

In the Free Fall investigation, you determined the acceleration of objects allowed to fall from rest. In this lab you will study this sort of motion with an initial velocity.

Predictions

If you throw a ball straight up into the air, releasing it at the level of the table-top with enough velocity so that it just reaches the top of a two-meter stick resting on the floor, how long will the ball's flight take from its release until it hits the floor? List all your assumptions and the measurements you will make, and show all your calculations in making this prediction.

Apparatus:

- 2-meter stick
- tennis ball
- 2 stop watches

Activity:

1. To test your predictions, stand the two-meter stick on its end. One of the experimenters should be positioned to view the high end of the stick.

2. A second experimenter throws the ball up in such a way that the ball is released at the level of the table-top and just reaches the top of the two-meter stick. You might try grasping the table-edge with your non-throwing arm extended out over the floor. Throw from under this extended arm, releasing the ball as your two arms make contact. One or more stop watches should be started at the instant of release. Repeat as necessary until the maximum height of the ball is as close to two-meters as possible.

3. Time the flight of the ball from its release until it hits the floor. When you get a good toss, record the value and compare it to your prediction.

Time predicted ______________________ Time measured ______________________

Questions:

1. You throw a ball straight up in the air. Some time later, the ball falls to the ground. Is the acceleration constant during the entire flight of the ball? Explain.

2. What is (are) the value(s) and direction(s) of the acceleration at the instant the ball is released? At its highest point? On the way down? Just before it hits the floor?

3. Does your measurement agree with your prediction? Explain any discrepancy.

4. What was the initial velocity of the ball?

5. What was the ball's velocity when it reached the level of the table-top on its way down?

6. What was the ball's velocity when it hit the ground?

19 Independence of Vertical and Horizontal Motion

Name ______________________ Section ____________ Date ____________

Objective:

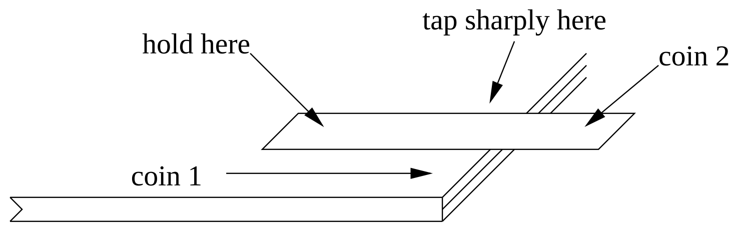
- Determine the relationship between vertical and horizontal motion

Apparatus:

- two coins
- ruler

Activity:

1. Place a coin in the far corner of the table nearest the center aisle of the room. (See figure.) Lay a ruler next to the coin, parallel to the edge of the table along the aisle, allowing several centimeters at the end to hang over the far edge. Place a coin on the portion of the ruler hanging over the edge. Hold the ruler in place with a finger on the opposite end.



2. With the ruler held, tap the side of the ruler opposite the coin sharply so that the coin on the table will be knocked over the edge of the table.
3. Both coins should land on the floor. Note, by sight or hearing, which hits the ground first.
4. Sketch the motion of the two coins.

Questions:

1. Which coin hit the floor first? Explain.
2. What was the acceleration of each of the coins? Explain.
3. Which coin had the greatest initial velocity? Explain.
4. Which coin had the greatest final velocity? Explain

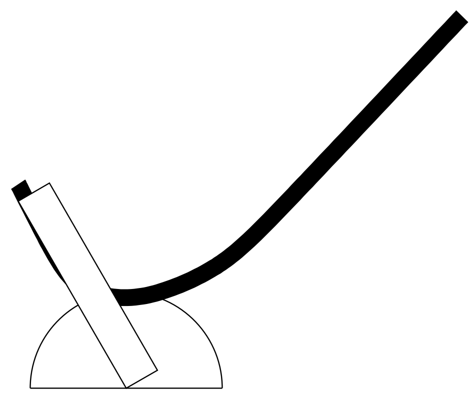
20 The Ski Jump

Name ______________________ Section ____________ Date ____________

Objectives:

- Introduce projectile motion
- Discover the angular dependence for trajectories

Note: The ski jump works by releasing the metal ball to roll down the channel and recording where it lands. For the purposes of laboratory exercise, the ball should be released from the same vertical height for all trials; use the horizontal support attached to the tall rod as a convenient reference point. The launch point is connected to a horizontal support attached to a short rod. This horizontal support rotates when you move the base of the tall rod nearer to or farther from the shorter rod. This rotation determines the angle at which the jump originates. The launching angle may be determined with a ruler and protractor by holding the ruler against the track in parallel with the launch point and then measuring the angle above the horizontal with the protractor (see figure below).



Apparatus:

- jump chute
- metal ball
- protractor and ruler
- two wooden boards
- sheets of white and carbon paper

Activity:

1. Set up the jump and landing surface so that the launch point and landing surface are at the same height.
2. With the benefit of several trial runs at different angles, decide where you should tape a piece of white paper down so all jumps will land on the paper. Cover the paper with a piece of carbon paper, ink side down.
3. Attempt several jumps each at a number of different angles, say 20, 30, 37, 45, 53, 60, and 70 degrees. Circle the impact points for a given angle and label them before proceeding to the next angle.
4. Note the relative ranges of the ball's flight as a function of the launching angle:

Questions:

1. Is the range always the same, or does it depend on the angle? Describe the dependence, if any.

2. At which angle is the range a maximum?

3. How are the ranges and angles related on either side of the maximum?

4. How do you think this angular dependence would change for landing surfaces at different heights? Test your predictions.

21 Projectile Motion[11]

Name ______________________ Section ____________ Date ____________

Objectives

To understand the experimental and theoretical basis for describing projectile motion as the superposition of two independent motions: (1) a body falling in the vertical direction, and (2) a body moving in the horizontal direction with no forces.

Apparatus

- A tennis ball.
- A video analysis system (*VideoPoint*).
- Graphing and curve fitting software (*Excel*).

Activity 1: Predicting the Two-Dimensional Motion of a Tossed Ball

(a) Toss a tennis ball up at an angle of about 60° with the horizon a couple of times. Sketch the motion and describe it in words below. What is the shape of the trajectory?

(b) Let's consider the horizontal and vertical components of the motion separately. What do you think is the horizontal motion of the ball? Is it motion with constant velocity? Constant acceleration? Or is it some other kind of motion? Hint: What is the force acting on the ball in the horizontal direction (after it is released)?

(c) What do you think is the vertical motion of the ball? Is it motion with constant velocity? Constant acceleration? Or is it some other kind of motion? Hint: What is the force acting on the ball in the vertical direction (after it is released)?

The two-dimensional motion of a tossed ball is too fast to observe carefully by eye without the aid of special instruments. In the next activity we will use a video analysis system to study the motion of a small ball launched at an angle of about 60° with respect to the horizontal. You are to use the video analysis software and mathematical modeling techniques to find the equations that describe: (a) the trajectory (y vs. x), (b) the horizontal motion (x vs. t), and (c) the vertical motion (y vs. t) of the projectile.

Activity 2: Analyzing Projectile Motion

(a) Make a movie of a tennis ball in flight by following these steps.

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1. Set up the video camera and center the field of view on the region where you will toss the ball. This region should be about 2 meters from the camera to get a large enough area for the flight of the ball. Place a ruler or meter stick somewhere in the field of view close to the plane of the motion of the ball where it won't interfere with the motion. This ruler will be user later to determine the scale.

2. Make a movie of the tennis ball flying through the air with a significant component of its initial velocity in the horizontal direction (i.e., don't toss it straight up). Make sure most of the complete trajectory is visible to the camera. See **Appendix D: Video Analysis** for details on making the movie.

(b) Determine the position of the projectile at different times during the motion. To do this task follow the instructions in the second section of **Appendix D: Video Analysis** for recording and calibrating the position data. When you are analyzing the movie place the origin at the position of the ball in your first frame. Do this by clicking on the arrow symbol on the menu bar to the left of the movie frame. The cursor will take the shape of an arrow when you place it over the frame. Move the point of the cursor's arrow to the origin. Click and drag the cursor and move the origin to the position of the ball in the first frame and release. This sets the origin at the location of the ball at the initial time. When you have placed the origin at the desired spot, click on the circular icon at the top of the menu bar to the left of the movie frame. This returns the cursor to a circle that marks the position of the ball. When you have finished marking the ball's position, export the data into an *Excel* file as described in **Appendix D**.

(c) Open your data in *Excel*. Launch *Excel*. See **Appendix C: Introduction to Excel** for details on using *Excel*. Make a plot of the vertical position (y) versus the horizontal position (x). Determine the equation that describes the trajectory of the projectile. When you have found a good fit to the data, print the graph and attach a copy to the unit. Write the equation for the trajectory of the projectile in the space below. What is the shape of the trajectory? Does the result agree with your earlier prediction?

(d) Determine the equation that describes the horizontal motion of the projectile by plotting the horizontal position (x) versus time (t) using *Excel*. Find a good fit to the data with *Excel*. When you have found a good fit to the data, print the graph and attach a copy to the unit. What kind of motion is it? Does the result agree with your earlier prediction?

1. The equation for the horizontal component of the motion with proper units is: $x =$

2. The horizontal component of the acceleration with proper sign and units is: $a_x =$

3. The horizontal component of the initial velocity with proper sign and units is: $v_{0x} =$

4. The initial x position with proper units is: $x_0 =$

(e) Determine the equation that describes the vertical motion of the projectile by plotting the vertical position (y) versus time (t). Find a good fit to the data. When you have found a good fit to the data, print the graph and attach a copy to the unit. What kind of motion is it? Does the result agree with your earlier prediction?

1. The equation for the vertical component of the motion with proper units is: $y =$

2. The vertical component of the acceleration with proper sign and units is: $a_y =$

3. The vertical component of the initial velocity with proper sign and units is: $v_{0y} =$

4. The initial y position with proper units is: $y_0 =$

(f) Does it appear that projectile motion is simply the superposition of two types of motion that we have already studied? Explain.

22 Uniform Circular Motion[12]

Name ______________________ Section ____________ Date ____________

Objective

To explore the phenomenon of uniform circular motion and the acceleration needed to maintain it.

Overview

You have recently studied projectile motion. In this unit we are going to explore another phenomenon in two dimensions, uniform circular motion. In particular, you will develop a mathematical description of the centripetal acceleration that keeps an object moving in a circle. You will start by watching a toy "airplane" suspended from a string fly in a circle.

Apparatus

- The "airplane."
- A video analysis system(*VideoPoint*).
- Graphing software (*Excel*).

Activity 1: Observing an Airplane Undergoing Circular Motion

(a) Put on your safety glasses and wear them until your instructor announces that they can be removed. Turn on the engine of the airplane suspended from the string. Be careful to stay clear of the propeller. Launch the airplane into circular motion by giving it a gentle push. If it doesn't quickly settle into steady circular motion, catch it and try launching it in the opposite direction. Sketch the motion and describe it in words below.

(b) How would you describe the speed of the airplane? How would you describe the velocity? Would you say that this is accelerated motion? Why?

(c) What is the definition of acceleration? (Remember that acceleration is a vector!)

(d) Are velocity and speed the same thing? Is the velocity of the airplane constant? (Hint: Velocity is a vector quantity!)

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(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

The two-dimensional motion of the airplane is too fast to observe carefully so we will use a video system to record the motion of the airplane and to analyze the movie frame-by-frame. You will use the video analysis system to investigate the direction of the velocity of the circling airplane and the direction of its acceleration.

Activity 2: Analyzing Circular Motion

(a) Make a movie of the airplane in flight by following these steps.

1. Turn on the video camera and center the field of view on the region where the plane will fly. This region should be about 1 meter from the camera to get a large enough area for the flight of the plane. Mount a ruler or meter stick somewhere in the field of view where it won't interfere with the motion. This ruler will be used later to determine the scale.
2. Launch the airplane into circular motion and wait until it settles into steady, circular flight. Record several revolutions of the airplane while holding the ruler in view of the camera and save the movie as the file Airplane. See **Appendix D: Video Analysis** for details on making movies and saving them in a file.

(b) Determine the position of the airplane during one complete revolution. To do this task follow the instructions in the second section of **Appendix D: Video Analysis** for recording and calibrating a data file entitled Airplane Data. The file should contain three columns with the values of time, x-position, and y-position for one complete revolution.

(c) Make a graph of the trajectory of the airplane during one full revolution. See **Appendix C: Introduction to Excel** for details on using the graphing software. When you make your plot make sure the x and y axes cover the same size range; otherwise you will distort the path of the airplane. Print the graph and attach a copy to the unit.

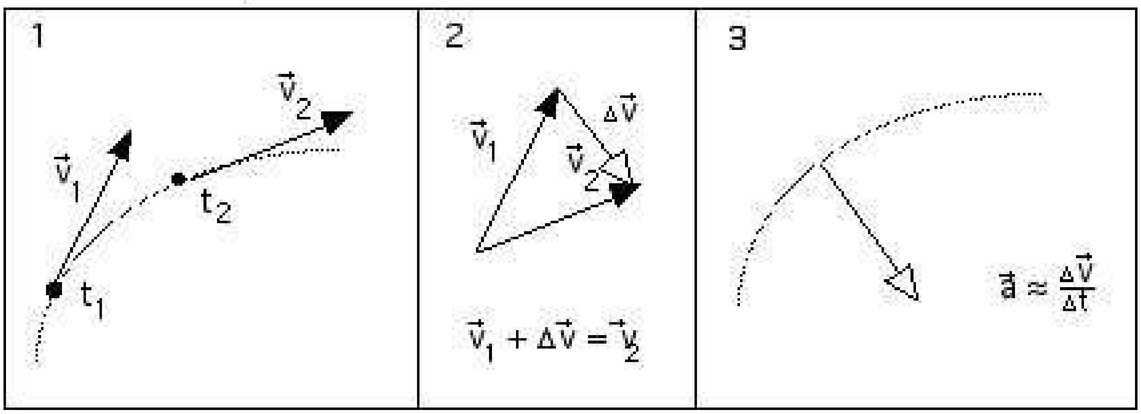
1. Is the motion circular? What is your evidence?
2. Make a sketch of the position vector and the velocity vector at one point on the trajectory. How did you arrive at this choice? How is the velocity vector related to the path of the airplane?
3. What would happen to the path of the plane if you cut the string? Explain.

(d) Launch the airplane into uniform circular motion. You are going to investigate what happens to the trajectory after the string is cut. Be sure you are wearing your eye protection. After the plane settles into steady, circular flight start recording a movie of the plane. After it completes one revolution use the scissors to cut the string just above the horizontal metal bar. BE CAREFUL to avoid letting the plane strike anyone or any object except the floor. Halt recording and save the movie.

(e) Make a plot of the trajectory of the airplane after the string was cut and for at least one full revolution before. Is the trajectory circular before the string was cut? Does the trajectory of the plane after the string was cut agree with the prediction you made earlier? Explain. Print the graph and attach a copy to the unit.

Using Vectors to Diagram How Velocity Changes

By now you should have concluded that since the direction of the motion of an object undergoing uniform circular motion is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the direction of the acceleration and its magnitude as a function of the speed v of an object such as a ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.



1. To Draw Velocities: Draw an arrow representing the velocity, $\mathbf{v}_1$, of the object at time t_1. Draw another arrow representing the velocity, $\mathbf{v}_2$, of the object at time t_2.

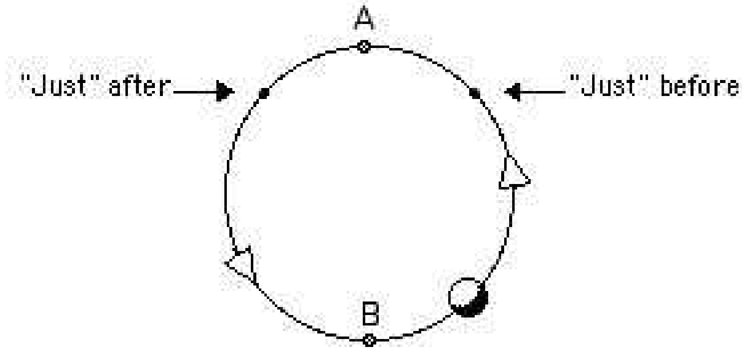
2. To Draw Velocity Change: Find the change in the velocity $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$ during the time interval described by $\Delta t = t_2 - t_1$. Start by using the rules of vector sums to rearrange the terms so that $\mathbf{v}_1 + \Delta\mathbf{v} = \mathbf{v}_2$. Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector that points from the head of the first velocity vector to the head of the second velocity vector.

3. To Draw Acceleration: The acceleration equals the velocity change $\Delta\mathbf{v}$ divided by the time interval t needed for the change. Thus, $\mathbf{a}$ is in the same direction as $\Delta\mathbf{v}$ but is a different length (unless $\Delta t = 1$). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as $\Delta\mathbf{v}$.

The acceleration associated with uniform circular motion is known as centripetal acceleration. You will use the vector diagram technique described above to find its direction.

Activity 3: The Direction of Centripetal Acceleration

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot just before it reaches point A. Label this vector $\mathbf{v}_1$.



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot just after it passes point A. Label this vector $\mathbf{v}_2$.

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. Next, draw the vector that must be added to vector $\mathbf{v}_1$ to add up to vector $\mathbf{v}_2$; label this vector $\Delta\mathbf{v}$. Be sure that vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ have the same magnitude and direction in this drawing that they had in your drawing in part (a)!

(d) Now, draw an exact copy of $\Delta\mathbf{v}$ on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original $\Delta\mathbf{v}$ in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration, $\mathbf{a}_c$?

(f) If you redid the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration, $\mathbf{a}_c$, would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed

You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use the rules of mathematics and the definition of acceleration to derive the relationship between speed, radius, and magnitude of centripetal acceleration.

Activity 4: How Does $\mathbf{a}_c$ Depend on v and r?

(a) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude, a_c, have to increase or decrease as r decreases if circular motion is to be maintained? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius r if the speed v is increased? In other words, would the magnitude, a_c, have to increase or decrease as v increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Lets use the definition of acceleration in two dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

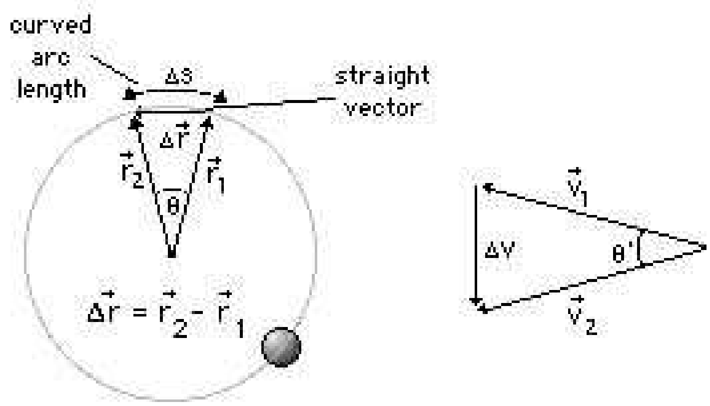
$$a_c = \frac{v^2}{r} \qquad [Eq.\,1]$$

In order to do this derivation you will want to use the following definition for acceleration

$$\langle \mathbf{a} \rangle = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad [Eq.\,2]$$

Activity 5: Finding the Equation for $\mathbf{a}_c$

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the position vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. Hint: In circular motion, velocity vectors are always perpendicular to their position vectors.



(b) Since the angles are the same and since the magnitudes of the displacements never change (i.e., $r = r_1 = r_2$) and the magnitudes of the velocities never change (i.e., $v = v_1 = v_2$), use the properties of similar triangles to explain why $\frac{\Delta v}{v} = \frac{\Delta r}{r}$.

(c) Now use the equation in part (b) and the definition of $\langle a \rangle$ to show that $\langle a_c \rangle = \frac{\Delta v}{\Delta t} = \frac{(\Delta r)}{(\Delta t)} \frac{v}{r}$.

(d) The speed of the object as it rotates around the circle is given by $v = \frac{\Delta s}{\Delta t}$. Is the change in arc length, Δs, larger or smaller than the magnitude of the change in the position vector, Δr? Explain why the arc length change and the change in the position vector are approximately the same when Δt is very small (so that the angle θ becomes very small) i.e., why is $\Delta s \simeq \Delta r$?

(e) If $\Delta s \simeq \Delta r$, then what is the equation for the speed in terms of Δr and Δt?

(f) Using the equation in part (c), show that as $\Delta t \to 0$, the instantaneous value of the centripetal acceleration is given by Eq. 1.

23 Force and Motion I[13]

Name ______________________ Section ____________ Date ____________

Objectives

- To learn how to use a force probe to measure force.
- To understand the relationship between forces applied to an object and its motions.
- To find a mathematical relationship between the force applied to an object and its acceleration.

Overview

In the previous labs, you have used a motion detector to display position-time, velocity-time and acceleration-time graphs of the motion of different objects. You were not concerned about how you got the objects to move, i.e., what forces (pushes or pulls) acted on the objects. From your experiences, you know that force and motion are related in some way. To start your bicycle moving, you must apply a force to the pedal. To start up your car, you must step on the gas pedal to get the engine to apply a force to the road through the tires.

But, exactly how is force related to the quantities you used in the previous unit to describe motion — position, velocity and acceleration? In this unit you will pay attention to forces and how they affect motion. You will apply forces to a cart, and observe the nature of its resulting motion graphically with a motion detector.

Apparatus

- Force probe
- Variety of hanging masses
- CS2000 compact scale (for measuring mass)
- Low friction pulley and string
- Motion detector
- Dynamics cart and track
- *Science Workshop 750 Interface*
- *DataStudio* software (V, A & F Graphs application)

Measuring Forces

In this investigation you will use a force probe to measure forces. The force probe puts out a voltage signal proportional to the force applied to the arm of the probe. Physicists have defined a standard unit of force called the newton, abbreviated N. For your work on forces and the motions they cause, it will be more convenient to have the force probe read directly in newtons rather than voltage. *Before each measurement you should zero the force probe by removing all forces and pushing the TARE button.*

Motion and Force

Now you can use the force probe to apply measured amounts of force to an object. You can also use the motion detector, as in the previous units, to examine the motion of the object. In this way you will be able to establish the relationship between motion and force.

Activity 1: Pushing and Pulling a Cart

In this activity you will move a cart by pushing and pulling it with your hand. You will measure the force, velocity and acceleration. Then you will be able to look for relationships between the applied force and the motion quantities, to see which is (are) related to force.

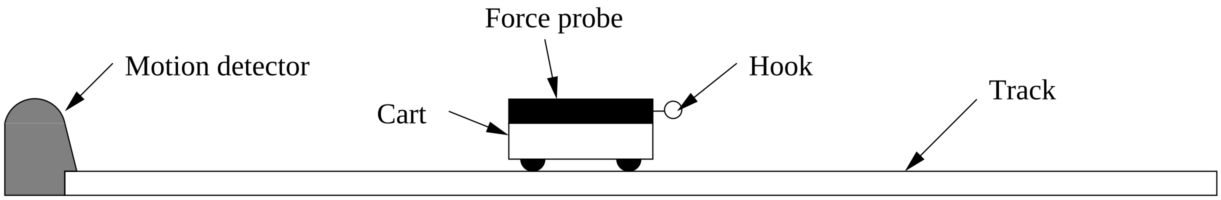


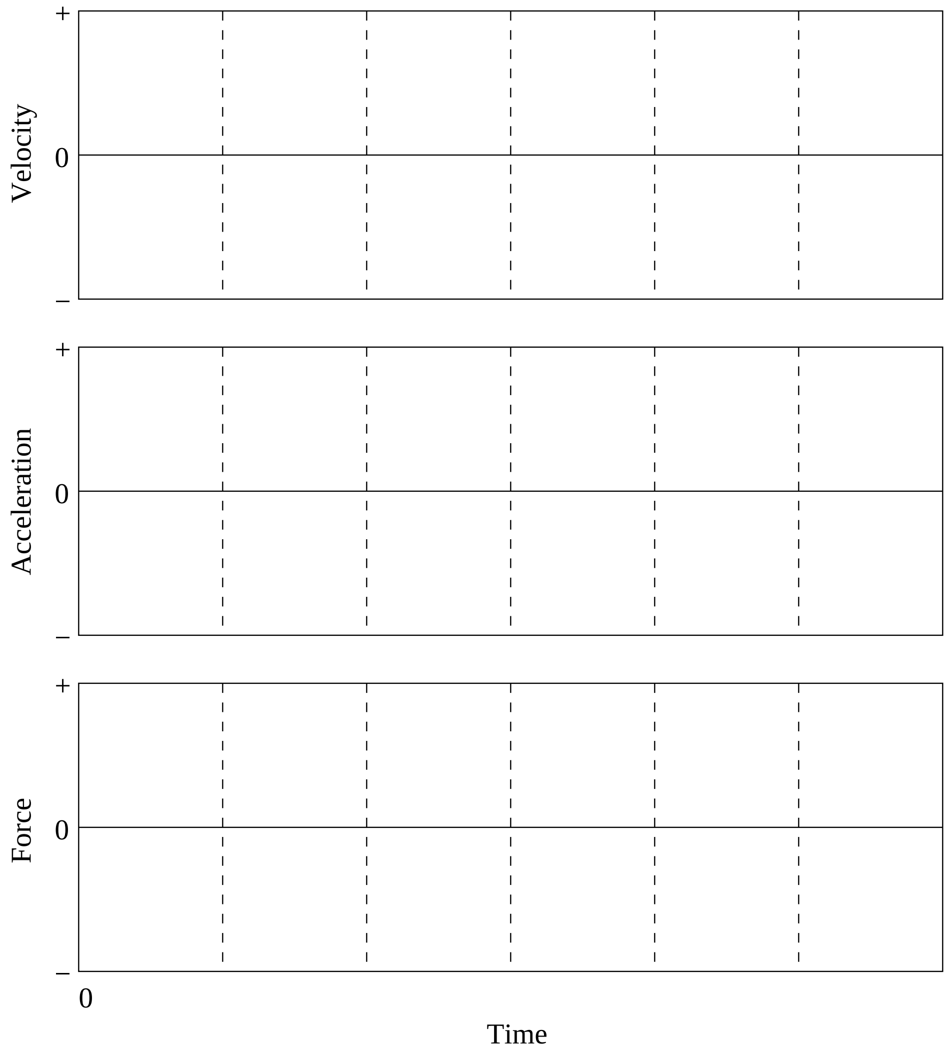
Figure 1: Equipment setup for qualitative measurements of force and motion.

(a) Set up the cart, force probe and motion detector on the level track as shown in Figure 1. Measure and record the mass of the cart and force probe assembly (using the compact scale).

(b) Suppose you grasp the hook on the force probe and move the cart forwards and backwards in front of the motion detector. Do you think that either the velocity or the acceleration graph will look like the force graph? Is either of these motion quantities related to force? That is to say, if you apply a changing force to the cart, will the velocity or acceleration change in the same way as the force?

(c) To test your predictions, open the V, A & F Graphs application. Grasp the end of the force probe arm and start acquiring data. When you hear the clicks, pull the cart away from the motion detector, and quickly stop it. Then push it back towards the motion detector, and again quickly stop it. Be sure that the cart never gets closer than 0.15 m away from the detector and be careful of the wires. Repeat until you get a good run, and adjust the sampling time and scale of the axes if necessary. Sketch your graphs on the axes that follow.

[13] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.



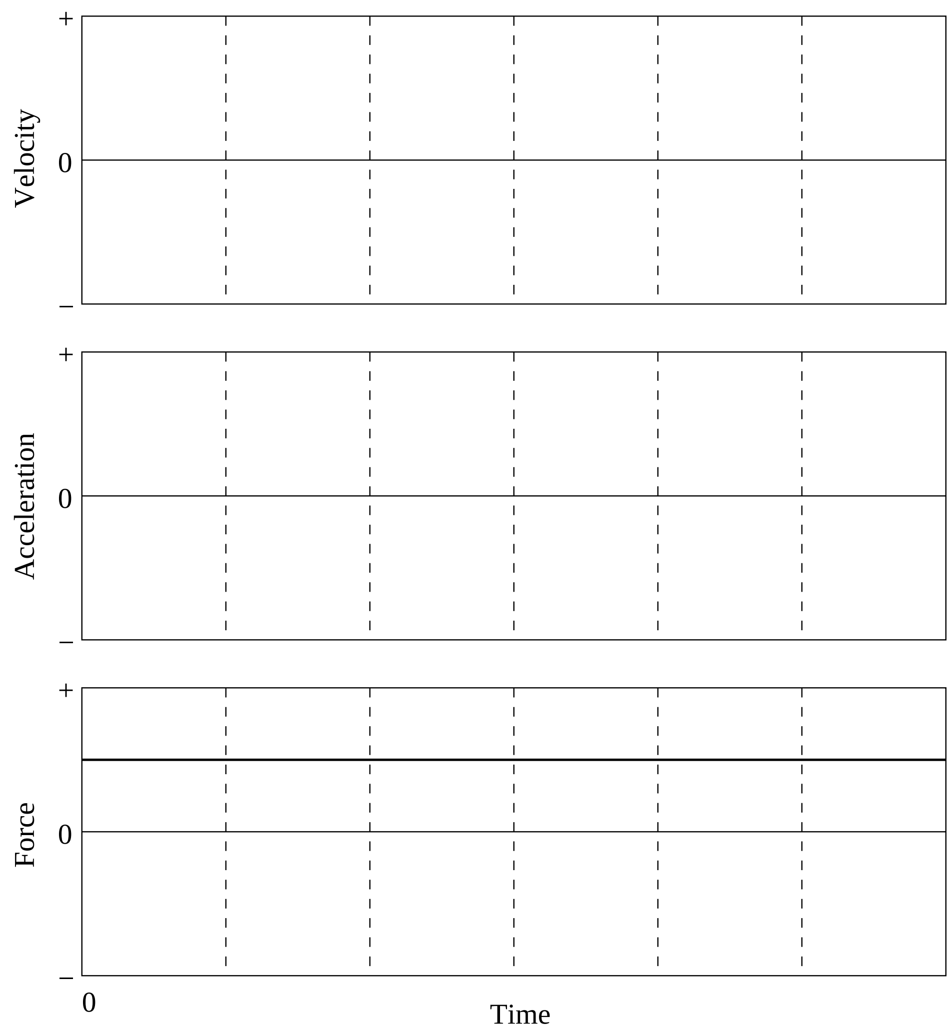
(d) Does either graph–velocity or acceleration–resemble the force graph? Which one? Explain.

(e) Based on your observations, does it appear that either the velocity or acceleration of the cart might be related to the applied force? Explain.

Activity 2: Speeding Up

You have seen in the previous activity that force and acceleration seem to be related. But just what is the relationship between force and acceleration?

(a) Suppose you have a cart with very little friction, and that you pull this cart with a constant force as shown below on the force-time graph. Sketch on the axes below the velocity-time and acceleration-time graphs of the cart's motion.



(b) Describe in words the predicted shape of the velocity vs. time and acceleration vs. time graphs for the cart.

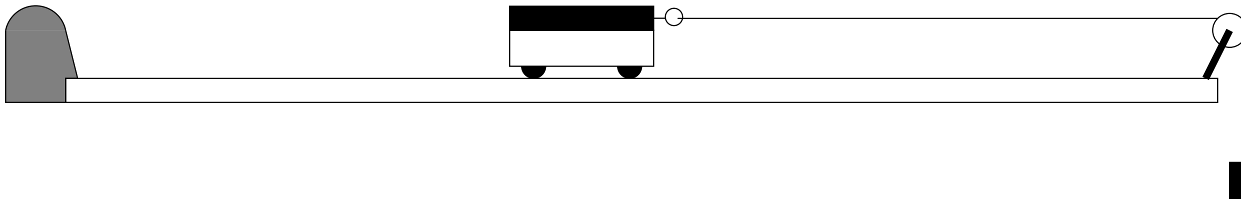
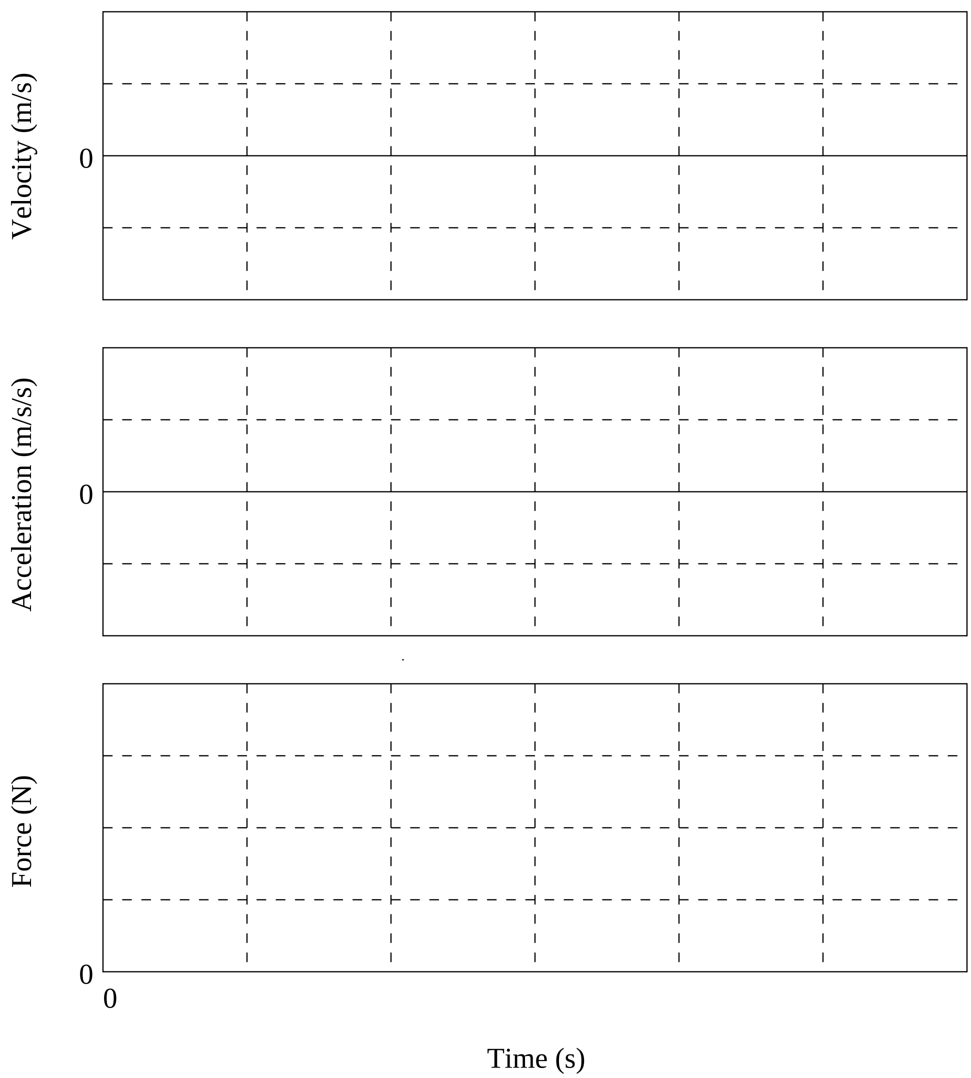


Figure 2: Equipment setup for quantitative measurements of force and motion.

(c) Test your predictions. Set up the pulley, cart, string, motion detector and force probe as shown in Figure 2. The cart should be the same mass as before. Zero the force probe. Hang 50 g from the end of the string. Start the data acquisition. Release the cart when you hear the clicks of the motion detector. Be sure that there is enough slack in the force probe cables to complete the motion and catch the cart before it crashes into the pulley. Repeat until you get good graphs in which the cart is seen by the motion detector over the entire motion. Sketch the actual velocity, acceleration and force graphs for the motion of interest on the axes below and indicate the scale on the axes. Draw smooth graphs; don't worry about small bumps.



(d) Is the force which is applied to the cart by the string constant, increasing or decreasing? Explain based on your graph.

(e) How does the acceleration graph vary in time? Does this agree with your prediction? What kind of acceleration corresponds to a constant applied force?

(f) How does the velocity graph vary in time? Does this agree with your prediction? What kind of velocity corresponds to a constant applied force?

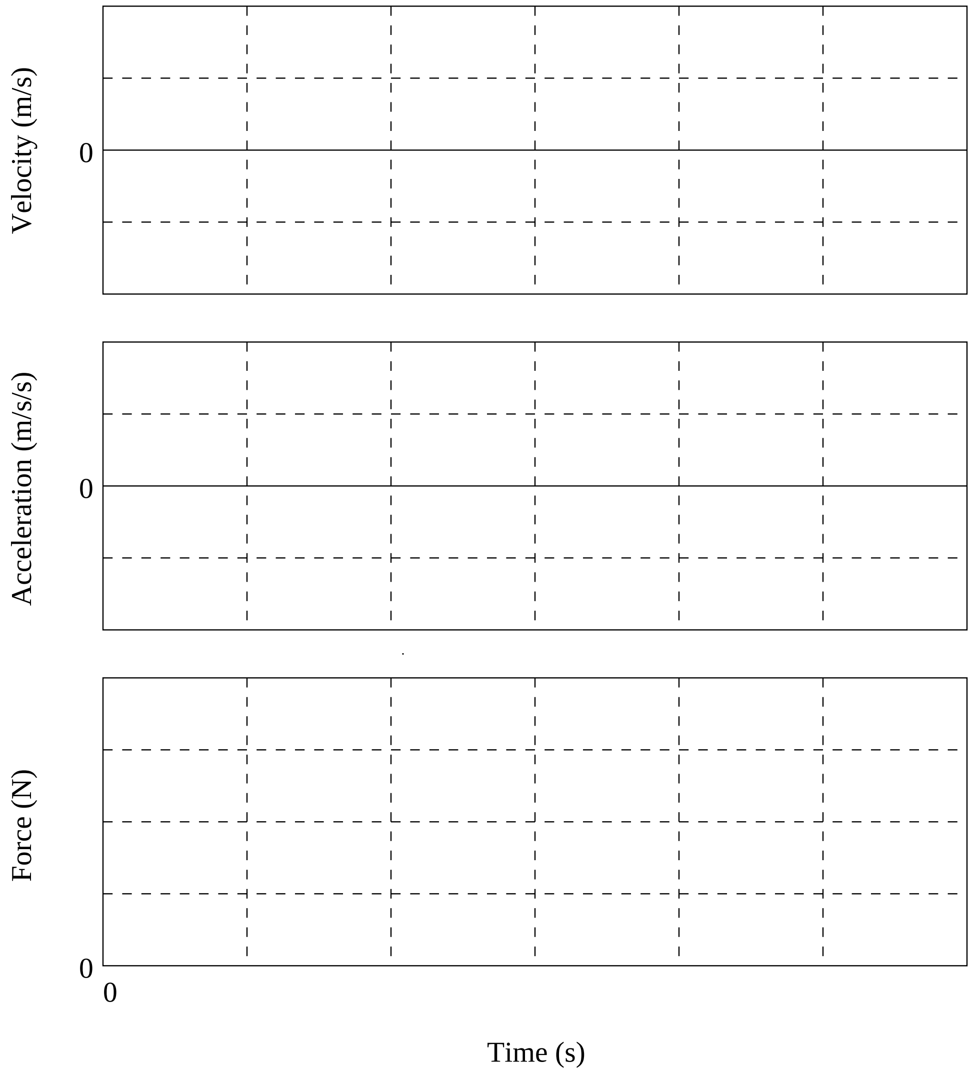
(g) Use the Smart Tool to determine the average force and the average acceleration and record them below. Find the mean values only during the time interval when the force and acceleration are nearly constant. See Appendix B for details on the use of the Smart Tool feature of DataStudio.

Activity 3: Acceleration from Different Forces

In the previous activity you have examined the motion of a cart with a constant force applied to it. But, what is the relationship between acceleration and force? If you apply a larger force to the same cart (same mass as before) how will the acceleration change? In this activity you will try to answer these questions by applying different forces to the cart, and measuring the corresponding accelerations.

(a) Suppose you pulled the cart with a force about twice as large as before. What would happen to the acceleration of the cart? Explain.

(b) Test your prediction by replacing the 50-g mass with a 100-g mass and creating graphs of the motion as before. Repeat until you have a good run. Sketch the results on the axes that follow. Don't forget to put the scale on the axes.



(c) Use the Smart Tool to find the average force and the average acceleration and record them below. Find the mean values only during the time interval when the force and acceleration are nearly constant.

(d) How did the force applied to the cart compare to that with the smaller force in Activity 2?

(e) How did the acceleration of the cart compare to that caused by the smaller force in Activity 2? Did this agree with your prediction? Explain.

Activity 4: The Relationship Between Acceleration and Force

If you accelerate the same cart (same mass) with another force, you will then have three data points–enough data to plot a graph of acceleration vs. force. You can then find the mathematical relationship between acceleration and force.

(a) Accelerate the cart with a force roughly midway between the other two forces tried. Use a hanging mass about midway between those used in the last two activities. Record the mass below.

(b) Graph velocity, acceleration and force. Sketch the graphs on the axes in Activity 3 using dashed lines.

(c) Find the mean acceleration and force, as before, and record the values in the table below (in the Activity 4 line). Also, enter the values from the previous two activities in the table.

	Average Force (N)	Average Acceleration (m/s^2)
Activity 2		
Activity 4		
Activity 3		

(d) Plot the average force applied to the cart as a function of the average acceleration of the cart by fitting the data with a linear function. Label and print the graph showing the best fit, and add it to this unit.

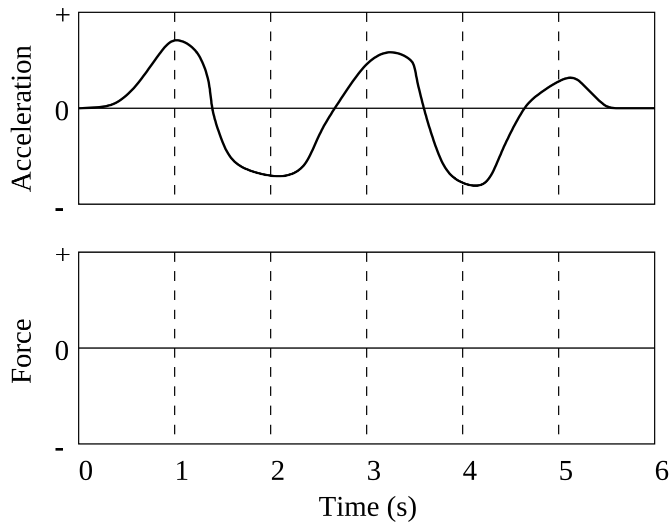
(e) Does there appear to be a simple mathematical relationship between the acceleration of a cart (with fixed mass) and the force applied to the cart (measured by the force probe mounted on the cart)? Write down the equation you found and describe the mathematical relationship in words.

(f) What is the slope of the graph? How does it compare with the mass of the cart and force probe assembly as measured in Activity 1?

Comment: The relationship which you have been examining between the acceleration of the cart and the applied force is known as Newton's Second Law, $F = ma$.

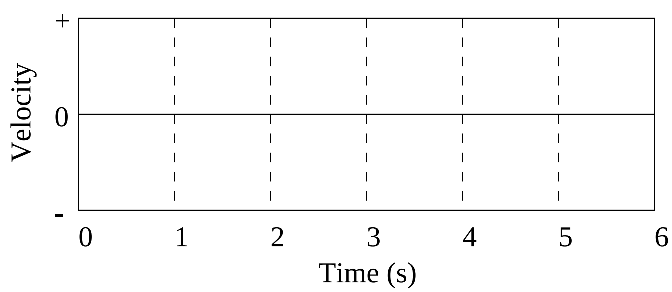
Homework

1. A force is applied which makes an object move with the acceleration shown below. Assuming that friction is negligible, sketch a force-time graph of the force on the object on the axes below.

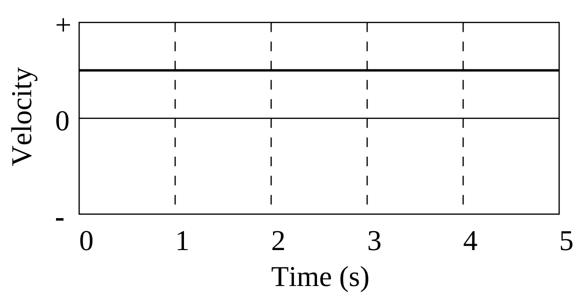


Explain your answer:

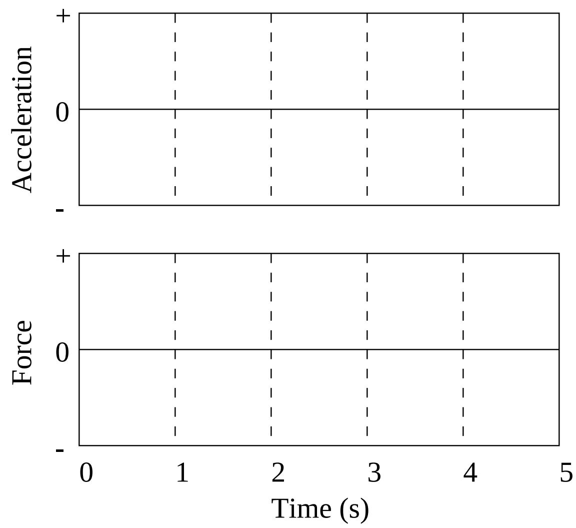
2. Roughly sketch the velocity-time graph for the object in question 1 on the axes below.



3. A cart can move along a horizontal line (the + position axis). It moves with the velocity shown below.



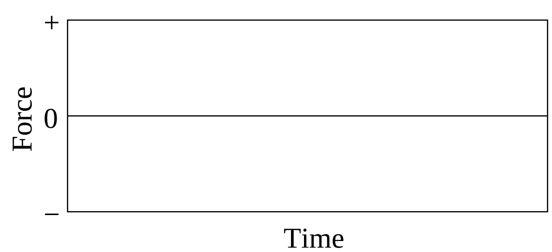
Assuming that friction is so small that it can be neglected, sketch on the axes that follow the acceleration-time and force-time graphs of the cart's motion.



Explain both of your graphs.

Questions 4-6 refer to an object which can move in either direction along a horizontal line (the + position axis). Assume that friction is so small that it can be neglected. Sketch the shape of the graph of the force applied to the object which would produce the motion described.

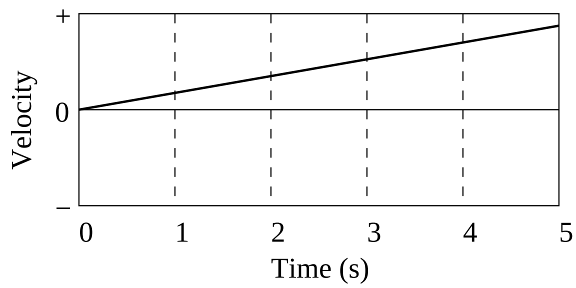
4. The object moves away from the origin with a constant acceleration.



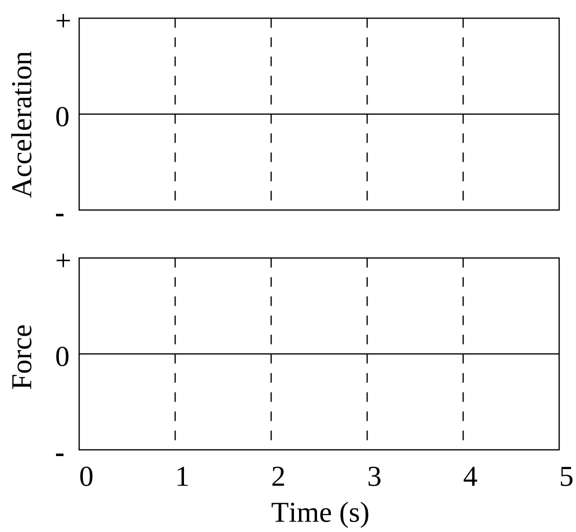
5. The object moves toward the origin with a constant acceleration.

6. The object moves away from the origin with a constant velocity.

Questions 7 and 8 refer to an object which can move along a horizontal line (the + position axis). Assume that friction is so small that it can be ignored. The object's velocity-time graph is shown below.

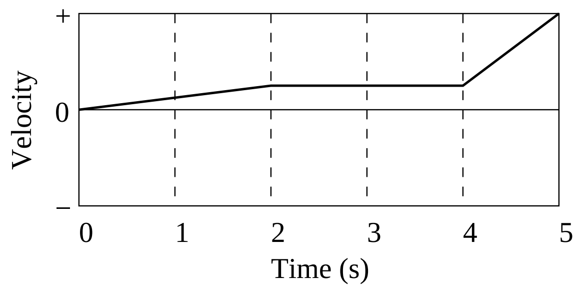


7. Sketch the shapes of the acceleration-time and force-time graphs on the axes below.

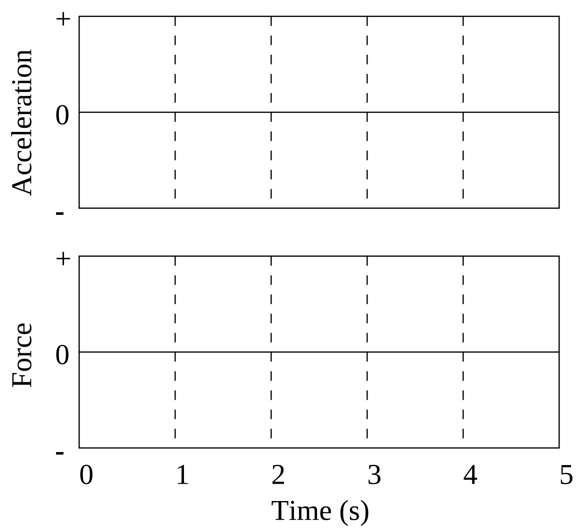


8. Suppose that the force applied to the object were twice as large. Sketch with dashed lines on the same axes above the force, acceleration, and velocity.

Questions 9 refer to an object which can move along a horizontal line (the + position axis). Assume that friction is so small that it can be ignored. The object's velocity-time graph is shown below.



9. Sketch the shapes of the acceleration and force graphs on the axes below.



24 Force and Motion II[14]

Name ______________________ Section ____________ Date ____________

Objective

To understand the relationship between the direction of the force applied to an object and the direction of the acceleration of the object.

Overview

In the previous lab you examined the one-dimensional motions of an object caused by a single force applied to the object. You have seen that when friction is so small that it can be ignored, a single constant applied force will cause an object to have a constant acceleration. (The object will speed up at a steady rate.)

Under these conditions, you have seen that the acceleration is proportional to the applied force, if the mass of the object is not changed. You saw that when a constant force is applied to a cart, the cart speeds up at a constant rate so that it has a constant acceleration. If the applied force is made larger, then the acceleration is proportionally larger. This allows you to define force more precisely not just in terms of the stretches of rubber bands and springs, but as the entity (the "thing") that causes acceleration.

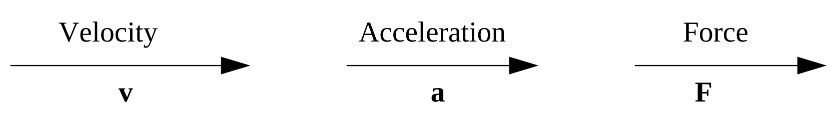
The goal of this lab is to continue to develop the relationship between force and acceleration, an important part of the first two of Newton's famous laws of motion. You will explore motions in which the applied force (and hence the acceleration of the object) is in a different direction than the object's velocity. In this case the object is slowing down in the sense that its speed is decreasing.

Apparatus

- *Science Workshop 750 Interface*
- Force probe
- Variety of hanging masses
- Low friction pulley and string
- Motion detector
- Dynamics cart (with flag) and track
- *DataStudio* software (V, A & F Graphs application)

Speeding Up and Slowing Down

So far you have looked at cases where the velocity, force and acceleration all have the same sign (all positive). That is, the vectors representing each of these three vector quantities all point in the same direction. For example, if the cart is moving toward the right and a force is exerted toward the right, then the cart will speed up. Thus the acceleration is also toward the right. The three vectors can be represented as:

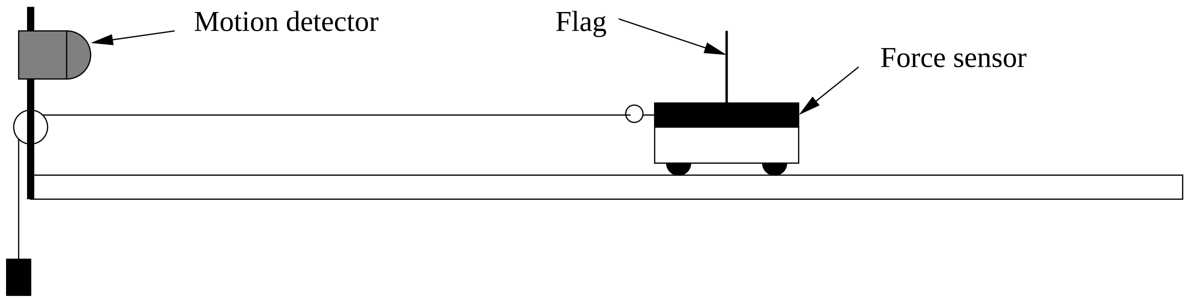


If the positive x direction is toward the right, then you could also say that the velocity, acceleration and also force are all positive. In this investigation, you will examine the vectors representing velocity, force and acceleration for other motions of the cart. This will be an extension of your earlier observations of changing motion.

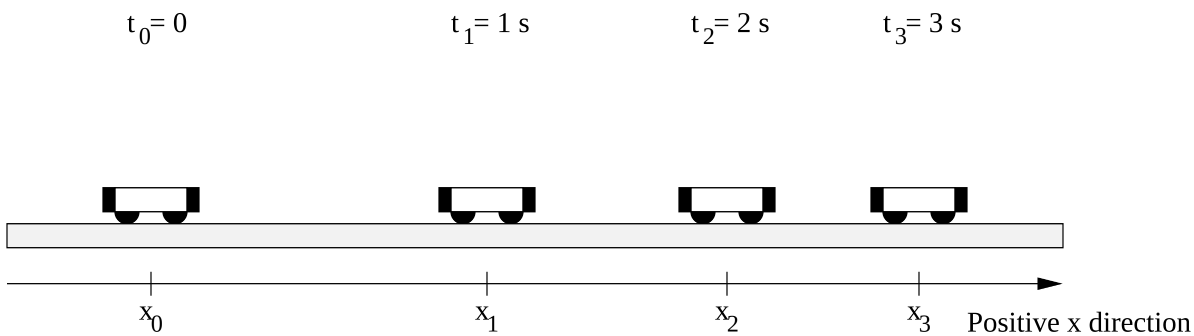
[14] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

Activity 1: Slowing Down

Set up the cart, pulley, hanging mass and motion detector as shown below. Now when you give the cart a push away from the motion detector, it will slow down after it is released. In this activity you will examine the acceleration and the applied force.



(a) Suppose that you position the cart 0.15 m from the motion detector and give it a push away from the motion detector and release it. Draw below vectors which might represent the velocity, force and acceleration of the cart at each time after it is released and is moving toward the right. Be sure to mark your arrows with **v**, **a**, or **F** as appropriate.

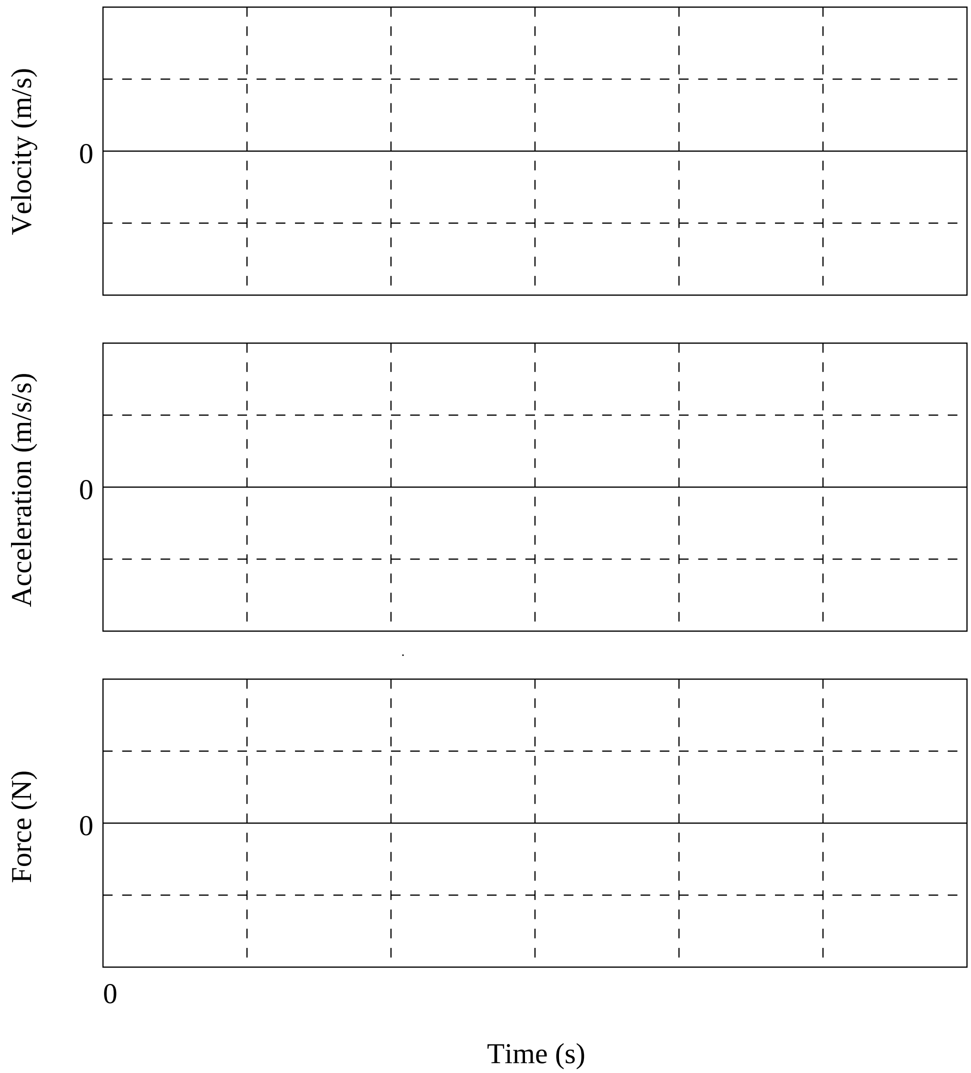


(b) If the positive x direction is toward the right, what are the signs of the velocity, force and acceleration after the cart is released and is moving toward the right?

(c) To test your predictions:

1. Zero the force probe.
2. Hang a 50-g mass from the end of the string.
3. Use the V, A & F Graphs application to graph the motion.
4. Test to be sure that the motion detector sees the cart during its complete motion, and that the string and force probe cable are not interfering with the motion detector. You may need to move the motion detector to the side slightly so that it does not see the string. Also make sure that the cables to the motion detector do not impede the motion of the cart. Remember that the back of the cart must always be at least 0.15 meter from the motion detector.
5. Start recording data. When the motion detector starts clicking, give the cart a short push away from the motion detector and then let it go. Stop the cart before it reverses its direction. Repeat until you get a good run.

6. Sketch your velocity, acceleration and force graphs on the axes below. Label the time scale on these axes. Indicate with an arrow the time when the push stopped.



(d) Did the signs of the velocity, force and acceleration agree with your predictions? If not, can you now explain the signs?

(e) Did the velocity and acceleration both have the same sign? Explain these signs based on the relationship between acceleration and velocity.

(f) Did the force and acceleration have the same sign? Were the force and acceleration in the same direction? Explain.

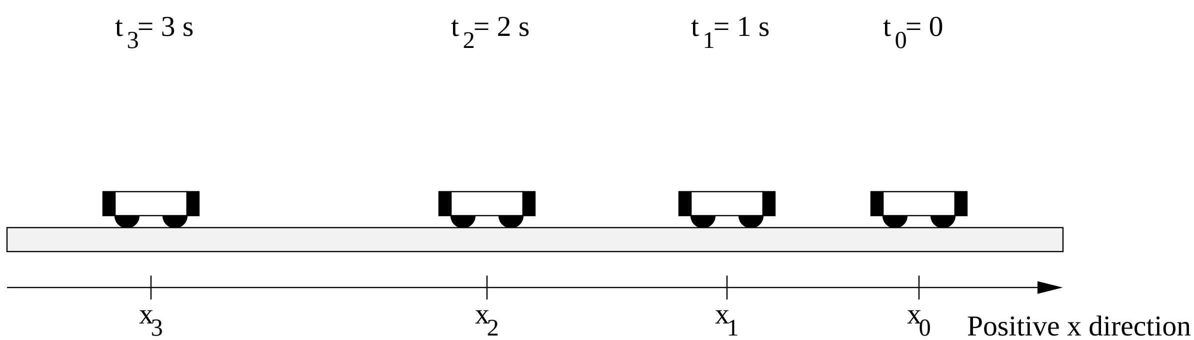
(g) Based on your observations, draw below vectors which might represent the velocity, force and acceleration for the cart at the same instant in time. Do these agree with your predictions? If not, can you now explain the directions of the vectors?

(h) After you released the cart, was the force applied by the falling mass constant, increasing or decreasing? Explain why this kind of force is necessary to cause the observed motion of the cart.

Activity 2: Speeding Up Toward the Motion Detector

Using the same setup as in the last activity, you can start with the cart at the opposite end of the table from the motion detector and release it from rest. It will then be accelerated toward the motion detector as a result of the force applied by the falling mass.

(a) Suppose that you release the cart from rest and let it move toward the motion detector. Draw on the diagram below vectors which might represent the velocity, force and acceleration of the cart at each time after it is released and is moving toward the left. Be sure to mark your arrows with **v**, **a**, or **F** as appropriate.



(b) What are the signs of the velocity, force and acceleration after the cart is released and is moving toward the motion detector? (The positive x direction is toward the right.)

(c) Test your predictions. Use a hanging mass of 100 g. Start recording data. When you hear the motion detector, release the cart from rest as far away from the motion detector as possible. Catch the cart before it hits the motion detector. Repeat until you get a good run. Sketch your graphs on the above axes with dashed lines.

(d) Which of the signs – velocity, force and/or acceleration – are the same as in the previous activity (where the cart was slowing down and moving away), and which are different? Explain any differences in terms of the differences in the motion of the cart.

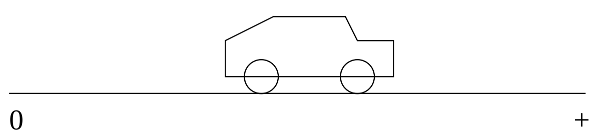
(e) Based on your observations, draw below vectors which might represent the velocity, force and acceleration for the cart at the same instant in time. Do these agree with your predictions? If not, can you now explain the directions of the vectors?

(f) Write down a simple rule in words which describes the relationship between the direction of the applied force and the direction of the acceleration for any motion of the cart.

(g) Is the direction of the velocity always the same as the direction of the force? Is the direction of the acceleration always the same as the direction of the force? In terms of its magnitude and direction, what is the effect of a force on the motion of an object?

Homework

Questions 1-6 refer to a toy car which can move in either direction along a horizontal line (the + position axis).



Assume that friction is so small that it can be ignored. Sketch the shape of the graph of the applied force which would keep the car moving as described in each statement.

1. The toy car moves away from the origin with a constant velocity.

2. The toy car moves toward the origin with a constant velocity.

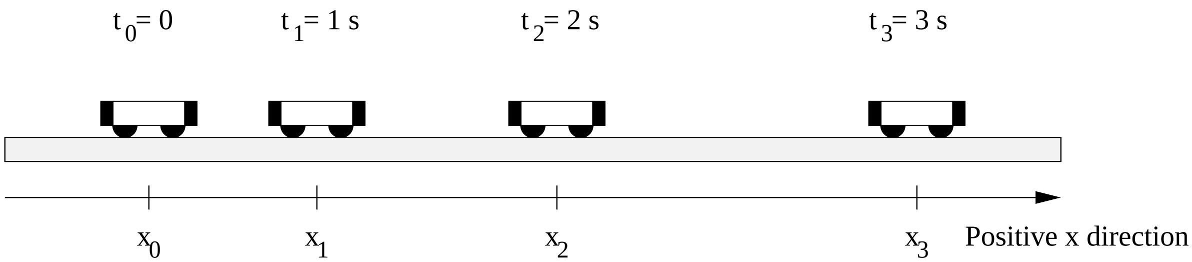
3. The toy car moves away from the origin with a steadily decreasing velocity (a constant acceleration).

4. The toy car moves away from the origin, speeds up and then slows down.

5. The toy car moves toward the origin with a steadily increasing speed (a constant acceleration).

6. The toy car is given a push away form the origin and released. It continues to move with a constant velocity. sketch the force after the car is released.

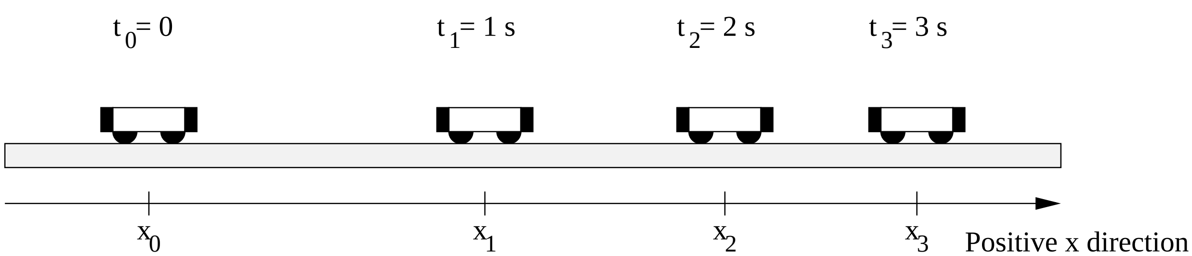
7. A cart is moving toward the right and speeding up, as shown in the diagram below. Draw arrows above the cart representing the magnitudes and directions of the net (combined) forces you think are needed on the cart at the times shown to maintain its motion with a steadily increasing velocity.



Explain the reasons for your answers.

8. If the positive direction is toward the right, what is the sign of the force at $t = 2$ sec in question 7? Explain.

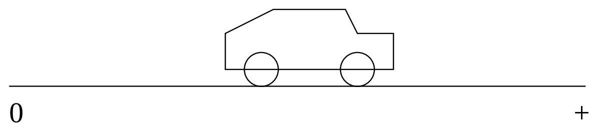
9. A cart is moving toward the right and slowing down, as shown in the diagrams below. Draw arrows above the cart representing the magnitudes and directions of the net (combined) forces you think are needed on the cart at $t = 0$ s, $t = 1$ s, etc. to maintain its motion with a steadily decreasing velocity.



Explain the reasons for your answers.

10. If the positive direction is toward the right, what is the sign of the force at $t = 2$ sec in question 9? Explain.

11. A toy car can move in either direction along a horizontal line (the + position axis).



Assume that friction is so small that it can be ignored. A force toward the right of constant magnitude is applied to the car. Sketch on the axes below using a solid line the shape of the acceleration-time graph of the car.

Explain the shape of your graph in terms of the applied force.

25 Combining Forces[15]

Name ______________________ Section ____________ Date ____________

Objectives

- To understand how different forces can act together to make up a combined force.
- To establish a definition of combined force as that which changes the motion of an object.
- To understand the motion of an object with no force applied to it and how Newton's First Law describes this motion.

Overview

In this unit, you will explore what happens when more than one force is applied to an object. Also, you will be asked to consider the special case when the object moves with a constant velocity, so that the object's acceleration is zero. What combination of forces must be applied to an object to keep it moving with a constant velocity when there is almost no friction? You can answer this question by collecting force and motion data again with the force probe and motion detector. The answer to this question will lead you to the discovery of Newton's First Law (in a situation where friction can be neglected).

Apparatus

- Force probe
- Motion detector
- Dynamics cart (with flag) and track
- Low friction pulleys (2) and string
- Variety of hanging masses
- *Science Workshop 750 Interface*
- *DataStudio* software (V, A & F Graphs application)

Net Force: Combining Applied Forces

As you know, vectors are mathematical entities which have both magnitude and direction. Thus a one-dimensional vector can have a direction along the positive x-axis or along the negative x-axis. Vectors pointing in the same direction add together and vectors pointing in opposite directions subtract from each other. Quantities which have vector behavior are often denoted by a letter with a little arrow above it or by a boldface letter (i.e., $\mathbf{F}$). The sum of several vectors is often denoted by placing a summation sign in front of a vector symbol (i.e., $\sum \mathbf{F}$). In the next two activities you will investigate how combined forces change the motion of an object.

Activity 1: Two Forces in the Same Direction

(a) Suppose you apply two 0.5-N forces to the cart in the same direction. How will the motion compare with the motion you observed in the previous unit for a 1-N force?

(b) Test your predictions. Set up a second pulley next the first one, attach a second string from the force probe hook over the second pulley, and hang 50 g from each string. Graph the motion as in the previous unit. Repeat until you get a good run and sketch the results on the axes below. Don't forget to label the axes.

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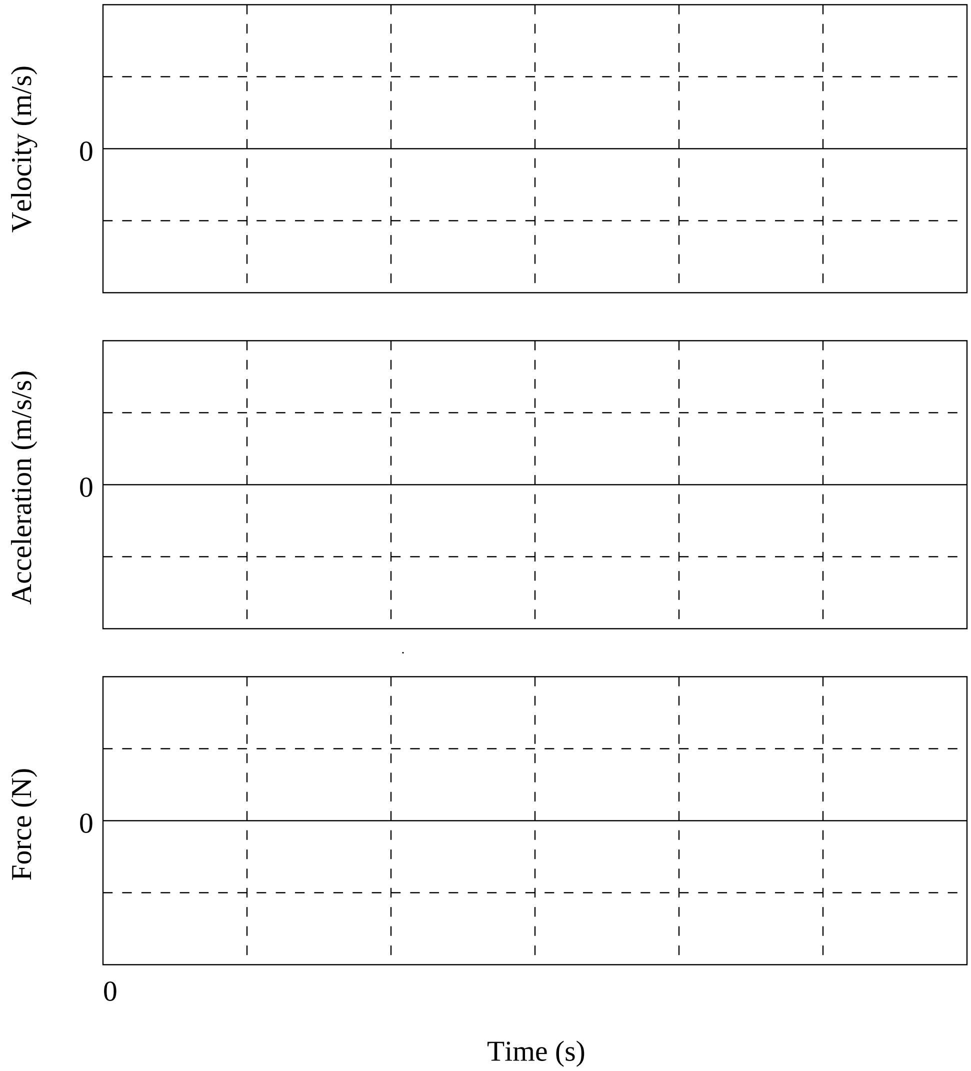


(c) Do the results agree with your predictions? Explain.

(d) Draw arrows in the space below that represent a scale drawing of the magnitudes and directions of the forces applied to the cart in this activity and the previous one.

Activity 2: Two Equal Forces in Opposite Directions

(a) Suppose two equal and opposite forces were applied to the cart. What would be the resulting motion?

(b) Test your predictions. Move one of the pulleys to the other end of the table and hang 100 g from each string. Release the cart. Does the resulting motion agree with your predictions? Explain.

(c) Draw a force diagram to scale below.

Activity 3: Two Unequal Forces in Opposite Directions

(a) Suppose you hang 150 g from the string above the motion detector and 50 g from the string at the other end of the table. What will be the resulting motion and how will it compare with the motions observed in Activities 1 and 2?

(b) Test your predictions by graphing the motion. Repeat until you get a good run and sketch your results as dashed lines on the axes in Activity 2.

(c) Do the results agree with your predictions? Explain.

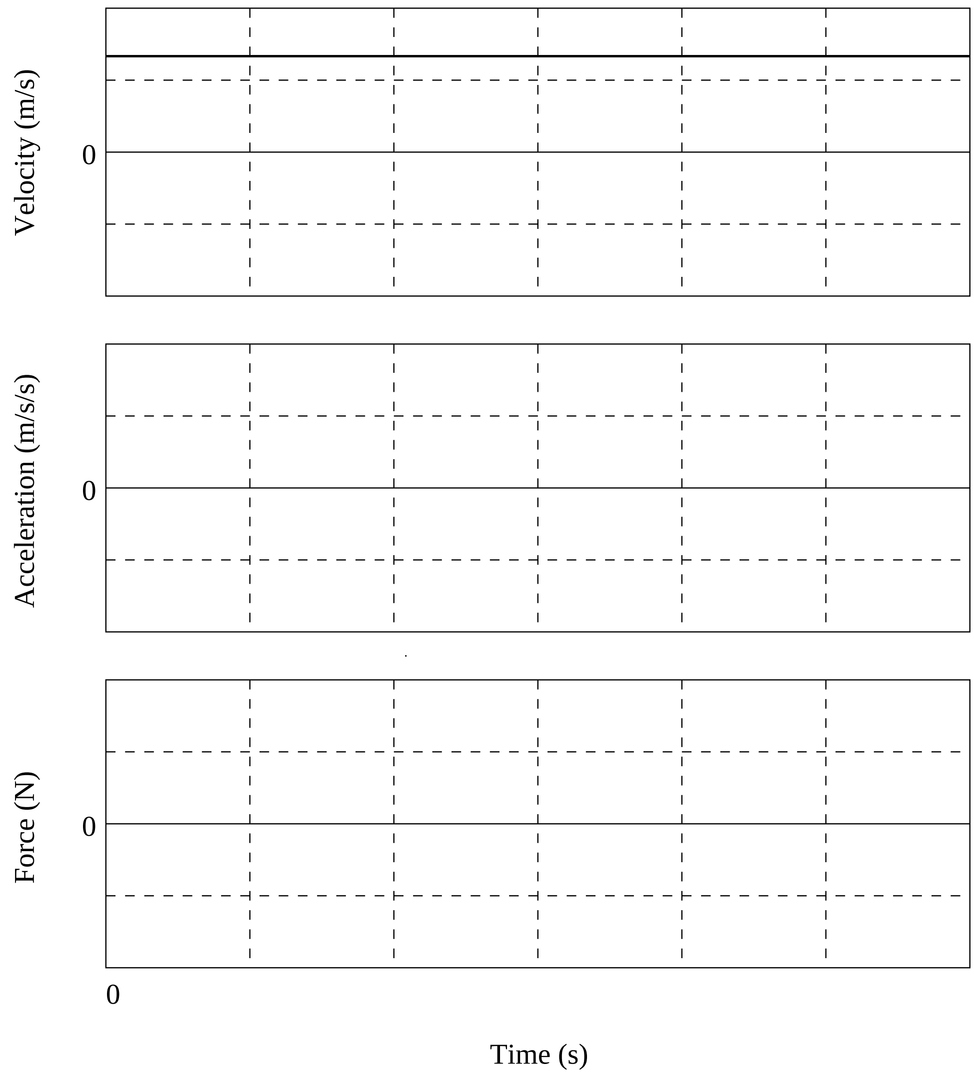
(d) Draw a force diagram (to scale) for this situation in the space below.

(e) Do one-dimensional forces seem to behave like one-dimensional vectors? Why or why not?

(f) When more than one force is acting on an object, what is it that determines the acceleration of the object?

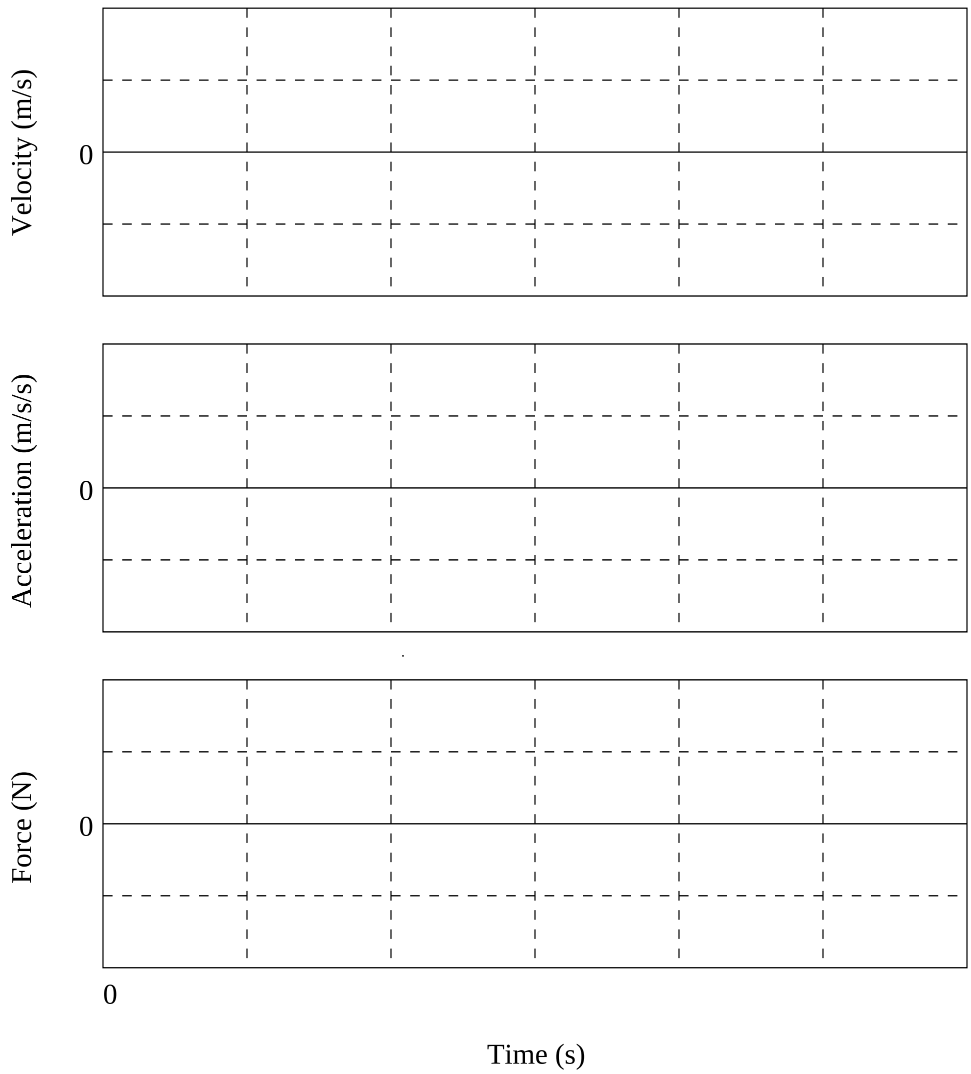
Activity 4: Motion at a Constant Velocity

(a) Suppose the cart is moving with the velocity which is shown on the velocity-time graph below. Sketch on the axes below the acceleration-time graph of the cart, and the force-time graph of the combined force after the cart begins moving.



(b) Describe in words the acceleration of the cart and the combined force needed to keep it moving at a constant velocity.

(c) Test your prediction. Remove the strings from the force probe and give the cart a push away from the motion detector. Notice that the cart slows down and eventually stops. This is due to the small amount of friction that exists. Replace one of the strings and hang a small mass from the string over the pulley at the far end of the table to balance the frictional force. Push the cart to see if it moves with approximately constant velocity. Adjust the hanging mass such that the cart moves with approximately constant velocity. Then graph the motion of the cart. Repeat until you get a good run and sketch the results on the axes below. Don't forget to label the time scale on the axes. If your hand was still in contact with the cart after the graphing started, indicate with an arrow the time when the push stopped.



(d) Do the graphs agree with your predictions? If not, how do they differ?

(e) What happened to the force of the push after you released the cart? Explain.

(f) If you give the cart a short push toward the motion detector, how will the graphs change compared to the ones in this activity for a short push away from the detector?

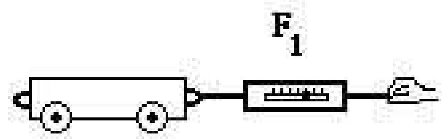
Comment: In this activity you have looked at a situation where the combined force acting on the cart is zero.

As you have seen, the velocity of the cart does not change. The cart either moves with a constant velocity, or remains at rest. The law, which you have examined in this investigation, describing the motion when the combined force acting on an object is zero is known as Newton's First Law.

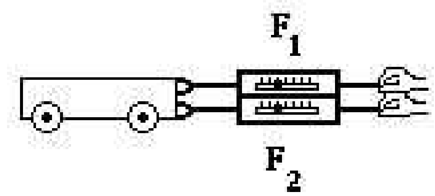
Homework

In questions 1-4, assume that friction is so small that it can be ignored.

1. The spring scale in the diagram below reads 10.5 N.

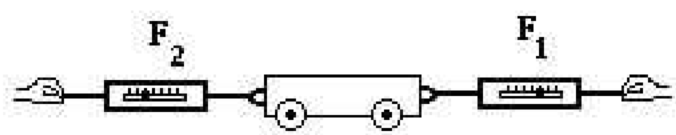


The cart moves toward the right with an acceleration toward the right of 3.50 (m/s)/s. Now two forces are applied to the cart with two different spring scales as shown below. The spring scale F_1 still reads 10.5 N.



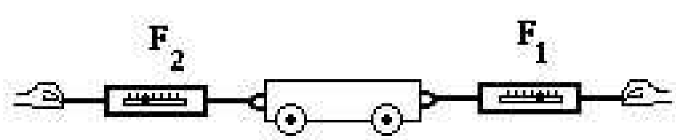
The cart now moves toward the right with an acceleration toward the right of 5.50 (m/s)/s. What does spring scale F_2 read? Show your calculations, and explain.

2. Now two forces are applied to the cart with two different spring scales as shown below. The spring scale F_1 still reads 10.5 N.



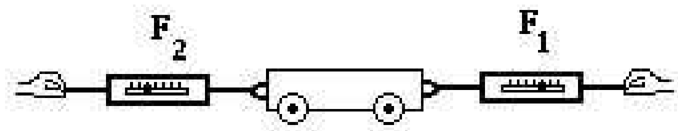
The cart now moves toward the right with an acceleration toward the right of 2.50 (m/s)/s. What does spring scale F_2 read? Show your calculations, and explain.

3. Again two forces are applied to the cart with two different spring scales as shown below. The spring scale F_1 still reads 10.5 N.



The cart moves with a constant velocity toward the right. What does spring scale F_2 read? Show your calculations, and explain.

4. Again two forces are applied to the cart with two different spring scales as shown below. The spring scale F_1 still reads 10.5 N.



The cart moves toward the left with an acceleration toward the left of 2.50 (m/s)/s. What does spring scale F_2 read? Show your calculations, and explain.

26 Force, Mass and Acceleration[16]

Name ______________________________ Section _______________ Date _______________

Objectives

- To develop a definition of mass in terms of an object's motion under the influence of a force.
- To understand the relationship among the force applied to an object, the mass of the object and its motion.
- To find a mathematical relationship between the acceleration of an object and its mass when a constant force is applied–Newton's Second Law.
- To examine the quantitative relationship between force, mass and acceleration–Newton's Second Law–in terms of the SI units (N for force, kg for mass, and (m/s)/s for acceleration).
- To pull all of the observations together and state Newton's First and Second Laws of Motion for motion in one dimension, along a straight line for any number of forces acting on an object.

Apparatus

- Force probe
- Motion detector
- Dynamics cart and track
- Variety of masses to increase the mass of the cart
- Triple-beam balance
- Low friction pulley and string
- Variety of hanging masses
- *Science Workshop 750 Interface*
- *DataStudio* software (V, A & F Graphs)

Overview

In this lab you will continue to develop the first two of Newton's famous laws of motion. You will do this by combining careful definitions of force and mass with observations of the mathematical relationships among force, mass and acceleration.

You have seen that the acceleration of an object is directly proportional to the combined or net force acting on the object. If the combined force is not zero, then the object will accelerate. If the combined force is constant, then the acceleration is also constant. These observations are part of Newton's Second Law of Motion.

You have also seen that for an object to move at a constant velocity (zero acceleration) when friction is negligible, the combined or net force on the object should be zero. The law which describes constant velocity motion of an object is Newton's First Law of Motion. Newton's First and Second Laws of Motion are very powerful! They allow you to relate the force on an object to its subsequent motion. Therefore, when the nature of the force(s) acting on an object is known, then Newton's laws of motion allow you to make mathematical predictions of the object's motion.

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In the first part of this lab you will study how the amount of “stuff” (mass) you are accelerating with a force affects the magnitude of the acceleration. What if the mass of the object were larger or smaller? How would this affect the acceleration of the object for a given combined force?

In the second part you will study more carefully the definitions of the units in which we express force, mass and acceleration.

Finally, in the last part, you will examine the motion of an object caused by a force applied to it when friction is large enough so that it cannot be ignored.

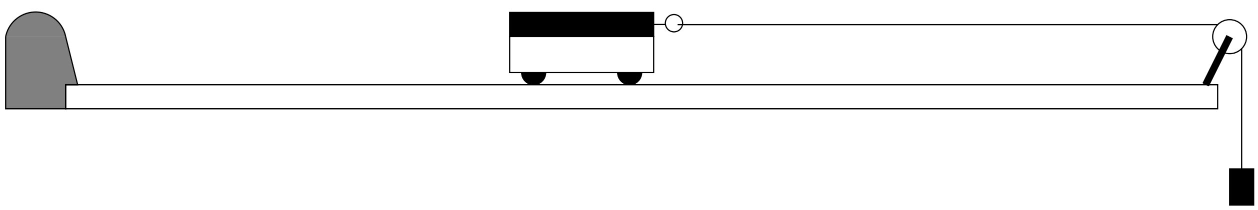
Force, Mass and Acceleration

In previous activities you have applied forces to a cart and examined its motion. Up until now, you have always used a cart with the same mass. But when you apply a force to an object, you know that its mass has a large effect on the object’s acceleration. For example, compare the different accelerations that would result if you pushed a 1000 kilogram (metric ton) automobile and a 1 kilogram cart, both with the same size force!

Activity 1: Acceleration and Mass

You can easily change the mass of the cart by attaching masses to it, and you can apply the same force to the cart by using appropriate hanging masses to accelerate the cart each time. By measuring the acceleration of different mass carts, you can find a mathematical relationship between the acceleration of the cart and its mass, with the applied force kept constant.

(a) Set up the pulley, cart, string, motion detector and force probe as shown below. (Be sure that the cable from the force probe doesn’t interfere with the motion of the cart, and is out of the way of the motion detector.)

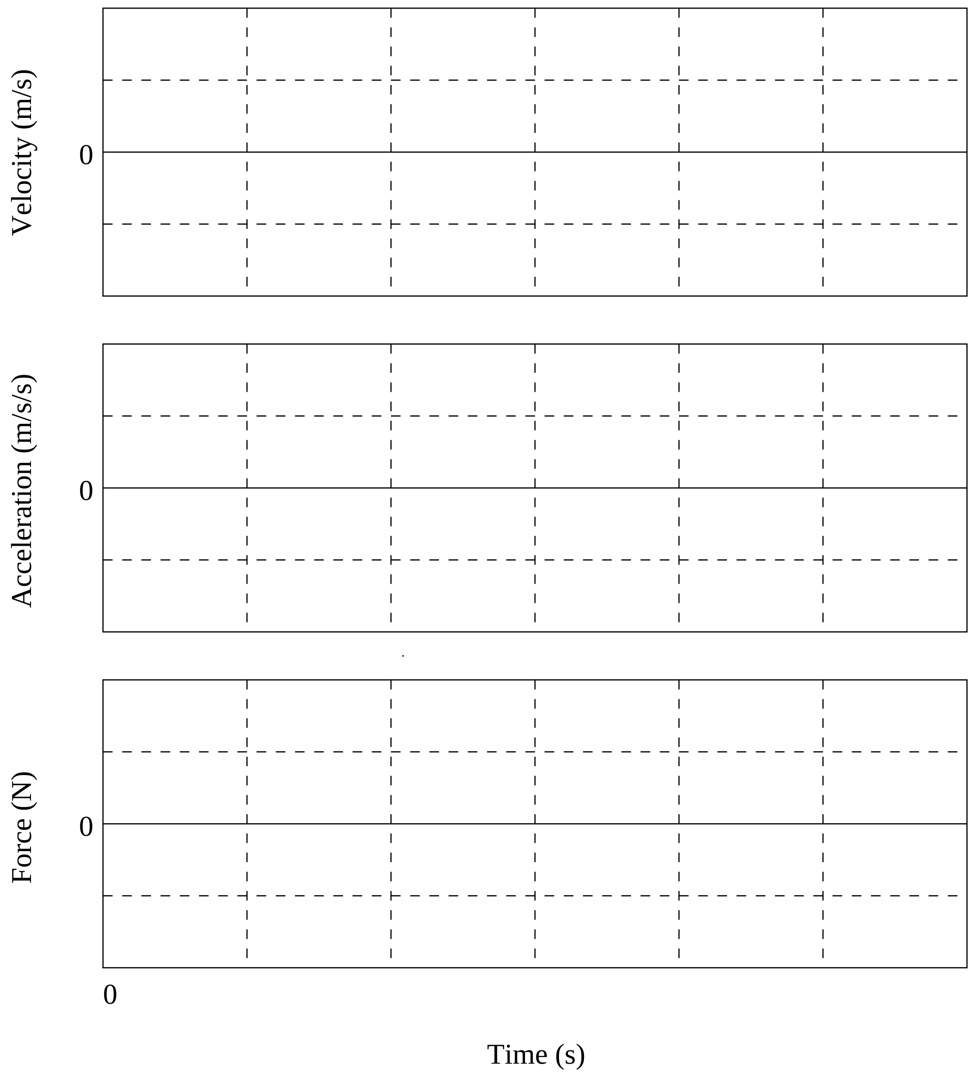


(b) We will define a mass scale in which the unit is the mass of the cart (including the force probe), called one cart mass. An equal arm balance can be used to assemble a combination of masses equal to one cart mass. If this combination of masses is divided in half, each half is 0.5 cart mass. Use the balance in this way to assemble masses which you can add to the cart to make the cart’s mass equal to 1.5, 2.0. 2.5 and 3.0 cart masses. Label these masses.

(c) Zero the force probe by pressing the TARE button on the probe.

(d) Now add masses to make the cart’s mass 2.0 cart masses.

(e) Open the V, A & F Graphs application. Hang 100 g from the end of the string. Start recording data and release the car from rest when you hear the clicks of the motion detector. Repeat until you get a good run. Sketch the graphs on the axes below. Don’t forget to label the axes.



(f) Use the Smart Tool to determine the average force and average acceleration during the time interval when they are nearly constant, and record them in the third row of the table below.

Mass of Cart (cart masses)	Average Applied Force (N)	Average Acceleration (m/s^2)
1.0		
1.5		
2.0		
2.5		
3.0		

(g) Suppose that you halve the mass of the cart back to 1.0 cart masses, and accelerate it with the same applied force. How would the acceleration compare with that of the 2.0 cart masses?

(h) Test your prediction. Remove the masses you added earlier which doubled the mass of the cart to 2.0 cart masses.

Comment: You want to accelerate the cart with the same applied force. As you may have noticed, the force applied to the force probe by the string decreases once the cart is released. (You will examine why this is so in a later lab.) This decrease depends on the size of the acceleration. Therefore, in order to keep the applied force constant, you may need to change the hanging mass.

Adjust the hanging mass until the force probe reading while the cart is accelerating is the same as the force you recorded in the third row of the table above. When you have found the correct hanging mass, graph the motion of the cart. Measure the average force and average acceleration of the cart during the time interval when the force and acceleration are nearly constant and record these values in the first row of the table.

(i) Did the acceleration agree with your prediction? Explain.

(j) Now make the mass of the cart 1.5 cart masses, and accelerate it again with the same size force. (Don't forget to adjust the hanging mass, if necessary.) Measure the average force and acceleration of the cart, and record these values in the table. Repeat for masses of 2.5 and 3.0 cart masses.

(k) Does the acceleration of the cart increase, decrease or remain the same as the mass of the cart is increased?

(l) Use Excel to plot the acceleration (y axis) as a function of the cart mass (x axis) and to find the best fit to the data. When you have found the best fit, print the graph and put a copy in your notebook.

(m) What appears to be the relationship between acceleration and mass of the cart, when the applied force is kept constant?

(n) In the previous lab, you found that the acceleration of the cart was proportional to the combined applied force, when the mass of the cart was not changed. State in words the general relationship between the applied force, the mass and the acceleration of the cart which you have found in these two labs. If the combined force is $\sum \mathbf{F}$, the mass is m and the acceleration is a, write a mathematical relationship which relates these three physical quantities.

Force & Mass Units

So far you have been measuring force in standard units called newtons. Where does this unit come from? By contrast, we have our own private units for mass, measuring mass in cart masses. If one group were using a large wooden cart in their force and motion experiments, and another group were using a small aluminum cart with smaller mass, they would have different values for mass, and would observe different accelerations for "one cart mass pulled by one newton." It's time to discuss standard units for force and mass.

It would be nice to be able to do a mechanics experiment in one part of the world and have scientists in another part of the world be able to replicate it or at least understand what actually happened. This requires that people agree on standard units. In 1960 an international commission met to agree upon units for fundamental quantities such as length, time, mass, force, electric current, pressure, etc. This commission agreed that the most fundamental units in the study of mechanics are length, time, and mass. All other units including force, work, energy, torque, rotational velocity, etc. which you encounter in your study of mechanics can be expressed as a combination of these basic quantities. The fundamental International System or SI units along with the standard unit for force are shown below.

Fundamental Units for Mechanics

Length: A **meter (m)** is the distance traveled by light in a vacuum during a time of 1/299,792,458 second.

Time: A **second (s)** is defined as the time required for a cesium-133 atom to undergo 9,192,631,770 vibrations.

Mass: A **kilogram (kg)** is defined as the mass of a platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures in Sévres France. It is kept in a special chamber to prevent corrosion.

The Force Unit Expressed in Terms of Length, Mass, and Time

Force: A newton (N) is defined as that force which causes a 1 kg mass to accelerate at 1 (m/s)/s .

We want to be able to measure masses in kilograms and forces in newtons in our own laboratory. The following activities are designed to give you a feel for standard mass and force units, and how they are determined in the laboratory. Our approach in these activities is to use a standard force scale to calibrate the force probe in newtons. Using your data from the previous investigation, you will see how you can establish a mass unit in terms of your force and acceleration measurements. Then you will use a standard mass scale to get enough stuff loaded on a cart to equal one kilogram of mass. Finally you can pull the cart with a force of about one newton and see if it accelerates at something close to one meter per second per second. **Warning:** There will probably be a noticeable amount of uncertainty associated with the necessary measurements.

Suppose you want to find the mass of an object in kilograms. You need to compare it to the one kilogram platinum-iridium alloy cylinders at the International Bureau of Weights and Measures in France. It would be nice to have a standard kilogram in your laboratory. You could go to France, but it is unlikely that they would let you take the standard home with you!

Suppose, however, that you accelerate the standard mass with a constant force and measure the force and also the resulting acceleration as accurately as possible. Next you would need to make a cylinder that seemed just like the standard one and add or subtract stuff from it until it undergoes exactly the same acceleration with the same constant force. Then within the limits of experimental uncertainty this new cylinder standard and the bureau standard would have the same mass. If the comparison could be made to three significant figures, then the mass of your new standard would be $m_{std} = 1.00$ kg.

Suppose you head home with your standard mass. You wish to determine the mass of another object. You could apply the same constant force, F, on the standard and on the other object, and measure both accelerations. Then, according to Newton's Second Law, F = ma,

$$m_{\text{std}} = 1.00\,\text{kg} = \frac{F}{a} \qquad m_{\text{other}} = \frac{F}{a_{\text{other}}}$$

Since the constant force, F, applied to both masses was the same,

$$m_{\text{other}} = 1.00\,\text{kg}\,\frac{a}{a_{\text{other}}}$$

In fact, you have already done something similar in the last investigation.

Activity 2: Calculating One "cart mass" in Standard Units

(a) In Activity 1, you measured the force applied to a cart and the acceleration of the cart with 1.0, 1.5, 2.0, 2.5 and 3.0 cart masses. Turn back to your table of information from that experiment, and copy the values of average force and average acceleration into the second and third columns of the table below.

Mass of Cart (cart masses)	Average Applied Force (N)	Average Acceleration (m/s^2)	Ratio of F/a (calculated mass)	Mass of cart from balance (kg)
1.0				
1.5				
2.0				
2.5				
3.0				

In the discussion above, the mass in standard units was calculated using Newton's second law by taking the ratio of the combined (net) force on the object in newtons to the acceleration of the object measured in meters per second per second. (b) For each row in the table, calculate the ratio of the force to acceleration and record it in the fourth column.

(c) According to the discussion, the values you just calculated should be the mass of the object in kilograms. Do your numbers seem to make sense? What do you get for the value of 1.00 cart masses in kilograms? What do you get for the value of 2.00 cart masses in kilograms?

Comment: Physicists call the quantity you have just calculated–the ratio of combined (net) force on an object to its acceleration–the inertial mass of the object. You could continue to determine and compare masses by accelerating them and taking force to acceleration ratios, but this process is pretty tedious. A simpler approach is to use an electronic scale or a mechanical balance that has already been calibrated in kilograms using a standard mass by somebody who is intelligent and knowledgeable! (The details of why such devices can give us correct masses in kg will not be easy to understand fully until after gravitational forces are studied.)

(d) Compare your inertial mass calculations for 1.0, 1.5, 2.0, 2.5 and 3.0 cart masses with the values you get by placing your cart on an electronic scale or mechanical balance. Record these values in the last column of the table.

(e) Are your inertial masses reasonably consistent with your scale masses?

Comment: In your experiments, you have seen that the physical quantities force, mass and acceleration are related through Newton's Second Law. In the activity you have just done, you have used this relationship to define inertial mass in terms of standard units of force, length and time. This is a good logical definition of inertial mass. Historically, however, the units of mass, length and time were defined first as standards and the unit of force was defined as a derived unit in terms of these standard units. Thus a newton of force is defined as the force needed to accelerate 1.00 kg at 1.00 (m/s)/s. In the next activity you will examine this definition.

Activity 3: Does a Force of 1.0 N Applied to a 1.0 kg Mass Really Cause an Acceleration of 1.0 meter/second/second?

You have used mass and force measuring devices that have been provided for you. You could now see if everything makes sense by accelerating one kilogram of mass with a force of about one newton and seeing if an acceleration of about one meter per second per second results. Unfortunately, the carts with force probes that we have been using typically have a mass greater than 1 kg. So we will accelerate a 2.0 kg cart with a 2.0 N force.

(a) What do you expect the acceleration to be in this case? Show the calculation below.

(b) Test your prediction. Add masses to the cart so that the total mass is 2.0 kg. Set up the pulley, weighted cart, string, motion detector and force probe as in Activity 1. Check the calibration of the force probe with a force of 2.0 N (hanging mass of 200 g). Graph the motion and measure the acceleration that results from an applied force on the force probe of 2 N. Adjust the hanging mass, if necessary, to get an applied force of close to 2 N while the cart is accelerating. (Comment: Be careful! Remember that when the cart is being held at rest the same hanging mass will exert more applied force on the cart than when it is accelerating.) Once you get a good run, measure the average values of force and acceleration, and record these values in a table below. Also record the mass of the cart and the hanging mass.

(c) How close is your result to the expected value of acceleration? Calculate the % difference.

(d) A force of 5.4 N is applied to an object, and the object is observed to accelerate with an acceleration of 3.0 (m/s)/s. If friction is so small that it can be ignored, what is the mass of the object in kg? Show your calculation.

(e) An object of mass 39 kg is observed to accelerate with an acceleration of 2.0 (m/s)/s. If friction is negligible, what is the force applied to the object in N? Show your calculation.

Comment: The main purpose of the last three labs has been to explore the relationship between the forces on an object, the object's mass, and its acceleration. You have been trying to develop Newton's First and Second Laws of Motion for one-dimensional situations in which all forces lie in a positive or negative direction along the same line.

Activity 4: Newton's Laws in Your Own Words

(a) Express Newton's First Law (the one about constant velocity) in terms of the combined (net) force applied to an object in your own words clearly and precisely.

(b) Express Newton's First Law in equations in terms of the acceleration vector, the combined (net) force vector applied to an object, and its mass.

$$\text{If } \sum \mathbf{F} = \qquad \text{then } \mathbf{a} = \qquad \text{and } \mathbf{v} =$$

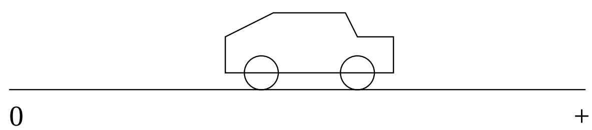
(c) Express Newton's Second Law (the one relating force, mass, and acceleration) in terms of the combined (net) force applied to an object in your own words clearly and precisely.

(d) Express Newton's Second Law in equations in terms of the acceleration vector, the combined (net) force vector applied to an object, and its mass.

$$\text{If } \sum \mathbf{F} \neq 0 \qquad \text{then } \mathbf{a} =$$

Homework

Questions 1 and 2 refer to a toy car which can move in either direction along a horizontal line (the + position axis).



Assume that friction is so small that it can be ignored. A force toward the right of constant magnitude is applied to the car.

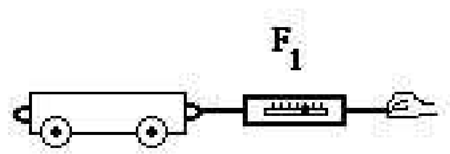
1. Sketch on the axes below using a solid line the shape of the acceleration-time graph of the car.

2. Suppose that the mass of the car were twice as large. The same constant force is applied to the car. Sketch on the axes above using a dashed line the acceleration-time graph of the car. Explain any differences in this graph compared to the acceleration-time graph of the car with the original mass.

3. When a force is applied to an object with mass equal to the standard kilogram, the acceleration of the mass is 3.30 (m/s)/s. (Assume that friction is so small that it can be ignored.) When the same magnitude force is applied to another object, the acceleration is 2.20 (m/s)/s. What is the mass of this object? What would the object's acceleration be if a force twice as large were applied to it? Show your calculations.

4. Given an object with mass equal to the standard kilogram, how would you determine if a force applied to it has magnitude just equal to one newton? (Assume that friction is so small that it can be ignored.)

5. The spring scale in the diagram below reads 10.5 N. (Assume that friction is so small that it can be ignored.)

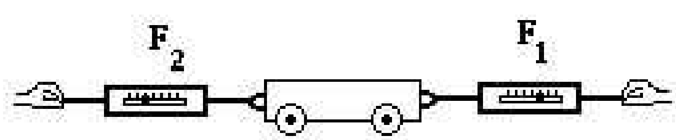


If the cart moves toward the right with an acceleration toward the right of 3.50 (m/s)/s, what is the mass of the cart? Show your calculations, and explain.

6. The force applied to the cart in (5) by spring scale F_1 is still 10.5 N. The cart now moves toward the right with a constant velocity. What are the magnitude and direction of the frictional force. Show your calculations and explain. (Assume that friction can not be ignored.)

7. The force applied to the cart in (5) by spring scale F_1 is still 10.5 N. The cart now moves toward the right with an acceleration toward the right of 1.75 (m/s)/s. What are the magnitude and direction of the frictional force. Show your calculations and explain.

8. The force applied to the cart by spring scale F_1 is 10.5 N.



The cart now moves toward the right with a constant velocity. The frictional force has the same magnitude as in (7). What does spring scale F_2 read? Show your calculations, and explain.

27 Newton's 3rd Law, Tension, and Normal Forces[17]

Name ______________________ Section ____________ Date ____________

Objectives

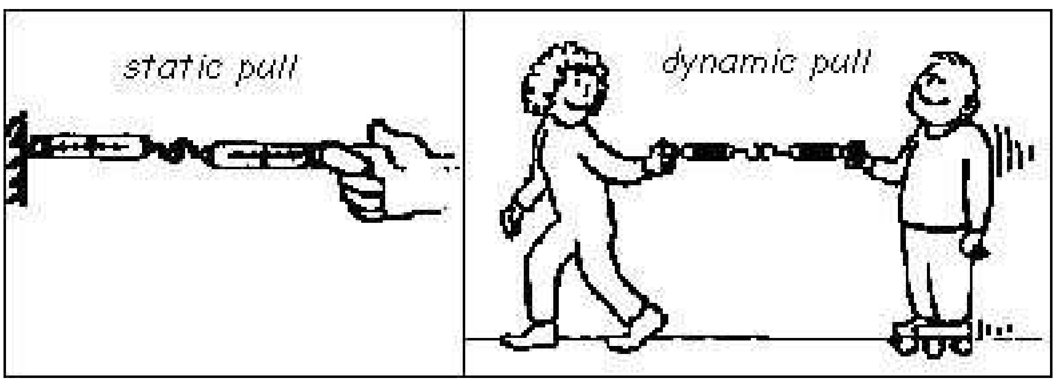
- To investigate Newton's 3rd law of motion.
- To explore the characteristics of two different types of forces: tension (in strings, ropes, springs, and chains), and normal forces (which support objects that are in contact with solid surfaces).

Apparatus

- Spring scales (2)
- Variety of masses
- Rubber band
- Various lengths of string
- Pulleys (2)

An Introduction to Newton's Third Law

In order to apply Newton's laws to complex situations with strings, pulleys, inclined planes and so forth, we need to consider a third force law formulated by Newton having to do with the forces of interaction between two objects. In order to "discover" some simple aspects of the third law, you should make some straightforward observations using 2 spring scales and a set of masses.



Activity 1: Newton's 3rd Law Forces of Interaction

Set up the situations shown in the diagram above and see if there are any circumstances in which the object that is pulling and the object that is being pulled exert different forces on each other. Describe your conclusions below. Note: You can drag the mass set block across the table for your dynamic observations.

In contemporary English, Newton's third law can be stated as follows:

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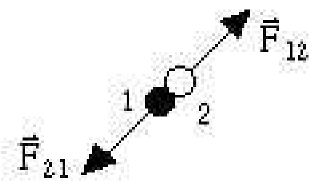
Newton's Third Law

If one object exerts a force on a second object, then the second object exerts a force back on the first object which is equal in magnitude and opposite in direction to that exerted on it by the first object.

In mathematical terms, using vector notation, we would say that the forces of interaction of object 1 on object 2 are related to the forces of interaction of object 2 on object 1 as follows:

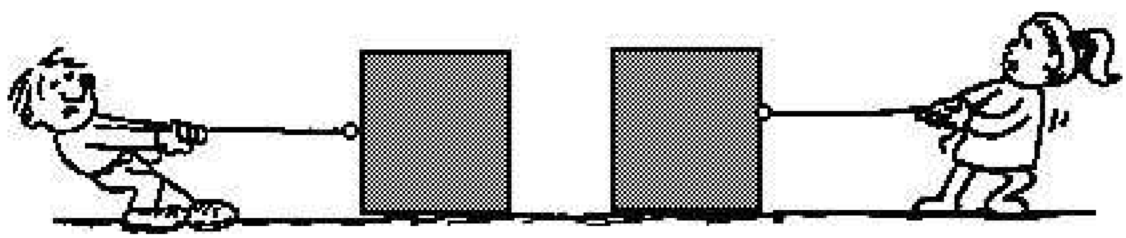
$$\vec{F}_{12} = -\vec{F}_{21}$$



Newton actually formulated the third law by studying the interactions between objects when they collide. It is difficult to understand the significance of this law fully without first studying collisions. Thus, we will consider this law again in the study of collision processes.

Tension Forces

When you pull on one end of a rope attached to a crate, a force is transmitted down the rope to the crate. If you pull hard enough the crate may begin to slide. Tension is the name given to forces transmitted in this way along devices that can stretch such as strings, ropes, rubber bands, springs, and wires.



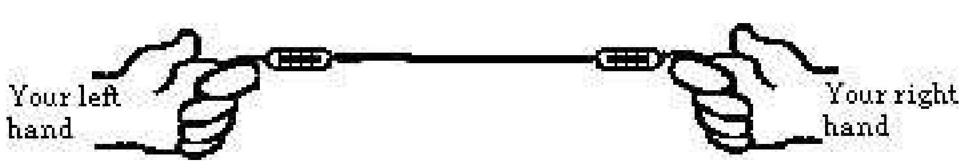
The end of the rope which is tied to the crate can apply a force to the crate only if you first pull on the other end of the rope.

In order to analyze situations in which objects are attached by strings, rubber bands, or ropes it is necessary to understand some attributes of tension forces. We need to answer the following related questions:

1. (a) What is the mechanism for creating tension in strings, ropes, and rubber bands? (b) If a string exerts a tension force on an object at one end, what is the magnitude and direction of the tension force it exerts on another object at its other end?
2. What happens to the magnitude and directions of the tension forces at each end of a string and in the middle of that string when the direction of the string is changed by a post or pulley?
3. Can a flexible force transmitter (like a string) support a lateral (or sideways) force?

Mechanisms for Tension and the Direction of Forces

For these observations you should stretch a rubber band and then a string between your hands as shown in the diagram below. First, just feel the directions of the forces. Then add the spring scales and both feel and measure the forces.



Activity 2: Tension Mechanisms & Force Directions

(a) Pull on the two ends of a rubber band. (Forget about the spring scales for now). Does the rubber band stretch? What is the direction of the force applied by the rubber band on your right hand? On your left hand?

(b) Does the magnitude of the forces applied by the rubber band on each hand feel the same?

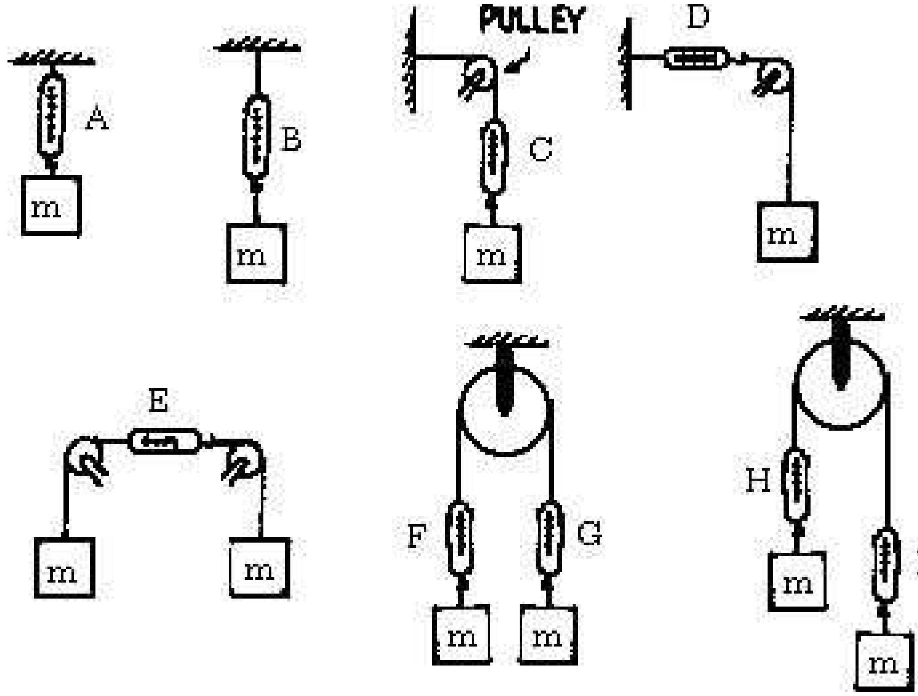
(c) Repeat this activity with a string instead of a rubber band. This time, use a spring scale at each end to measure the forces at the ends of the string. Does the string stretch? (Look carefully!)

(d) If you pull by the same amount on the string as you did on the rubber band, does substituting the string for the rubber band change anything about the directions and magnitudes of the tension forces exerted on each hand?

(e) If the forces caused by the string on your left and right hands respectively are given by F_{T1} and F_{T2}, what is the equation that relates these two forces?

Tension Forces when a String Changes Direction

Suppose you were to hang equal masses of $m = 0.5$ kg in the various configurations shown below. Predict and measure the tension in the string for each of the following situations.

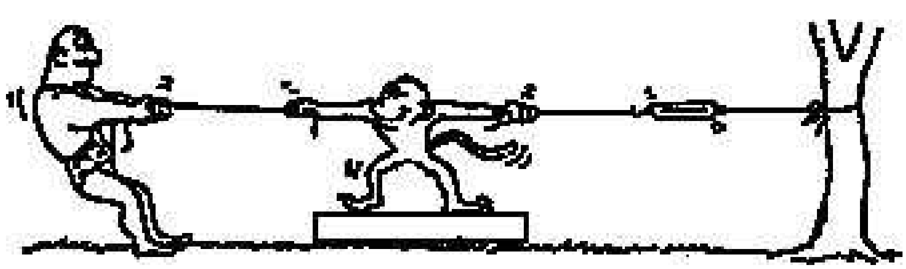


Activity 3: Tension and Direction Changes

(a) For each configuration shown above, predict the reading in newtons on each of the spring scales; these readings indicate the forces that are transmitted by the tensions at various places along the string. Then measure all of the forces and record their values. Note: Remember that m = 0.5 kg.

Predicted Force Magnitudes	Measured Force Magnitudes
F_A = ______________ N	F_A = ______________ N
F_B = ______________ N	F_B = ______________ N
F_C = ______________ N	F_C = ______________ N
F_D = ______________ N	F_D = ______________ N
F_E = ______________ N	F_E = ______________ N
F_F = ______________ N	F_F = ______________ N
F_G = ______________ N	F_G = ______________ N
F_H = ______________ N	F_H = ______________ N
F_I = ______________ N	F_I = ______________ N

(b) Based on Newton's third law and the observations you just made, answer the following questions using vector notation. If the muscle man in the diagram below is pulling to the left on a rope with a force of $\mathbf{F}$ = -(150 N)$\mathbf{i}$.



(1) What is the magnitude and direction of the force that the rope is exerting on the man? ______________

(2) What force is the left-hand rope exerting on the monkey's right arm? ______________

(3) What force is the spring scale experiencing on its left end? ______________

(4) What force is the spring scale experiencing on its right end? ______________

(5) What is the reading on the spring scale? ______________

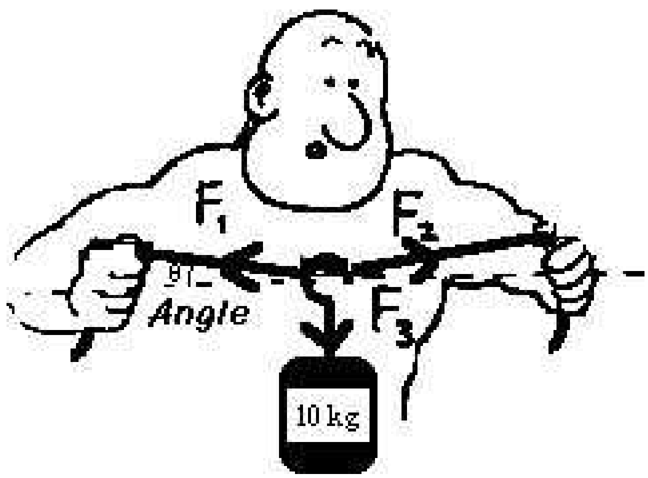
(6) What force is the rope exerting on the tree? ______________

(7) What force is the tree exerting on the rope? ______________

(c) Summarize what your observations reveal about the nature of tension forces everywhere along a string.

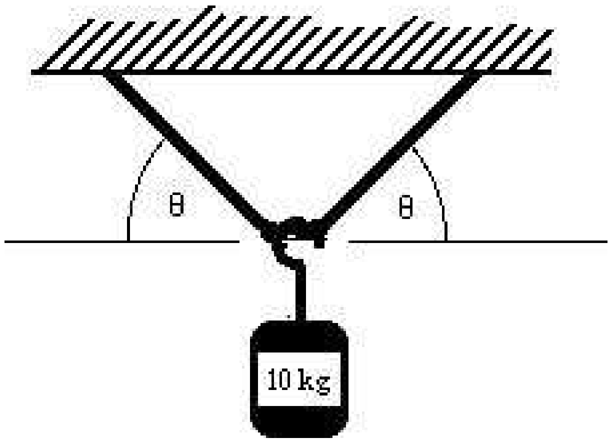
Can a String Support Lateral Forces?

Take a look at the diagram below. Can the strongest member of your group stretch a string or rope so that it is perfectly horizontal when a 10 kg mass is hanging from it? In other words, can the string provide a force that just balances the force exerted by the mass?



Activity 4: Can a String Support a Lateral Force?

(a) Draw a vector diagram showing the directions of the forces exerted by the strings on the mass hook in the diagram below. What would happen to the direction of the forces as θ goes to zero? Do you think it will be possible to support the mass when $\theta = 0$? Why?



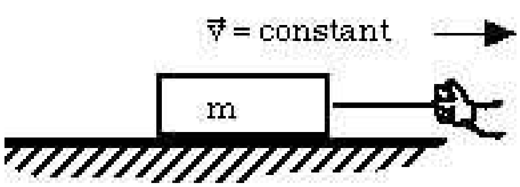
(b) Now, experiment with holding a mass horizontally with a string. What do you conclude about the ability of a string to support a mass having a force which is perpendicular to the direction of the string?

Normal Forces

A book resting on a table does not move; neither does a person pushing against a wall. According to Newton's first law the net force on the book and on the person's hand must be zero. We have to invent another type of force to explain why books don't fall through tables and hands don't usually punch through walls. The force exerted by any surface always seems to act in a direction perpendicular to that surface; such a force is known as a normal force.

Activity 5: Normal Forces

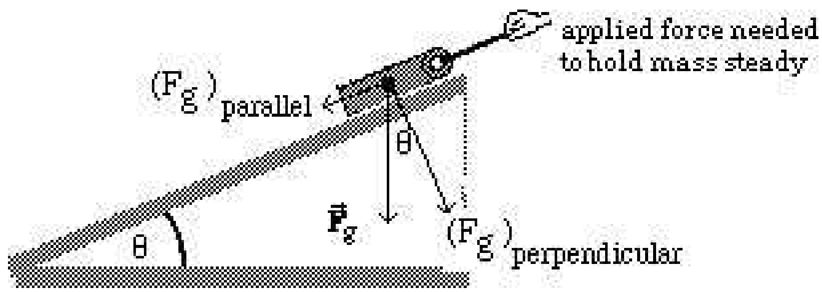
(a) The diagram below shows a block sliding along a table near the surface of the earth at a constant velocity. According to Newton's first law, what is the net force on the block? In other words, what is the vector sum of all the forces on the block?



(b) The net force is made up of four forces. In what direction does each one act? Draw a diagram indicating the direction of each of the forces.

Gravitational Force on a Mass on an Incline

Suppose that a block of mass m is perched on an incline of angle θ as shown in the diagram below. Also suppose that you know the angle of the incline and the magnitude and direction of the gravitational force vector. What do you predict the magnitude of the components of the force vector will be parallel to the plane? Perpendicular to the plane?



Activity 6: Components of $\mathbf{F}_g$ on an Incline

(a) The angle that the incline makes with the horizontal and the angle between $\mathbf{F}_{\text{perpendicular}}$ and F_g are the same. Explain why.

(b) Choose a coordinate system with the x-axis parallel to the plane with the positive direction up the plane. Using normal mathematical techniques for finding the components of a vector, find the values of F_{parallel} and $F_{\text{perpendicular}}$ as a function of F_g and the angle of the incline θ.

(c) What is the equation for the magnitude of the normal force exerted on the block by the surface of the incline? Hint: Use Newton's first law and the knowledge that the block is not moving in a direction perpendicular to the plane.

28 Atwood's Machine[18]

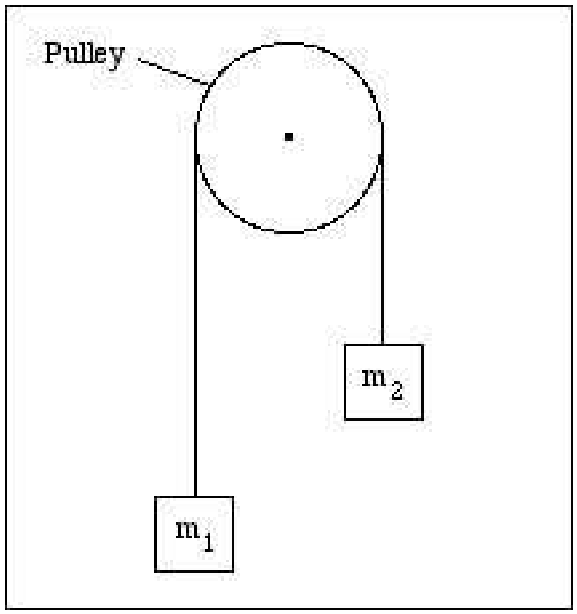
Name ______________________________ Section _______________ Date _______________

Objective

To apply tension concepts and Newton's laws of motion to the analysis of Atwood's machine.

Overview

Sometime before 1780, a physicist at Cambridge University named George Atwood devised a marvelous machine for measuring the acceleration of a falling mass without the aid of high speed timers, motion detectors, or video cameras. It consists of two masses connected to each other by means of a light string passing over a relatively frictionless light pulley, as shown in the diagram below.



Atwood's machine is not only historically important but it allows us to practice applying Newton's laws and the kinematic equations to the analysis of motion.

Apparatus

- Smart pulley with photogate
- String with 2 hooks
- Small washers (15)
- *Science Workshop 750 Interface*
- *DataStudio* software (Atwood's Machine application)

Activity 1: Predictions and Qualitative Observations

(a) Assume that the two masses are equal. If you pull down gently on one of them, what motion do you predict will result? Explain your reasoning.

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(b) Set up Atwood's Machine with combinations of equal masses (same number of washers on each hook), pull on one of them gently, and describe what you observe. How does your observation compare with your prediction?

(c) Suppose that m_1 is greater than m_2. What do you expect to observe and why?

(d) Set up Atwood's Machine with combinations of unequal masses, and describe what you observe. How do your observations compare with your prediction?

Activity 2: Quantitative Observations

(a) Perform the following procedure to measure the acceleration of Atwood's machine for different combinations of masses.

1. Determine and record the mass of the two wire hooks using a laboratory balance.

2. Determine and record the mass of fifteen (15) washers. Calculate and record the average mass of one washer.

3. Construct a data table below with the column headings: Trial #, m_1, m_2, $m_1 - m_2$, Experimental Acceleration, Theoretical Acceleration, and % Difference. Leave enough room to record data for seven trials.

4. Launch the Atwood's Machine application.

5. Place eight washers on one wire hook and seven on the other. Record the appropriate values in your data table including the masses of the wire hooks. Let m_1 be the heavier mass.

6. Move m_1 upward until m_2 almost touches the floor. Make sure the string is over the pulley and that the two masses are motionless.

7. Start recording data and release m_1, which will fall downward, pulling m_2 up. The computer will display a graph of velocity versus time. Fit the data with a straight line. The slope of the line and mean-squared error (MSE) of the fit will also be displayed. Examine the graph. If you feel the line provides a good fit to the data, record the slope in your data table. This is the acceleration in units of m/s^2. If the fit is not good, repeat the run.

8. Move one washer from m_2 to m_1, record the appropriate masses in your data table, and repeat steps 6 and 7. Note that the total mass of the system remains constant.

9. Repeat step 8 until m_2 consists of only one washer (and one wire hook). You will have a total of seven trials.

(b) Construct a graph of experimental acceleration (vertical axis) versus $m_1 - m_2$ (horizontal axis). Find the best fit to the data to determine the mathematical relationship between the mass difference and the acceleration. Print the graph showing the fit and put a copy in your notebook. Write the equation (with appropriate units) that describes the data in the space below.

Activity 3: Derivation of Atwood's Equation

(a) If the tension on the string is denoted by T, draw a diagram describing the forces on m_1. Draw another diagram showing the forces on m_2. Hint: Include the gravitational forces acting in the negative y-direction on masses 1 and 2 and the tension forces acting upwards. Assume that m_1 is greater than m_2.

(b) Write down an equation for the net force on m_1 in terms of T, g, and m_1. Use Newton's second law to relate this net force to the acceleration, a_1, of m_1.

(c) Next, write down an equation for the net force on m_2 in terms of T, g, and m_2. Use Newton's second law to relate this net force to the acceleration a_2 of m_2.

(d) Why can we say the acceleration of m_1 is $a_1 = -a$ if the acceleration of m_2 is $a_2 = a$?

(e) Eliminate T from the two equations to show that the net acceleration for the Atwood's machine is given by the equation $a = \frac{m_1 - m_2}{m_1 + m_2} g$.

(f) You have just derived the famous Atwood's equation. Use this expression to calculate the theoretical acceleration for each trial and record the values in the table in Activity 2.

(g) Compare the experimental acceleration with the theoretical acceleration by determining the % difference between the two for each trial. Is there good agreement between the two values? Do the results verify Newton's Second Law of motion? Is there a systematic difference? If so, what might this be due to?

(h) Rearrange the expression you derived for the acceleration so that it is the equation of the graph you constructed in Activity 2. Note that the slope of the graph contains g. Calculate g from the slope determined from your fit and find the % difference between this value and the accepted value. How well do the two values agree? Can you account for any difference?

29 Friction and Applying the Laws of Motion[19]

Name ____________________ Section __________ Date __________

Objectives

- To explore the characteristics of friction.
- To learn to use free-body diagrams to make predictions about the behavior of systems which are acted on by multiple forces in two and three dimensions.

Predicting and Measuring Friction Factors

If Newton's laws are to be used to describe the sliding of a block in contact with a flat surface, we must postulate the existence of a frictional force that crops up to oppose the applied force. There are two kinds of frictional forces: static friction and kinetic or sliding friction, which is the friction between surfaces in relative motion. We will concentrate on the study of kinetic friction for a sliding block.

Apparatus

- Wooden block with hook
- Spring scale
- Variety of masses
- Triple-beam balance

Activity 1: Prediction of Friction Factors

(a) List several parameters that might influence the magnitude of the kinetic frictional force.

(b) Describe how you might do an experiment to determine the effect of mass on the magnitude of the frictional force.

Measuring the Effect of Mass on Kinetic Friction

Let's determine how mass actually influences the frictional force. Perform the experiment that you designed in the previous activity to determine how the frictional force varies with mass.

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Activity 2: Friction Data and Analysis

(a) Create a data table for the frictional force as a function of mass in the space below. You should make at least 5-7 measurements.

(b) Graph the frictional force (vertical axis) versus the mass and find the best fit to the data. Print the graph showing the fit and put a copy in your notebook. Write the equation (with appropriate units) that describes the data in the space below.

(c) Look up kinetic friction in the index of your text. Read about the coefficient of sliding friction, μ_k, and figure out how to determine μ_k from the fit to your data. Calculate μ_k for the block sliding on the table. How does your value compare with the appropriate value in the table in your text?

Theories of Friction

No material is perfectly "smooth and flat." Any surface when examined under a microscope is full of irregularities. It is usually assumed that sliding friction forces result from the rubbing of rough surfaces, i.e., from the interlocking of surface bumps during the sliding process. How reasonable is this explanation for sliding friction?

Activity 3: What Surfaces Have High Friction?

(a) Which kinds of surfaces do you think will have the most friction rough ones or smooth ones? Why?

(b) Examine the table of coefficients of friction in a text. Do the values listed in this table support your predictions? Are you surprised?

The fact that smooth surfaces sometimes have more sliding friction associated with them than rough surfaces has led to the modern view that other factors such as adhesion (i.e., the attraction between molecules on sliding surfaces) also play a major role in friction. Predicting the coefficient of sliding friction for different types of surfaces is not always possible and there is much yet to be learned about the nature of the forces that govern sliding friction. Most authors of introductory physics texts still tend, incorrectly, to equate smooth surfaces with "frictionless ones" and to claim that the rubbing of rough surfaces is the cause of friction.

Free-Body Diagrams: Putting It All Together

You have made a series of observations which hopefully led you to reconstruct Newton's three laws of motion and some of its ramifications for yourself. In summary the laws are:

Newton's First Law: If the net force acting on an object is zero its acceleration is zero. [If $\sum \mathbf{F} = 0$ then $\mathbf{a} = 0$ so that $\mathbf{v}$ = constant or zero.]

Newton's Second Law: The net force on an object can be calculated by multiplying its mass times its acceleration. [$\sum \mathbf{F} = m\mathbf{a}$.]

Newton's Third Law: Any two objects that interact exert forces on each other which are equal in magnitude and opposite in direction. [$\mathbf{F}_{12} = -\mathbf{F}_{21}$]

These three laws are incredibly powerful because an understanding of them allows you to either: (1) use a complete knowledge of forces on a system of objects to predict motions in the system or (2) identify the forces on a system of objects based on observations of its motions.

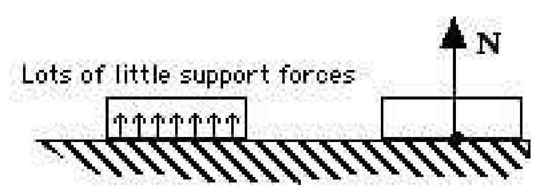
Using Free-Body Diagrams to Predict Motions and Calculate Forces

The key to the effective application of Newton's laws is to identify and diagram all the forces acting on each object in a system of interest. The next step is to define a coordinate system and break the forces down into components to take advantage of the fact that if $\sum \mathbf{F} = m\mathbf{a}$ then $\sum F_x = ma_x$ and $\sum F_y = ma_y$.

A free-body diagram consists of a set of arrows representing all the forces on an object, but NOT the forces that the object exerts on other objects. To create a free-body diagram you should do as follows:

1. Draw arrows to represent all force acting on the object or objects in the system of interest.
2. Place the tail of each arrow at the point where the force acts on the object.
3. Each arrow should point in the direction of the force it represents.
4. The relative lengths of the arrows should, if possible, be made to correspond to the magnitudes of the forces.
5. A set of coordinate axis should be chosen and indicated and all of the arrows should be labeled using standard notation to indicate the type of force involved.

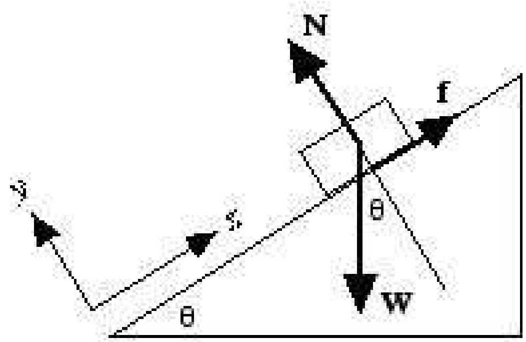
Important Note: The idea of using a single force vector to summarize external forces that act in the same direction is a useful simplification which is not real. For example, when a block rests on a table, we will say that the table exerts a normal force on the block. It is conventional to draw a single upward arrow at the point where the middle of the bottom surface of the block touches the table. This arrow actually represents the sum of all the smaller forces at each point where the block touches the table. This is shown in the diagram below:



An Example of a Free-Body Diagram

Consider a block of mass m sliding on a rough inclined plane as shown in the diagram below. It has three forces on it: (1) a gravitational force, (2) a normal force perpendicular to the surface of the plane, and (3) the friction force opposing its motion down the plane.

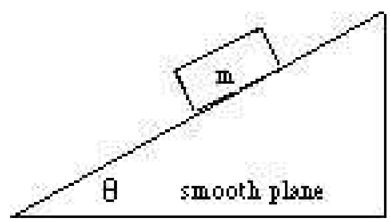
Since the block is sliding it could either be moving at a constant velocity or with a constant acceleration. Thus, it is possible that the vector sum of forces on it is not equal to zero in some cases. For example, if there is accelerated motion, then: $\sum \mathbf{F} = \mathbf{N} + \mathbf{W} + \mathbf{f} = m\mathbf{a}$



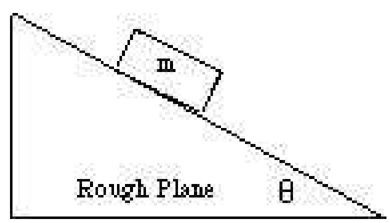
Based on this example, try your hand at drawing the free-body diagrams for the situations described below. In each case, write the equation for the vector sum of forces. As always, be sure to put arrows over symbols representing vector quantities.

Activity 4: Some Free-Body Diagrams

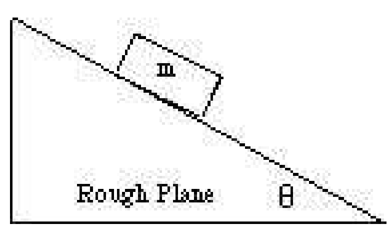
(a) A block slides freely down a smooth inclined plane.



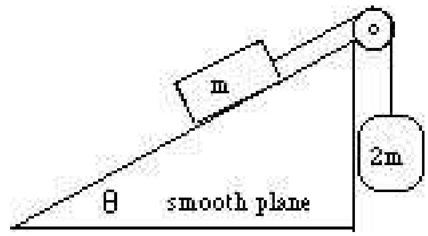
(b) A block is on a rough plane but is not moving due to static friction.



(c) A block is on a rough plane but the coefficient of friction between the block and the plane is small. The block is sliding down the plane at a constant velocity so that kinetic friction is acting.

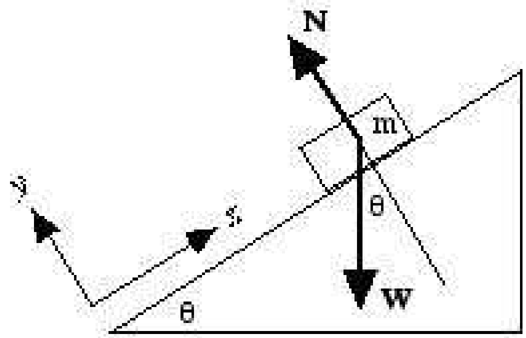


(d) The block is on a frictionless plane but is attached to a hanging mass of $2m$ by one of our famous "massless" strings over a pulley. Construct free-body diagrams for the forces on the mass m and for the forces on the mass $2m$.



Breaking the Forces into Components: An Example

Consider the case of the block sliding down a "smooth" plane with a negligible amount of friction. The free-body diagram and coordinate system chosen for analysis are shown in the figure below.



Taking components: $W_x = -W \sin\theta$ and $W_y = -W \cos\theta$.

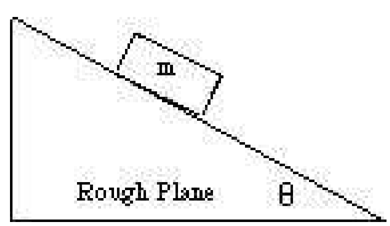
There is no motion perpendicular to the surface so $\sum F_y = 0 = N + W_y$ so that $N = -W_y = W \cos\theta$.

There is no balancing force for the x-component of W so according to Newton's second law $\sum F_x = ma_x = -W \sin\theta = -mg \sin\theta$ and therefore $a_x = -g \sin\theta$.

Activity 5: Solving an Inclined Plane Problem

(a) Consider a block sliding down a rough inclined plane at a constant velocity. What is the net force on it?

(b) Refer to the free-body diagram you created for this situation in the last activity. Break the forces up into components and apply Newton's first law to find the equation for the coefficient of kinetic friction as a function of m, θ, and g. You may need to consult a standard textbook for hints on how to tackle this problem.



30 The Electrical and Gravitational Forces[20]

Name ______________________________ Section _______________ Date _______________

> I began to think of gravity extending to the orb of the moon, and . . . I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since. — Isaac Newton

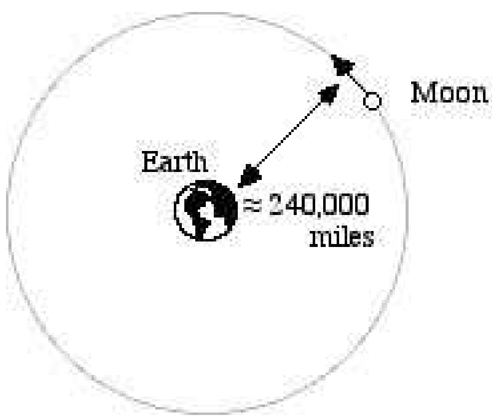
Objective

To understand the similarities of the gravitational and electrical forces.

Overview

The enterprise of physics is concerned ultimately with mathematically describing the fundamental forces of nature. Nature offers us several fundamental forces, which include a strong force that holds the nuclei of atoms together, a weak force that helps us describe certain kinds of radioactive decay in the nucleus, the force of gravity, and the electromagnetic force.

Two kinds of force dominate our everyday reality: the gravitational force acting between masses and the Coulomb force acting between electrical charges. The gravitational force allows us to describe mathematically how objects near the surface of the earth are attracted toward the earth and how the moon revolves around the earth and planets revolve around the sun. The genius of Newton was to realize that objects as diverse as falling apples and revolving planets are both moving under the action of the same gravitational force.



Similarly, the Coulomb force allows us to describe how one charge "falls" toward another or how an electron orbits a proton in a hydrogen atom.

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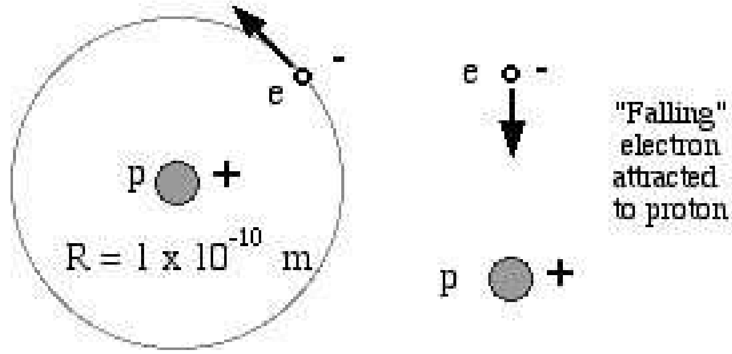
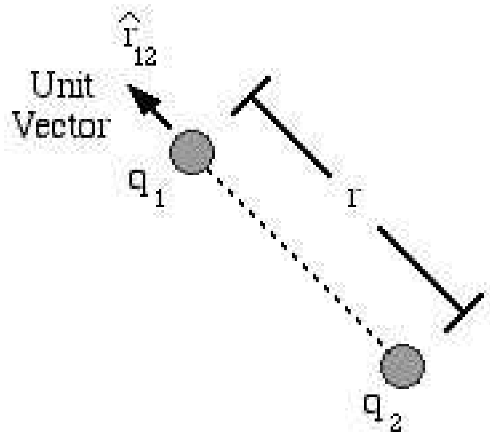


The fact that both the Coulomb and the gravitational forces lead to objects falling and to objects orbiting around each other suggests that these forces might have the same mathematical form.

In this unit we will explore the mathematical symmetry between electrical and gravitational forces for two reasons. First, it is beautiful to behold the unity that nature offers us as we use the same type of mathematics to predict the motion of planets and galaxies, the falling of objects, the flow of electrons in circuits, and the nature of the hydrogen atom and of other chemical elements. Second, what you have already learned about the influence of the gravitational force on a mass can be applied to aid your understanding of the forces on charged particles.

Activity 1: Comparison of Electrical and Gravitational Forces

Let's start our discussion of this comparison with the familiar expression of the Coulomb force exerted on charge 1 by charge 2.

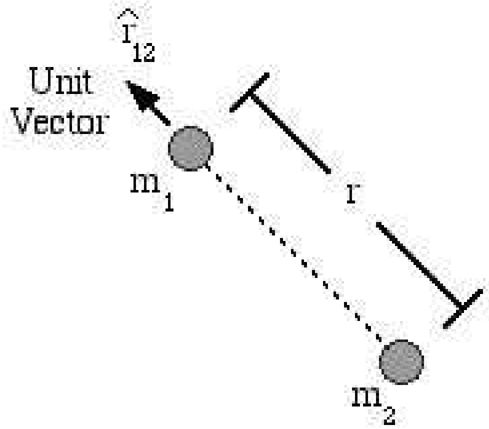


$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$k_e = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$$

Charles Coulomb did his experimental investigations of this force in the 18th century by exploring the forces between two small charged spheres. Much later, in the 20th century, Coulomb's law enabled scientists to design cyclotrons and other types of accelerators for moving charged particles in circular orbits at high speeds.

Newton's discovery of the universal law of gravitation came the other way around. He thought about orbits first. This was back in the 17th century, long before Coulomb began his studies. A statement of Newton's universal law of gravitation describing the force experienced by mass 1 due to the presence of mass 2 is shown below in modern mathematical notation:



$$\mathbf{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

$$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$$

About the time that Coulomb did his experiments with electrical charges in the 18th century, one of his contemporaries, Henry Cavendish, did a direct experiment to determine the nature of the gravitational force between two spherical masses in a laboratory. This confirmed Newton's gravitational force law and allowed him to determine the gravitational constant, G. A fact emerges that is quite amazing. Both types of forces, electrical and gravitational, are very similar. Essentially the same mathematics can be used to describe orbital and linear motions due to either electrical or gravitational interactions of the tiniest fundamental particles or the largest galaxies. This statement needs to be qualified a bit when electrons, protons and other fundamental particles are considered. A new field called quantum mechanics was developed in the early part of the last century to take into account the wave nature of matter, which we don't actually study in introductory physics. However, even in wave mechanical calculations electrical forces like those shown above are used.

Activity 2: The Electrical vs. the Gravitational Force

Examine the mathematical expression for the two force laws.

(a) What is the same about the two force laws?

(b) What is different? For example, is the force between two like masses attractive or repulsive? How about two like charges? What part of each equation determines whether the like charges or masses are attractive or repulsive?

(c) Do you think negative mass could exist? If there is negative mass, would two negative masses attract or repel?

Which Force is Stronger– Electrical or Gravitational?

Gravitational forces hold the planets in our solar system in orbit and account for the motions of matter in galaxies. Electrical forces serve to hold atoms and molecules together. If we consider two of the most common fundamental particles, the electron and the proton, how do their electrical and gravitational forces compare with each other?

Let's peek into the hydrogen atom and compare the gravitational force on the electron due to interaction of its mass with that of the proton to the electrical force between the two particles as a result of their charge. In order to do the calculation you'll need to use some well known constants.

Electron: $m_e = 9.1 \times 10^{-31}$ kg, $q_e = -1.6 \times 10^{-19}$ C

Proton: $m_p = 1.7 \times 10^{-27}$ kg, $q_p = +1.6 \times 10^{-19}$ C

Distance between the electron and proton: $r = 1.0 \times 10^{-10}$ m

Activity 3: The Electrical vs. the Gravitational Force in the Hydrogen Atom

(a) Calculate the magnitude of the electrical force on the electron. Is it attractive or repulsive?

(b) Calculate the magnitude of the gravitational force on the electron. Is it attractive or repulsive?

(c) Which is larger? By what factor (i.e., what is the ratio)?

(d) Which force are you more aware of on a daily basis? If your answer does not agree with that in part (c), explain why.

Activity 4: The Gravitational Force of the Earth

(a) Use Newton's law of universal gravitation to show that the magnitude of the acceleration due to gravity on an object of mass m at a height h above the surface of the earth is given by the following expression.

$$\frac{GM_e}{(R_e + h)^2}$$

Hint: Because of the spherical symmetry of the Earth you can treat the mass of the Earth as if it were all concentrated at a point at the Earth's center.

(b) Calculate the acceleration due to gravity of a mass m at the surface of the earth (h=0). The radius of the earth is $R_e \approx 6.38 \times 10^3$ km and its mass $M_e \approx 5.98 \times 10^{24}$ kg. Does the result look familiar? How is this acceleration related to the gravitational acceleration g?

(c) Use the equation you derived in part (a) to calculate the acceleration due to gravity at the ceiling of the room you are now in. How does it differ from the value at the floor? Can you measure the difference in the lab using the devices available?

(d) Suppose you travel halfway to the moon. What is the new value of the acceleration due to gravity (neglecting the effect of the moon's pull)? (Recall that the earth-moon distance is about 384,000 km.)

(e) Is the gravitational acceleration "constant," g, really a constant? Explain.

(f) In part (d) you showed that there is a significant gravitational attraction halfway between the earth and the moon. Why, then, do astronauts experience "weightlessness" when they are orbiting a mere 120 km above the earth?

31 Centripetal Force[21]

Name ______________________ Section ____________ Date ____________

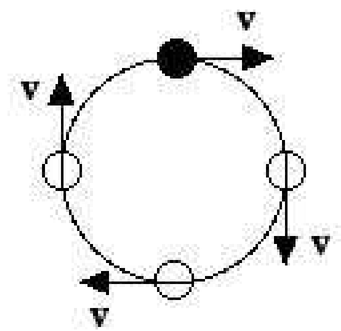
Objective

To explore the phenomenon of uniform circular motion and the accelerations and forces needed to maintain it.

Overview

In a previous unit you began the study of the application of Newton's laws to projectile motion. In this unit we are going to consider the application of Newton's laws to another phenomenon in two dimensions. Since Newton's laws can be used to predict types of motion or the conditions for no motion, their applications are useful in many endeavors including human body motion, astrophysics, and engineering.

You will explore uniform circular motion, in which an object moves at a constant speed in a circle. In particular, you will develop a mathematical description of centripetal acceleration and the force needed to keep an object moving in a circle.



Apparatus

- An airplane.
- A video analysis system (VideoPoint).
- A spring scale.
- Graphing software (Excel).

Moving in a Circle at a Constant Speed

When a race car speeds around a circular track, or when David twirled a stone at the end of a rope to clobber Goliath, or when a planet like Venus orbits the sun, they undergo uniform circular motion. Understanding the forces which govern orbital motion has been vital to astronomers in their quest to understand the laws of gravitation.

But we are getting ahead of ourselves, for as we have done in the case of linear and projectile motion we will begin our study by considering situations involving external applied forces that lead to circular motion in the absence of friction. We will then use our belief in Newton's laws to see how the circular motions of the planets can be used to help astronomers discover the laws of gravitation.

[21] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

Let's begin our study with some very simple considerations. Suppose an astronaut goes into outer space, ties a ball to the end of a rope, and spins the ball so that it moves at a constant speed.

Activity 1: Uniform Circular Motion

(a) Consider the figure above. What is the speed of a ball that moves in a circle of radius $r = 2.5$ m if it takes 0.50 s to complete one revolution?

(b) The speed of the ball is constant! Would you say that this is accelerated motion?

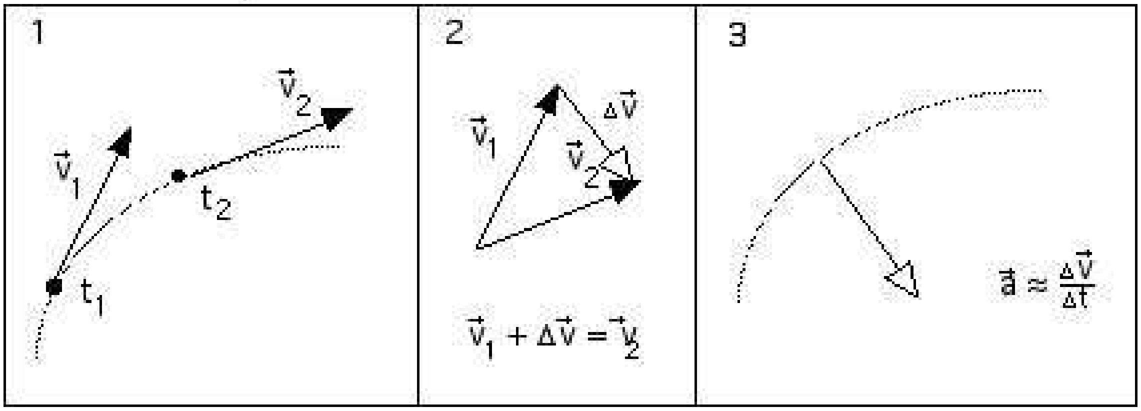
(c) What is the definition of acceleration? (Remember that acceleration is a vector!)

(d) Are velocity and speed the same thing? Is the velocity of the ball constant? (Hint: Velocity is a vector quantity!)

(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

Using Vectors to Diagram How Velocity Changes

By now you should have concluded that since the direction of the motion of an object undergoing uniform circular motion is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the direction of the acceleration and its magnitude as a function of the speed v of an object such as a ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.

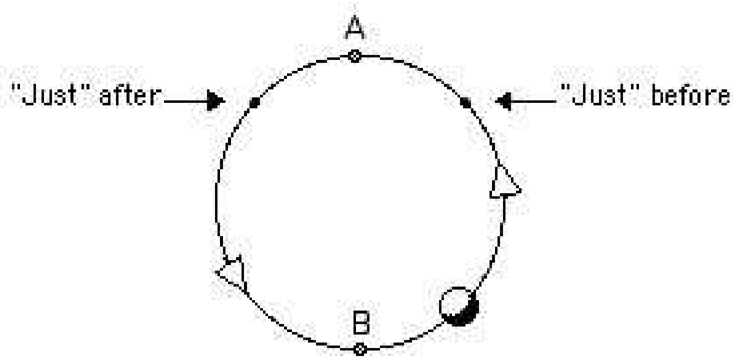


1. To Draw Velocities: Draw an arrow representing the velocity, $\mathbf{v}_1$, of the object at time t_1. Draw another arrow representing the velocity, $\mathbf{v}_2$, of the object at time t_2.
2. To Draw Velocity Change: Find the change in the velocity $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$ during the time interval described by $\Delta t = t_2 - t_1$. Start by using the rules of vector sums to rearrange the terms so that $\mathbf{v}_1 + \Delta\mathbf{v} = \mathbf{v}_2$. Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector that points from the head of the first velocity vector to the head of the second velocity vector.
3. To Draw Acceleration: The acceleration equals the velocity change $\Delta\mathbf{v}$ divided by the time interval t needed for the change. Thus, $\mathbf{a}$ is in the same direction as $\Delta\mathbf{v}$ but is a different length (unless $\Delta t = 1$). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as $\Delta\mathbf{v}$.

The acceleration associated with uniform circular motion is known as centripetal acceleration. You will use the vector diagram technique described above to find its direction.

Activity 2: The Direction of Centripetal Acceleration

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot just before it reaches point A. Label this vector $\mathbf{v}_1$.



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot just after it passes point A. Label this vector $\mathbf{v}_2$.

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. Next, draw the vector that must be added to vector $\mathbf{v}_1$ to add up to vector $\mathbf{v}_2$; label this vector $\Delta\mathbf{v}$. Be sure that vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ have the same magnitude and direction in this drawing that they had in your drawing in part (a)!

(d) Now, draw an exact copy of $\Delta\mathbf{v}$ on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original $\Delta\mathbf{v}$ in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration, $\mathbf{a}_c$?

(f) If you re did the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration, $\mathbf{a}_c$, would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

(h) Use Newton's second law in vector form ($\sum \mathbf{F} = m\mathbf{a}$) to describe the direction of the net force on the ball as it moves around the circle.

(i) If the ball is being twirled around on a string, what is the source of the net force needed to keep it moving in a circle?

Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed

You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use the rules of mathematics and the definition of acceleration to derive the relationship between speed, radius, and magnitude of centripetal acceleration.

Activity 3: How Does $\mathbf{a}_c$ Depend on v and r?

(a) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude, a_c, have to increase or decrease as r decreases if circular motion is to be maintained? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius r if the speed v is increased? In other words, would the magnitude, a_c, have to increase or decrease as v increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Lets use the definition of acceleration in two dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

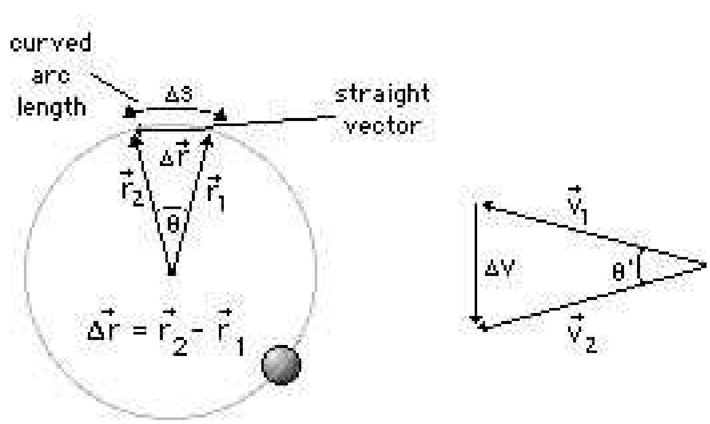
$$a_c = \frac{v^2}{r} \qquad [Eq.\,1]$$

In order to do this derivation you will want to use the following definition for acceleration

$$\langle \mathbf{a} \rangle = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad [Eq.\,2]$$

Activity 4: Finding the Equation for $\mathbf{a}_c$

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the position vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. Hint: In circular motion, velocity vectors are always perpendicular to their position vectors.



(b) Since the angles are the same and since the magnitudes of the displacements never change (i.e., $r = r_1 = r_2$) and the magnitudes of the velocities never change (i.e., $v = v_1 = v_2$), use the properties of similar triangles to explain why $\frac{\Delta v}{v} = \frac{\Delta r}{r}$.

(c) Now use the equation in part (b) and the definition of $\langle a \rangle$ to show that $\langle a_c \rangle = \frac{\Delta v}{\Delta t} = \frac{(\Delta r)}{(\Delta t)} \frac{v}{r}$.

(d) The speed of the object as it rotates around the circle is given by $v = \frac{\Delta s}{\Delta t}$. Is the change in arc length, Δs, larger or smaller than the magnitude of the change in the position vector, Δr? Explain why the arc length change and the change in the position vector are approximately the same when t is very small (so that the angle θ becomes very small) i.e., why is $\Delta s \simeq \Delta r$?

(e) If $\Delta s \simeq \Delta r$, then what is the equation for the speed in terms of Δr and Δt?

(f) Using the equation in part (c), show that as $\Delta t \to 0$, the instantaneous value of the centripetal acceleration is given by Eq. 1.

(g) If the object has a mass m, what is the equation for the magnitude of the centripetal force needed to keep the object rotating in a circle (in terms of v, r, and m)? In what direction does this force point as the object rotates in its circular orbit?

Experimental Test of the Centripetal Force Equation

The theoretical considerations in the last activity should have led you to the conclusion that, whenever you see an object of mass m moving in a circle of radius r at a constant speed v, it must at all times be experiencing a net centripetal force directed toward the center of the circle that has a magnitude of

$$F_c = ma_c = m\frac{v^2}{r} \qquad [Eq. 3]$$

Let's check this out. Does this rather odd equation really work for an external force?

To test the validity of the derivation we must compare it to experience. We will use a "toy" airplane suspended from a string and flying in a circular path. We will use the video analysis system to measure the properties of the motion and determine the horizontal and vertical components of the force exerted on the airplane using Equation 3. We will compare that result with a direct measurement of the tension in the string.

Activity 5: Verifying the $\mathbf{F}_c$ Equation

(a) Measure the mass of the airplane and the length of the portion of the string that hangs below the horizontal post with the hole in it. Record the values below.

(b) The airplane is suspended from a spring scale. The string should pass straight down from the scale through the small hole in the horizontal post. The camera should be placed about 1 meter above the airplane. Turn on the camera and center the spring scale in the frame by using the pendulum. Is the reading of the spring scale consistent with the mass of the airplane? Mount a ruler somewhere in the camera's field of view to serve as a scaling object. Launch the plane into uniform circular motion. When the motion is steady record a movie of at least one complete revolution and record the reading of the spring scale. See **Appendix D: Video Analysis** for details.

$F_{scale} =$

(c) Determine the position of the airplane during one complete revolution. To do this task follow the instructions in the second section of **Appendix D: Video Analysis** for recording and calibrating a data file. As you analyze the movie frame by frame, estimate to the nearest fraction of a frame the number of frames for one complete revolution. Record your result below. When you calibrate the time and position data, note the number of frames per second of the movie and convert that number to the time interval between frames, Δt. Record this result below. Now you can determine the time interval for one complete revolution and record it below. The resulting data file should contain three columns with the values of time, x-position, and y-position.

$N_{frame} =$

$\Delta t =$

$t_{rev} =$

(d) Make a graph of the trajectory of the airplane during one full revolution. See **Appendix C: Introduction to Excel** for details on using the graphing software. When you make your plot make sure the x and y axes cover the same size range; otherwise, you will distort the path of the airplane. Print your plot and attach it to your write-up.

1. Is the motion circular? What is your evidence?

2. What is the radius of the motion? $r =$

(e) To test the validity of the expression for F_c we must know the speed. Use the measurements of the radius of the airplane's trajectory and the time for one complete revolution to calculate the average speed.

$v_{ave} =$

(f) Use your results for the mass, the average velocity of the airplane, and the radius of the circular motion to predict the centripetal force exerted by the string.

$F_c =$

(g) We need one more piece of the puzzle to predict the tension in the string, namely, the vertical component of the force exerted on the airplane by the string. Recall that we know r, the radius of the airplane's circular path, and R, the total length of the string that is actually rotating.

1. Using these two distances (r and R), calculate the angle the string makes with the horizontal.

$\theta =$

2. We determined the horizontal component of the force on the airplane in part (f). Now with the angle θ generate an expression for the vertical component exerted by the string and calculate it. Make a vector diagram of the different components. Generate an expression for the total force acting on the airplane due to the string and calculate the result.

   $F_y =$

   $F_{plane} =$

(h) Compare your result for F_{plane} with the measurement of the spring scale F_{scale}. Within experimental uncertainty, how well does your data support the hypothesis that $F_c = mv^2/r$?

(i) Discuss the major sources of uncertainty in this experiment.

32 Kepler's Laws[22]

Name ______________________ Section ____________ Date ____________

Objectives:

- The applicability of Kepler's Laws
- The mass of the moon

Introduction

Based on data provided by Tycho Brahe, Johannes Kepler devised three laws of planetary motion:

1. The orbits of the planets are ellipses with the sun at one focus.

2. A line segment connecting the sun and a given planet sweeps out equal areas in equal time intervals.

3. The square of the period is proportional to the cube of the semi-major axis of the orbit.

Newton's Law of Gravitation generalized these laws to apply to any two bodies in orbital motion about one another.

Newton's analysis led to the following formulation of Kepler's Third Law:

$$T^2 = \frac{4\pi^2}{G(M+m)}a^3 \tag{1}$$

where M and m are the two masses, T is the period of mutual revolution, G is the universal gravitational constant, and a is the semi-major axis of their relative orbit.

Apparatus:

- ruler

Activity 1: The ellipse

Discussion An ellipse is the set of points in the plane for which the sum of the distances from two fixed points is constant. The two points are called foci (the plural of focus). If the center of the ellipse is at the origin, then the standard form of the equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2}$$

where the curve intercepts the x-axis at $(a, 0)$ and $(-a, 0)$, and the y-axis at $(0, b)$ and $(0, -b)$. If $a > b$, then the foci are at $(\pm c, 0)$, where $c = \sqrt{a^2 - b^2}$. If $a < b$, then the foci are at $(0, \pm c)$, where now $c = \sqrt{b^2 - a^2}$. If $a = b$, the ellipse is a circle. The departure from circularity is quantified with the term eccentricity, defined symbolically as

$$\varepsilon = \frac{c}{a} \tag{3}$$

when $a > b$. If P is any point on the ellipse, then the sum of its distances from the foci is $2a$ for $a > b$, or $2b$ for $a < b$. An ellipse can be drawn with a string tacked at the foci, pulling the string taut with a pencil, and running the pencil around.

[22] Adapted from Activities in Astronomy, D. Hoff, L. Kelsey, and J. Neff, Kendall/Hunt Publishing Co., 1978.

1. Sketch an ellipse with an eccentricity of 1.

2. Sketch an ellipse with an eccentricity of 0.

Activity 2: Explorer 35 and Kepler's Laws

Discussion Explorer 35 lifted off from Cape Kennedy on 19 July 1967, and entered orbit around the moon on 22 July. The satellite, weighing 230 lbs. (1.04.4 kg), carried instruments for measuring solar x-rays and energetic particles; the solar wind in interplanetary space and its interaction with the moon; and the magnetic properties of the moon. It continued operation until June 1973, having accomplished all its mission objectives.

The data[23] below form a sample set of positions of Explorer 35 in its elliptical orbit about the moon. The time interval between position measurements is 15 minutes. The unit of length is the radius of the moon. The center of the coordinate system is the center of the moon.

1. Using Excel, plot the data and find the major axis and foci.

2. Find the semi-major axis, the minor axis, the semi-minor axis, the eccentricity, and the period of the orbit (this last one, by inspection).

3. Verify Kepler's First Law. [Hint: refer to the definition of an ellipse.]

4. Determine the areas swept out in a time interval at different points of the orbit and show that Kepler's Second Law is valid. [Hint: use triangles as an approximation.]

5. Using the Newtonian form of Kepler's Third Law, find the mass of the moon in kilograms. [radius of the moon = 1738 km; and $G = 6.668 \times 10^{-8}$ cm^3/gm sec^2 $= 8.642 \times 10^{-13}$ km^3/kg hr^2]

[23] From the NASA-Goddard Space Flight Center.

Questions:

1. Discuss variations in your measurements used to validate the First and Second Laws.

2. Noting the number of significant figures in the data, how do the variations in your measurements compare to the uncertainties in the data?

3. Estimate the uncertainty in your determination of the mass of the moon. What has a greater effect, uncertainties in a or those in T?

Elapsed time (h)	(m)	X (lunar radii)	Y (lunar radii)	Elapsed time (h)	(m)	X (lunar radii)	Y (lunar radii)
0	00	-3.62	1.04	6	00	0.27	4.86
0	15	-3.46	0.63	6	15	-0.56	4.95
0	30	-3.25	0.20	6	30	-0.84	5.01
0	45	-2.97	-0.22	6	45	-1.12	5.03
1	00	-2.60	-0.65	7	00	-1.38	5.04
1	15	-2.14	-1.03	7	15	-1.64	5.00
1	30	-1.55	-1.37	7	30	-1.89	4.95
1	45	-0.85	-1.58	7	45	-2.14	4.87
2	00	-0.03	-1.59	8	00	-2.37	4.77
2	15	+0.78	-1.32	8	15	-2.59	4.65
2	30	1.45	-0.79	8	30	-2.80	4.50
2	45	1.87	-0.11	8	45	-2.99	4.33
3	00	2.09	+0.58	9	00	-3.17	4.14
3	15	2.16	1.22	9	15	-3.33	3.93
3	30	2.11	1.82	9	30	-3.49	3.69
3	45	1.99	2.35	9	45	-3.59	3.42
4	00	1.82	2.81	10	00	-3.69	3.15
4	15	1.61	3.22	10	15	-3.77	2.85
4	30	1.37	3.59	10	30	-3.81	2.52
4	45	1.11	3.90	10	45	-3.83	2.20
5	00	0.85	4.16	11	00	-3.81	1.83
5	15	0.58	4.40	11	15	-3.76	1.46
5	30	0.28	4.58	11	30	-3.65	1.06
5	45	0.00	4.74	11	45	-3.51	0.65

33 Work and Power[24]

Name ______________________ Section ____________ Date ____________

Objectives

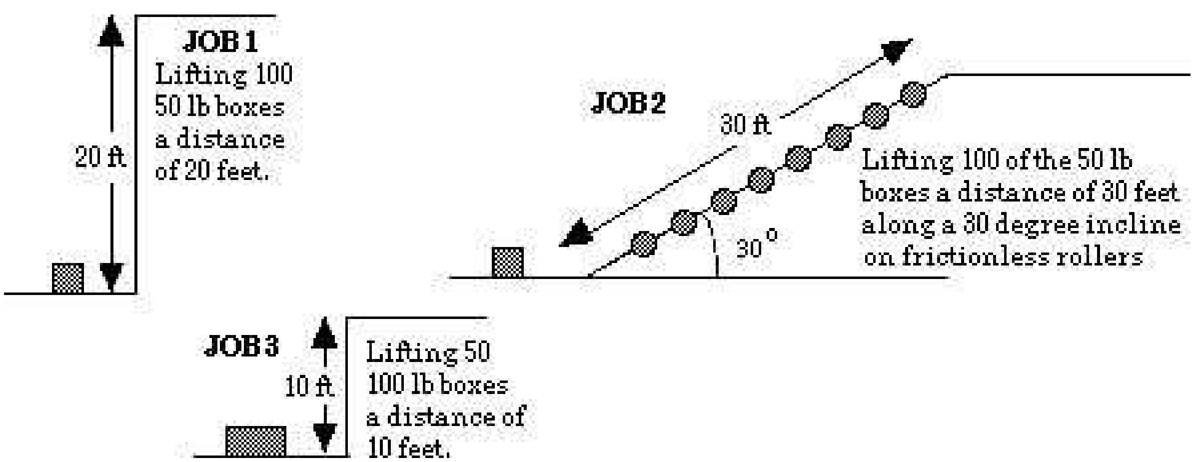
- To extend the intuitive notion of work as physical effort to a formal mathematical definition of work as a function of force and distance.
- To understand the concept of power and its relationship to work.

Apparatus

- Spring scale
- Wooden block with hook
- Variety of masses

The Concept of Physical Work

Suppose you are president of the Richmond Load 'n' Go Co. A local college has three jobs available and will allow you to choose which one you want before offering the other two jobs to rival companies. All three jobs pay the same amount of money. Which one would you choose for your crew?

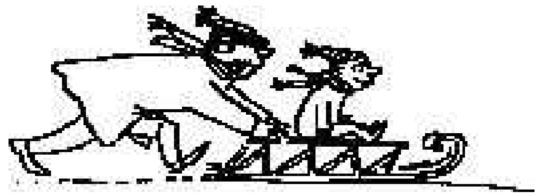


Activity 1: Choosing Your Job

Examine the descriptions of the jobs shown in figure above. Which one would you be most likely to choose? Least likely to choose? Explain the reasons for your answer.

You obviously want to do the least amount of work for the most money. Before you reconsider your answers later in this unit, you should do a series of activities to get a better feel for what physicists mean by work and how the president of Load 'n' Go can make top dollar.

[24] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.



In everyday language we refer to doing work whenever we expend effort. In order to get an intuitive feel for how we might define work mathematically, you should experiment with moving your textbook back and forth along a table top and a rougher surface such as a carpeted floor.

Activity 2: This is Work!

(a) Pick a distance of a meter or so. Sense how much effort it takes to push a heavy book that distance. How much more effort does it take to push it twice as far?

(b) Pile another similar book on top of the original one and sense how much effort it takes to push the two books through the distance you picked. Comment below.

(c) From your study of sliding friction, what is the relationship between the mass of a sliding object and the friction force it experiences? On the basis of your experience with sliding friction, estimate how much more force you have to apply to push two books compared to one book.

(d) If the "effort" it takes to move an object is associated with physical work, guess an equation that can be used to define work mathematically when the force on an object and its displacement (i.e., the distance it moves) lie along the same line.

In physics, work is not simply effort. In fact, the physicist's definition of work is precise and mathematical. In order to have a full understanding of how work is defined in physics, we need to consider its definition in a very simple situation and then enrich it later to include more realistic situations.

A Simple Definition of Physical Work: If an object that is moving in a straight line experiences a constant force in the direction of its motion during the time it is undergoing a displacement, the work done by the external force, F_{ext}, is defined as the product of the force and the displacement of the object,

$$W = F_{ext}\Delta x$$

where W represents the work done by the external force, F_{ext} is the magnitude of the force, and Δx is the displacement of the object.

What if the force of interest and the displacement are in opposite directions? For instance, what about the work done by the force of sliding friction, F_f, when a block slides on a rough surface? The work done by the friction force is

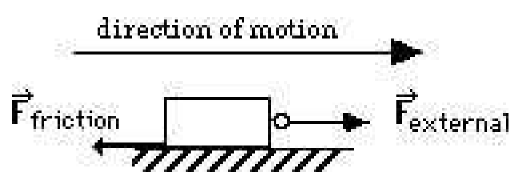
$$W_f = -F_f \Delta x$$

Activity 3: Applying the Physics Definition of Work

(a) Does effort necessarily result in physical work? Suppose two guys are in an evenly matched tug of war. They are obviously expending effort to pull on the rope, but according to the definition of physical work, are they doing any physical work? Explain.

(b) A wooden block with a mass of .30 kg is pushed along a sheet of ice that has no friction with a constant external force of 10 N which acts in a horizontal direction. After it moves a distance of 0.40 m how much work has been done on the block by the external force?

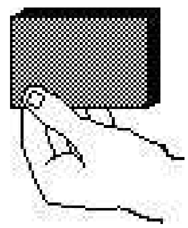
(c) The same wooden block with a mass of .30 kg is pushed along a table with a constant external force of 10 N which acts in a horizontal direction. It moves a distance of 0.40 m. However, there is a friction force opposing its motion. Assume that the coefficient of sliding friction, μ_k, is 0.20.



1. According to the definition of work done by a force, what is the work associated with the external force? Is the work positive or negative? Show your calculation.

2. According to our discussion above of the work done by a friction force, what is the work associated with the friction force? Is the work positive or negative? Show your calculation.

(d) Suppose you lift a 0.3 kg object through a distance of 1.0 m at a constant velocity.

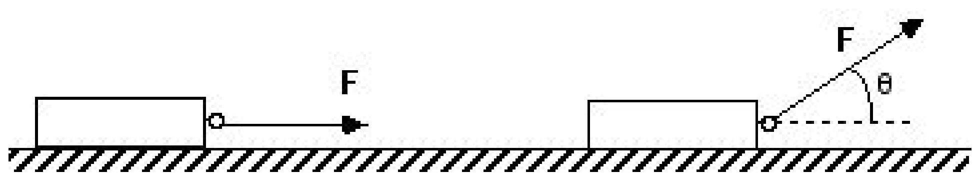


1. What is the work associated with the force that the earth exerts on the object? Is the work positive or negative? Show your calculation.

2. What is the work associated with the external force you apply to the object? Is the work positive or negative? Show your calculation.

Pulling at an Angle What Happens When the Force and the Displacement Are Not Along the Same Line?

Let's be more quantitative about measuring force and distance and calculating the work. How should work be calculated when the external force and the displacement of an object are not in the same direction?



To investigate this, you will use a spring scale to measure the force necessary to slide a block along the table at a constant speed. Before you make your simple force measurements, you should put some weights on your block so that it slides along a smooth surface at a constant velocity even when it is being pulled with a force that is 30 or 60 degrees from the horizontal.

Activity 4: Calculating Work

(a) Hold a spring scale horizontal to the table and use it to pull the block a distance of 0.5 meters along the horizontal surface in such a way that the block moves at a constant speed. Record the force in newtons and the distance in meters in the space below and calculate the work done on the block in joules. (Note that there is a special unit for work, the joule, or J for short. One joule is equal to one newton times one meter, i.e., J = N m.)

(b) Repeat the measurement, only this time pull on the block at a 30° angle with respect to the horizontal. Pull the block at about the same speed. Is the force needed larger or smaller than you measured in part (a)?

(c) Repeat the measurement once more, this time pulling the block at a 60° angle with respect to the horizontal. Pull the block at about the same speed as before.

(d) Assuming that the actual physical work done in part (b) is the same as the physical work done in part (a) above, how could you enhance the mathematical definition of work so that the forces measured in part (b) could be used to calculate work? In other words, use your data to postulate a mathematical equation that relates the physical work, W, to the magnitude of the applied force, F, the magnitude of the displacement, Δs, and the angle, θ, between F and Δs. Explain your reasoning. Hint: sin 30° = .500, sin 60° = .866, cos 30° = .866, cos 60° = .500.

Work as a Dot Product

Review the definition of dot (or scalar) product as a special product of two vectors in your textbook, and convince yourself that the dot product can be used to define physical work in general cases when the force is constant but not necessarily in the direction of the displacement resulting from it.

$$W = \mathbf{F} \cdot \mathbf{\Delta s}$$

Activity 5: How Much Work Goes with Each Job?

(a) Re-examine the descriptions of the jobs shown in the figure on the first page of this section. What is the minimum physical work done in job 1? Note that the data are given in British units, so the work will be expressed in foot pounds (ft lbs), not newton meters (joules).

(b) What is the minimum physical work done in job 2?

(c) What is the minimum physical work is done in job 3?

(d) Was your original intuition about which job to take correct? Which job should Richmond Load 'n' Go try to land?

The Concept of Power

People are interested in more than physical work. They are also interested in the rate at which physical work can be done. Average power, $\langle P \rangle$, is defined as the ratio of the amount of work done, ΔW, to the time interval, Δt, it takes to do the work, so that

$$\langle P \rangle = \frac{\Delta W}{\Delta t}.$$

Instantaneous power is given by the derivative of work with respect to time, or

$$P = \frac{dW}{dt}.$$

If work is measured in joules and time in seconds then the fundamental unit of power is in joules/second where 1 joule/second equals one watt. However, a more traditional unit of power is the horsepower, which represents the rate at which a typical work horse can do physical work. It turns out that *1 horsepower (or hp) = 746 watts = 746 joules/second.*

Those of you who are car buffs know that horsepower is a big deal in rating high performance cars. The hp in a souped-up car is in the hundreds. How does your stair climbing ability stack up? Let's see how long it takes you to climb the two stories of stairs in the science center.

Activity 6: Rate the Horsepower in Your Legs

(a) Determine the time it takes you to climb the two flights of stairs in the science center. Then measure the height of the climb and compute the work done against the force of gravity.

(b) Compute the average power, $\langle P \rangle$, you expended in hp. How does this compare to the horsepower of your favorite automobile? If you're not into cars, how do you stack up against a horse?

34 Work and Kinetic Energy[25]

Name ______________________ Section ____________ Date ____________

Objectives

- To discover Hooke's law.
- To understand the concept of kinetic energy and its relationship to work as embodied in the work-energy theorem.
- To develop an understanding of the physical significance of mathematical integration.

The Force Exerted on a Mass by an Extended Spring

So far we have pushed and pulled on an object with a constant force and calculated the work needed to displace that object. In most real situations the force on an object can change as it moves.

What happens to the average force needed to stretch a spring from 0 to 1 cm compared to the average force needed to extend the same spring from 10 to 11 cm? How does the applied force on a spring affect the amount by which it stretches, i.e., its displacement?

Apparatus

- A large spring
- A support rod to hang the spring
- A 2-meter stick
- A variety of masses

Activity 1: Are Spring Forces Constant?

Hang the spring from a support rod with the large diameter coils in the downward position. Extend the spring from 0 to 1 cm. Feel the force needed to extend the spring. Extend the spring from 10 to 11 cm. Feel the force needed to extend the spring again. How do the two forces compare? Are they the same?

The Force and Work Needed to Stretch a Spring

Now we would like to be able to quantify the force and work needed to extend a spring as a function of its displacement from an equilibrium position (i.e., when it is "unstretched").

Activity 2: Force vs. Displacement for a Spring

(a) Measure the distance from the floor to the lower end of the spring and record this distance as s_0 below. Create a data table in the space below with the column headings m (kg), s (m), F_{ext} (N), x (m), Δx (m), $\langle F_{ext} \rangle$ (N), ΔW (J), and W_{total} (J). (The last four columns will not be filled in until you get to Activities 3 and 4.) Hang different masses in 0.100 kg increments up to 1.000 kg and measure the distance from the floor to the lower end of the spring, s, for each mass. Record the values of m and s in the data table. Calculate and record the external force, F_{ext}, and the stretch of the spring, x $(= s_0 - s)$, for each mass.

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(b) Using *Excel*, create a graph of F_{ext} (vertical axis) vs. x. Is the graph linear? If the force, F_{ext}, increases with the displacement in a proportional way, fit the data to find the slope of the line. Insert a copy of the graph into your notebook. Use the symbol k to represent the slope of the line. What is the value of k? What are its units? Note: k is known as the spring constant.

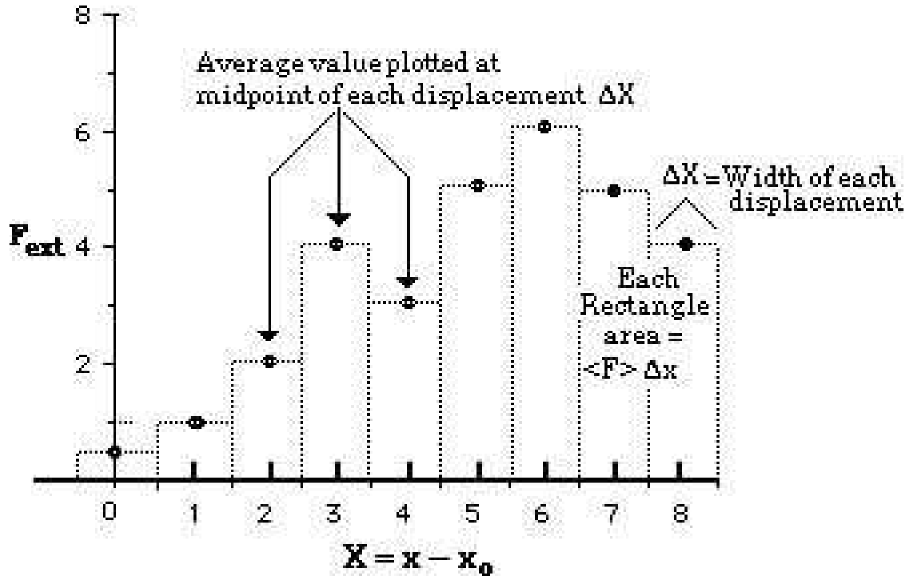
(c) Write the equation describing the relationship between the external force, F_{ext}, and the total displacement, x, of the spring from its equilibrium using the symbols F_{ext}, k, and x.

Note: Any restoring force on an object which is proportional to its displacement is known as a Hooke's Law Force. There was an erratic, contentious genius named Robert Hooke who was born in 1635. He played with springs and argued with Newton.

Calculating Work when the Force is not Constant

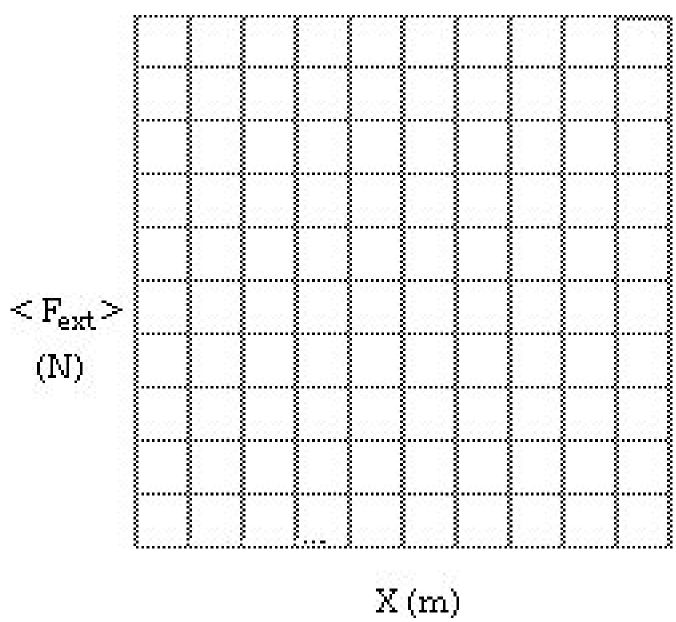
We would like to expand the definition of work so it can be used to calculate the work associated with stretching a spring and the work associated with other forces that are not constant. A helpful approach is to plot the average force needed to move an object for each successive displacement Δx as a bar graph like that shown in the figure below. The figure shows a graph representing the average applied force causing each unit of displacement of an object. This graph represents force that is not constant but not the force vs. displacement of a typical spring.

Note: The bar graph below is intended to illustrate mathematical concepts. Any similarity between the values of the forces in the bar graph and any real set of forces is purely coincidental. In general, the force causing work to be done on an object is not constant.



Activity 3: Force vs. Distance in a Bar Graph

(a) Using your data from Activity 2, calculate the width of each displacement, Δx, and the average external force, $\langle F_{ext} \rangle$, for each displacement, and record the values in the table above. Plot $\langle F_{ext} \rangle$ vs. x as a bar graph on the grid below. (Choose appropriate scales for the axes before making the graph.)



How can we calculate the work done in stretching the spring? We can use several equivalent techniques: (1) adding up little pieces of $\langle F_{ext} \rangle \Delta x$ from the above bar graph, (2) finding the area under the "curve" you created in Activity 2, or (3) using mathematical integration.

All three methods should yield about the same result. If you have not yet encountered integrals in a calculus course, you can compare the results of using the first two methods. If you have studied integrals in calculus you may want to consult your instructor or the textbook about how to set up the appropriate definite integral to calculate the work needed to stretch the spring.

Activity 4: Calculation of Work

(a) Calculate the work needed to stretch the spring by adding up small increments of $\langle F_{ext} \rangle \Delta x$ in your table. Also record the running sum in the table and summarize the result below. Don't forget to specify units.

$W_{total} =$

(b) Calculate the work needed to stretch the spring by computing the area under the curve in the graph of F_{ext} vs. x that you created in Activity 2.

(c) How does adding up the little rectangles in part (a) compare to finding the area under the curve in part (b)?

Note that in the limit where the x values are very small the sum of $\langle F_{ext} \rangle \Delta x$, known by mathematicians as the Riemann sum, converges to the mathematical integral and to the area under the curve.

Defining Kinetic Energy and Its Relationship to Work

What happens when you apply an external force to an object that is free to move and has no friction forces on it? Obviously it should experience an acceleration and end up being in a different state of motion. Can we relate the change in motion of the object to the amount of work that is done on it?

Let's consider a fairly simple situation. Suppose an object is lifted through a distance s near the surface of the earth and then allowed to fall. During the time it is falling it will experience a constant force as a result of the attraction between the object and the earth glibly called the force of gravity. You can use the theory we have already developed for the gravitational force to compare the velocity of the object to the work done on it by the gravitational field as it falls through a distance y. This should lead naturally to the definition of a new quantity called kinetic energy, which is a measure of the amount of "motion" gained as a result of the work done on the mass.

Activity 5: Equations for Falling v vs. y

(a) An object of mass m is dropped near the surface of the earth. What are the magnitude and direction of its acceleration g?

(b) If the object has no initial velocity and is allowed to fall for a time t under the influence of the gravitational force, what kinematic equation describes the relationship between the distance the object falls, y, and its time of fall, t? Assume $y_0 = 0$.

(c) Do you expect the magnitude of the velocity to increase, decrease or remain the same as the distance increases? Note: This is an obvious question!!

(d) Differentiate the equation you wrote down in part (b) to find a relationship between v, the acceleration g, and time t.

(e) Eliminate t from the equations you obtained in parts (b) and (d) to get an expression that describes how the velocity, v, of the falling object depends on the distance, y, through which it has fallen.

You can use the kinematic equations to derive the functional relationship you hopefully discovered experimentally in the last activity. If we define the kinetic energy (K) of a moving object as the quantity $K = \frac{1}{2}mv^2$, then we can relate the change in kinetic energy as an object falls to the work done on it. Note that for an object initially at rest the initial kinetic energy is $K_i = 0$, so the change in kinetic energy is given by the difference between the initial and final kinetic energies. $\Delta K = K_f - K_i = \frac{1}{2}mv^2$.

Activity 6: Computing Work and Kinetic Energy of a Falling Mass

(a) Suppose the mass of your falling object is 0.35 kg. What is the value of the work done by the gravitational force when the mass is dropped through a distance of $y = 1.2$ m?

(b) Use the kinematic equation you derived in Activity 5(e) that relates v and y to find the velocity of the falling object after it has fallen 1.2 m.

(c) What is the kinetic energy of the object before it is dropped? After it has fallen 1.2 m? What is the change in kinetic energy, ΔK, as a result of the fall?

(d) How does the work done by the gravitational force compare to the kinetic energy change, ΔK, of the object?

Activity 7: The Mathematical Relationship between Work and Kinetic Energy Change During a Fall

(a) Since our simplified case involves a constant acceleration, write down the equation you derived in Activity 5(e) to describe the speed, v, of a falling object as a function of the distance y which it fell.

(b) Using the definition of work, show that $W = mgy$ when the object is dropped through a distance y.

(c) By combining the equations in parts (a) and (b) above, show that in theory the work done on a mass falling under the influence of the gravitational attraction exerted on it by the earth is given by the equation $W = \Delta K$.

You have just proven an example of the work-energy theorem which states that the change in kinetic energy of an object is equal to the net work done by all the forces acting on it.

$$W = \Delta K \qquad \text{[Work-Energy Theorem]}$$

Although you have only shown the work-energy theorem for a special case where no friction is present, it can be applied to any situation in which the net force can be calculated. For example, the net force on an object might be calculated as a combination of applied, spring, gravitational, and friction forces.

35 Conservation of Mechanical Energy[26]

Name ______________________ Section ____________ Date ____________

Objectives

- To understand the concept of potential energy.
- To investigate the conditions under which mechanical energy is conserved.

Overview

The last unit on work and energy culminated with a mathematical proof of the work-energy theorem for a mass falling under the influence of the force of gravity. We found that when a mass starts from rest and falls a distance y, its final velocity can be related to y by the familiar kinematic equation

$$v_f^2 = v_i^2 + 2gy \qquad \text{or} \qquad gy = \frac{1}{2}\left(v_f^2 - v_i^2\right) \qquad [Eq.\,1]$$

where v_f is the final velocity and v_i is the initial velocity of the mass.

We believe this equation is valid because: (1) you have derived the kinematic equations mathematically using the definitions of velocity and constant acceleration, and (2) you have verified experimentally that masses fall at a constant acceleration. We then asked whether the transformation of the mass from a speed v_i to a speed v_f is related to the work done on the mass by the force of gravity as it falls.

The answer is mathematically simple. Since $F_g = mg$, the work done on the falling object by the force of gravity is given by

$$W_g = F_g y = mgy \qquad [Eq.\,2]$$

But according to Equation 1, $gy = \frac{1}{2}v_f^2 - \frac{1}{2}v_i^2$, so we can re-write Equation 2 as

$$W_g = mgy = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad [Eq.\,3]$$

The $\frac{1}{2}mv_f^2$ is a measure of the motion resulting from the fall. If we define it as the energy of motion, or, more succinctly, the kinetic energy, we can define a work-energy theorem for falling objects:

$$W = \Delta K \qquad [Eq.\,4]$$

or, the work done on a falling object by the earth is equal to the change in its kinetic energy as calculated by the difference between the final and initial kinetic energies.

If external work is done on the mass to raise it through a height y (a fancy phrase meaning "if some one picks up the mass"), it now has the potential to fall back through the distance y, gaining kinetic energy as it falls. Aha! Suppose we define *potential energy* to be *the amount of external work, W_{ext}, needed to move a mass at constant velocity through a distance y against the force of gravity.* Since this amount of work is positive while the work done by the gravitational force has the same magnitude but is negative, this definition can be expressed mathematically as

$$U = W_{ext} = mgy \qquad [Eq.\,5]$$

Note that when the potential energy is a maximum, the falling mass has no kinetic energy but it has a maximum potential energy. As it falls, the potential energy becomes smaller and smaller as the kinetic energy increases. The kinetic and potential energy are considered to be two different forms of mechanical energy. What about the total mechanical energy, consisting of the sum of these two energies? Is the total mechanical energy constant

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during the time the object falls? If it is, we might be able to hypothesize a law of conservation of mechanical energy as follows: *In some systems, the sum, E, of the kinetic and potential energy is a constant.* This hypothesis can be summarized mathematically by the following statement.

$$E = K + U = \text{constant} \qquad [Eq.\,6]$$

The idea of mechanical energy conservation raises a number of questions. Does it hold quantitatively for falling masses? How about for masses experiencing other forces, like those exerted by a spring? Can we develop an equivalent definition of potential energy for the mass-spring system and other systems and re-introduce the hypothesis of conservation of mechanical energy for those systems? Is mechanical energy conserved for masses experiencing frictional forces, like those encountered in sliding?

In this unit, you will explore whether or not the mechanical energy conservation hypothesis is valid for a falling mass.

Activity 1: Mechanical Energy for a Falling Mass

Suppose a ball of mass m is dropped from a height h above the ground.

(a) Where is U a maximum? A minimum?

(b) Where is K a maximum? A minimum?

(c) If mechanical energy is conserved what should the sum of $K + U$ be for any point along the path of a falling mass?

Mechanical Energy Conservation

How do people in different reference frames near the surface of the earth view the same event with regard to mechanical energy associated with a mass and its conservation? Suppose the president of your college drops a 2.0-kg water balloon from the second floor of the administration building (10.0 meters above the ground). The president takes the origin of his or her vertical axis to be even with the level of the second floor. A student standing on the ground below considers the origin of his coordinate system to be at ground level. Have a discussion with your classmates and try your hand at answering the questions below.

Activity 2: Mechanical Energy and Coordinate Systems

(a) What is the value of the potential energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations and don't forget to include units!

The president's perspective is that $y = 0.0$ m at $t = 0$ s and that $y = -10.0$ m when the balloon hits the student):

$U_i =$

$U_f =$

The student's perspective is that $y = 10.0$ m at $t = 0$ s and that $y = 0.0$ m when the balloon hits:

$U_i =$

$U_f =$

Note: If you get the same potential energy value for the student and the president, you are on the wrong track!

(b) What is the value of the kinetic energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations. Hint: Use a kinematic equation to find the velocity of the balloon at ground level.

President's perspective:

$K_i =$

$K_f =$

Student's perspective:

$K_i =$

$K_f =$

Note: If you get the same values for both the student and the president for values of the kinetic energies you are on the right track!

(c) What is the value of the total mechanical energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations. Note: If you get the same values for both the student and the president for the total energies you are on the wrong track!!!!!

President's perspective:

$E_i =$

$E_f =$

Student's perspective:

$E_i =$

$E_f =$

(d) Why don't the two observers calculate the same values for the mechanical energy of the water balloon?

(e) Why do the two observers agree that mechanical energy is conserved?

36 Conservative and Non-Conservative Forces[27]

Name ____________________ Section ____________ Date ____________

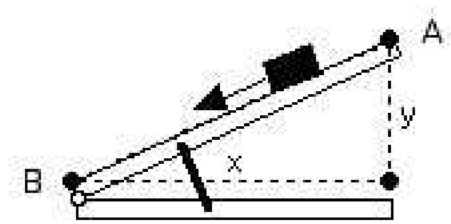
Objectives

- To investigate the conditions under which mechanical energy is conserved.
- To relate conservative and non-conservative forces to the net work done by a force when an object moves in a closed loop.

Is Mechanical Energy Conserved for a Sliding Object?

In the last two sessions you should have found that mechanical energy is conserved for a freely falling object. Let's investigate whether mechanical energy is conserved when an object slides down an inclined plane in the presence of a friction force.

In this activity you will raise the incline just enough to allow the block to slide at a constant velocity (see the figure below). By measuring the velocity you can determine if mechanical energy is conserved.



Apparatus

- Wooden block with hook
- Board to use as an incline S
- Spring scale
- Meter stick
- Variety of masses
- Triple-beam balance
- Stop watch

Activity 1: Is Mechanical Energy Conserved for a Sliding Block?

(a) Raise the incline until the block slides at a constant velocity once it is pushed gently to overcome the static friction force. What is the potential energy of the block at point A relative to the bottom of the ramp? Show your data and calculation.

(b) What is the velocity of the block as it travels down the incline? Show your data and calculation.

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(c) What is the kinetic energy of the block at the bottom of the incline? Does it change as the block slides? Show your calculation.

(d) What is the potential energy change, ΔU, of the block when it reaches point B? Explain.

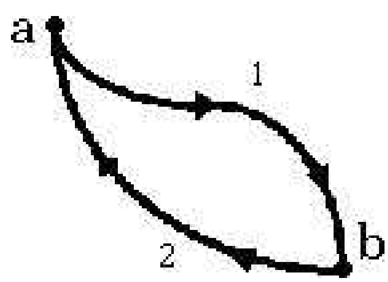
(e) Assuming the initial kinetic energy (just after your starter shove) is the same as the final kinetic energy, what is the kinetic energy change, ΔK, of the block when it reaches point B?

(f) Is mechanical energy conserved as a result of the sliding? Cite the evidence for your answer.

As you examine the activities you just completed you should conclude that the conservation of mechanical energy will probably only hold in situations where there are no friction forces present.

Conservative and Non-Conservative Forces

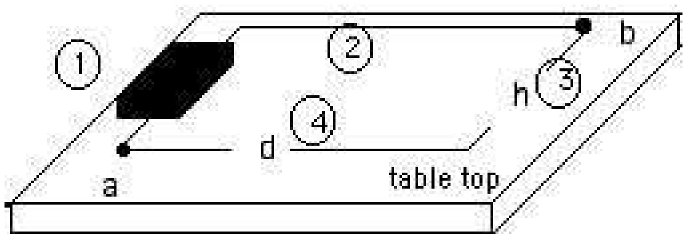
Physicists have discovered that certain conservative forces such as gravitational forces and spring forces do no total work on an object when it moves in a closed loop. Other forces involving friction are not conservative and hence the total work these forces do on an object moving in a closed loop is not zero. In this next activity you will try to determine the validity of this assertion.



It is not hard to see that a gravitational force does no net work when an object moves at a constant speed through a complete round trip. In the above figure, it takes negative external work to lower a mass from point a to point b, as the force of gravity takes care of the work. On the other hand, raising the mass from point b to point a requires positive external work to be done against the force of gravity, and thus the net work done by the gravitational force for the complete trip is zero. When a frictional force is present it always does net work on an object as it undergoes a round trip. For example, when a block slides from point a to point b on a horizontal surface, it takes work to overcome the frictional force that opposes the motion. When the block slides back from point b to point a, the frictional force still opposes the motion of the block so that net work is done. Let's make this idea more concrete by sliding a block around a horizontal loop on your lab table in the presence of a frictional force and computing the work it does. Then you can raise and lower the block around a similar vertical loop and calculate the work the gravitational force does.

Activity 2: Are Frictional Forces Conservative According to the Loop Rule?

(a) Put masses on the wooden block and use the spring scale to pull the block around a rectangular path on your table top at constant velocity. Draw arrows along the path for the direction of motion and the direction of the force you exert on the block. List the measured forces and distances for each of the four segments of the path to calculate the work you do around the entire path a to b to a as shown.

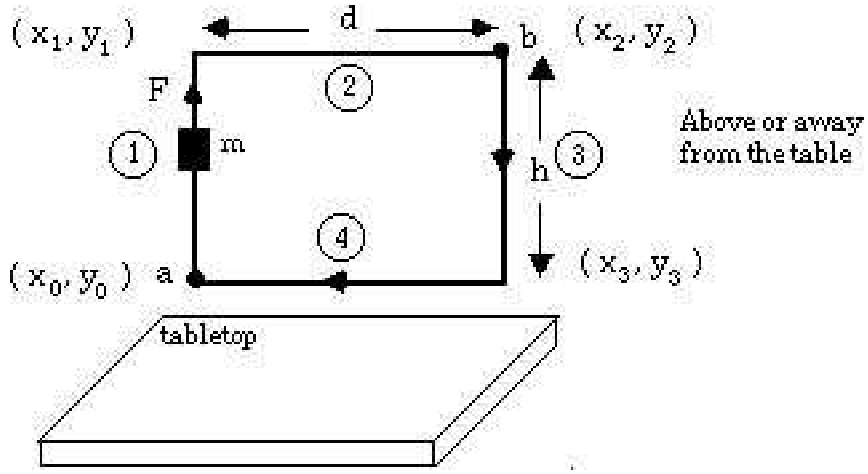


(b) What is the change in K as you progress around the loop?

(c) Is the frictional force conservative or non-conservative (i.e., is the total work done by the friction force zero or non-zero)? Explain.

(d) Is mechanical energy conserved? Explain.

Let's return once again to our old friend the gravitational force and apply an external force to move the same block (without the masses) in a vertical loop above the table. Be very, very careful to pay attention to the direction of the gravitational force relative to the direction of the motion. Remember that work is the dot product of this force and the displacement in each case so that the work done by the gravitational force when the block moves up and when it moves down are not the same. What happens to the work when the gravitational force is perpendicular to the direction of motion, as is the case in moving from left to right and then later from right to left?



Activity 3: Are Gravitational Forces Conservative?

(a) Use the spring scale and raise and lower the wooden block around a rectangular path above your tabletop at a constant speed without allowing it to slide at all. Draw arrows along the path for the direction of motion and the direction of the gravitational force exerted on the block. Use your measured distances, the measurement of the mass of the block and the dot product notation to calculate the work done by the gravitational force on the block over the entire path a to b to a as shown. Be careful not to let the block slide on the table or rub against it. Don't forget to specify the units! Hints: (1) In path 1 $\Delta x = x_1 - x_0$, $\Delta y = y_1 - y_0$, etc. (2) Keep track of the signs. For which paths is the work negative? Positive?

(b) Is the gravitational force conservative or non-conservative according to the loop rule? Explain.

The "Missing" Energy

We have seen in the case of the sliding block that the Law of Conservation of Mechanical Energy does not seem to hold for forces involving friction. The question is, where does the "missing" energy ΔE go when frictional forces are present? All the energy in the system might not be potential energy or kinetic energy. If we are clever and keep adding new kinds of energy to our collection, we might be able to salvage a Law of Conservation of Energy. If we can, this law has the potential to be much more general and powerful than the Law of Conservation of Mechanical Energy.

Activity 4: What Happens to the Missing Energy?

(a) Rub the sliding block back and forth vigorously against your hand? What sensation do you feel?

(b) How might this sensation account for the missing energy?

Physicists call the energy which is lost by a system as a result of work done against frictional forces thermal energy. This thermal energy may lead to an increase in the system's internal energy. Using the symbol ΔE_{int} to represent the change in internal energy of a system that experiences frictional forces allows us to express the Law of Conservation of Total Energy mathematically with the expression

$$E = U + K + \Delta E_{int} = \text{constant}$$

We are not prepared in this part of the course to consider the nature of internal energy or its actual measurement, so the Law of Conservation of Energy will for now remain an untested hypothesis. However, we will re-consider the concept of internal energy later in the course when we deal with heat and temperature.

37 Momentum and Momentum Change[28]

Name ______________________________ Section _______________ Date _______________

Objectives

- To understand the definition of momentum and its vector nature as it applies to one- dimensional collisions.
- To reformulate Newton's second law in terms of change in momentum, using the fact that Newton's "motion" is what we refer to as momentum.
- To develop the concept of impulse to explain how forces act over time when an object undergoes a collision.
- To use Newton's second law to develop a mathematical equation relating impulse and momentum change for any object experiencing a force.

Overview

In the next few units we will explore the forces of interaction between two or more objects and study the changes in motion that result from these interactions. We are especially interested in studying collisions and explosions in which interactions take place in fractions of a second or less. Early investigators spent a considerable amount of time trying to observe collisions and explosions, but they encountered difficulties. This is not surprising, since the observation of the details of such phenomena requires the use of instrumentation that was not yet invented (such as the high speed camera). However, the principles of the outcomes of collisions were well understood by the late seventeenth century, when several leading European scientists (including Sir Isaac Newton) developed the concept of "quantity of motion" to describe both elastic collisions (in which objects bounce off each other) and inelastic collisions (in which objects stick together). These days we use the word momentum rather than motion in describing collisions and explosions.

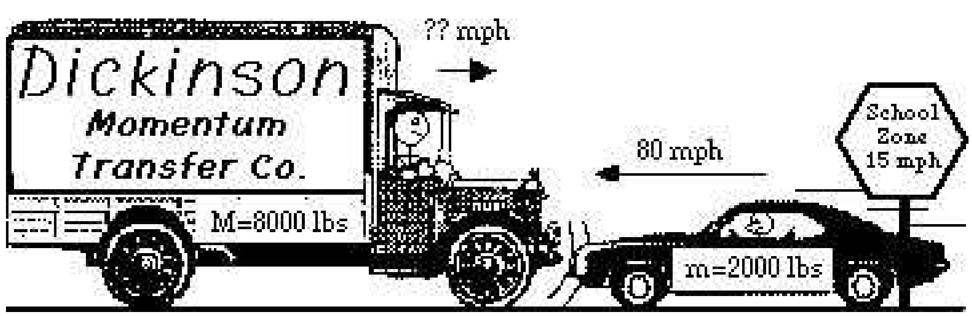
We will begin our study of collisions by exploring the relationship between the forces experienced by an object and its momentum change. It can be shown mathematically from Newton's laws and experimentally from our own observations that the integral of force experienced by an object over time is equal to its change in momentum. This time-integral of force is defined as a special quantity called impulse, and the statement of equality between impulse and momentum change is known as the impulse-momentum theorem.

Apparatus

- Dynamics carts (2) and track

Defining Momentum

In this session we are going to develop the concept of momentum to predict the outcome of collisions. But you don't officially know what momentum is because we haven't defined it yet. Lets start by predicting what will happen as a result of a simple one-dimensional collision. This should help you figure out how to define momentum to enable you to describe collisions in mathematical terms.



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It's early fall and you are driving along a two lane highway in a rented moving van. It is full of all of your possessions so you and the loaded truck were weighed in at 8000 lbs. You have just slowed down to 15 MPH because you're in a school zone. It's a good thing you thought to do that because a group of first graders is just starting to cross the road. Just as you pass the children you see a 2000 lb sports car in the oncoming lane heading straight for the children at about 80 MPH. What a fool the driver is! A desperate thought crosses your mind. You figure that you just have time to swing into the oncoming lane and speed up a bit before making a head-on collision with the sports car. You want your truck and the sports car to crumple into a heap that sticks together and doesn't move. Can you save the children or is this just a suicidal act? For simulated observations of this situation you can use two carts of different masses set up to stick together in trial collisions.

Activity 1: Can You Stop the Car?

(a) Predict how fast you would have to be going to completely stop the sports car. Explain the reasons for your prediction.

(b) Try some head on collisions with the carts of different masses to simulate the event. Describe some of your observations. What happens when the less massive cart is moving much faster than the more massive cart? Much slower? At about the same speed?

(c) Based on your intuitive answers in parts (a) and (b) and your observations, what mathematical definition might you use to describe the momentum (or motion) you would need to stop an oncoming vehicle traveling with a known mass and velocity?

Just to double check your reasoning, you should have come to the conclusion that momentum is defined by the vector equation

$$\mathbf{p} = m\mathbf{v}.$$

Expressing Newton's Second Law Using Momentum

Originally Newton did not use the concept of acceleration or velocity in his laws. Instead he used the term "motion," which he defined as the product of mass and velocity (a quantity we now call momentum). Let's examine a translation from Latin of Newton's first two laws (with some parenthetical changes for clarity).

Newton's First Two Laws of Motion

1. *Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed on it.*

2. *The (rate of) change of motion is proportional to the motive force impressed: and is made in the direction in which that force is impressed.*

The more familiar contemporary statement of the second law is that the net force on an object is the product of its mass and its acceleration where the direction of the force and of the resulting acceleration are the same. Newton's statement of the law and the more modern statement are mathematically equivalent, as you will show.

Activity 2: Re-expressing Newton's Second Law

(a) Write down the contemporary mathematical expression for Newton's second law relating net force to mass and acceleration. Please use vector signs and a summation sign where appropriate.

(b) Write down the definition of instantaneous acceleration in terms of the rate of change of velocity. Again, use vector signs.

(c) It can be shown that if an object has a changing velocity and a constant mass then $m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt}$. Explain why.

(d) Show that $\sum \mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt}$.

(e) Explain in detail why Newton's statement of the second law and the mathematical expression $\sum \mathbf{F} = \frac{d\mathbf{p}}{dt}$ are two representations of the same statement, i.e., are logically equivalent.

Momentum Change and Collision Forces

What's Your Intuition?

You are sleeping in your sister's room while she is away at college. Your house is on fire and smoke is pouring into the partially open bedroom door. The room is so messy that you cannot get to the door. The only way to close the door is to throw either a blob of clay or a super ball at the door — there's not enough time to throw both.

Activity 3: What Packs the Biggest Wallop-A Clay Blob or a Super ball?

Assuming that the clay blob and the super ball have the same mass, which would you throw to close the door: the clay blob (which will stick to the door) or the super ball (which will bounce back with almost the same velocity it had before it collided with the door)? Give reasons for your choice, using any notions you already have or any new concepts developed in physics such as force, momentum, Newton's laws, etc. Remember, your life depends on it!

Momentum Changes

It would be nice to be able to use Newton's formulation of the second law of motion to find collision forces, but it is difficult to measure the rate of change of momentum during a rapid collision without special instruments.

However, measuring the momenta of objects just before and just after a collision is usually not too difficult. This led scientists in the seventeenth and eighteenth centuries to concentrate on the overall changes in momentum that resulted from collisions. They then tried to relate changes in momentum to the forces experienced by an object during a collision. In the next activity you are going to explore the mathematics of calculating momentum changes.

Activity 4: Predicting Momentum Changes

Which object undergoes the most momentum change during the collision with a door: the clay blob or the super ball? Explain your reasoning carefully.

Let's check your reasoning with some formal calculations of the momentum changes for both inelastic and elastic collisions. This is a good review of the properties of one-dimensional vectors. Recall that momentum is defined as a vector quantity that has both magnitude and direction. Mathematically, momentum change is given by the equation

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

where $\mathbf{p}_i$ is the initial momentum of the object just before and $\mathbf{p}_f$ is its final momentum just after a collision.

Activity 5: Calculating 1D Momentum Changes

(a) Suppose a dead ball (or clay blob) is dropped on a table and "sticks" in such a way that it has an initial momentum just before it hits of $\mathbf{p}_i = -p_{iy}\widehat{\mathbf{j}}$ where $\widehat{\mathbf{j}}$ is a unit vector pointing along the positive y axis. Express the final momentum of the dead ball in the same vector notation.

(b) What is the change in momentum of the clay blob as a result of its collision with the table? Use the same type of unit vector notation to express your answer.

(c) Suppose that a live ball (or a super ball) is dropped on a table and "bounces" on the table in an elastic collision so that its speed just before and just after the bounce are the same. Also suppose that just before it bounces it has an initial momentum $\mathbf{p}_i = -p_{iy}\widehat{\mathbf{j}}$, where $\widehat{\mathbf{j}}$ is a unit vector pointing along the positive y-axis. What is the final momentum of the ball in the same vector notation? Hint: Does the final $\mathbf{p}$ vector point along the +y or $-y$ axis?

(d) What is the change in momentum of the ball as a result of the collision? Use the same type of unit vector notation to express your result.

(e) The answer is not zero. Why? How does this result compare with your prediction? Discuss this situation.

(f) Suppose the mass of each ball is 0.2 kg and that they are dropped from 1 m above the table. Using this value for mass of the balls and a calculated value for the velocity of each of the balls just before they hit the table, you can calculate the momentum just before the collision $\mathbf{p}_i$ for each of the balls. Also calculate the momentum of the balls just after the collision $\mathbf{p}_f$ and the change in momentum $\Delta\mathbf{p}$ for each ball. Show your calculations in the space below.

Applying Newton's Second Law to the Collision Process (The Egg Toss)

Suppose somebody tosses you a raw egg and you catch it. In physics jargon, one would say (in a very official tone of voice) that "the egg and the hand have undergone an inelastic collision." What is the relationship between the force you have to exert on the egg to stop it, the time it takes you to stop it, and the momentum change that the egg experiences? You ought to have some intuition about this matter. In more ordinary language, would you catch an egg slowly or fast?

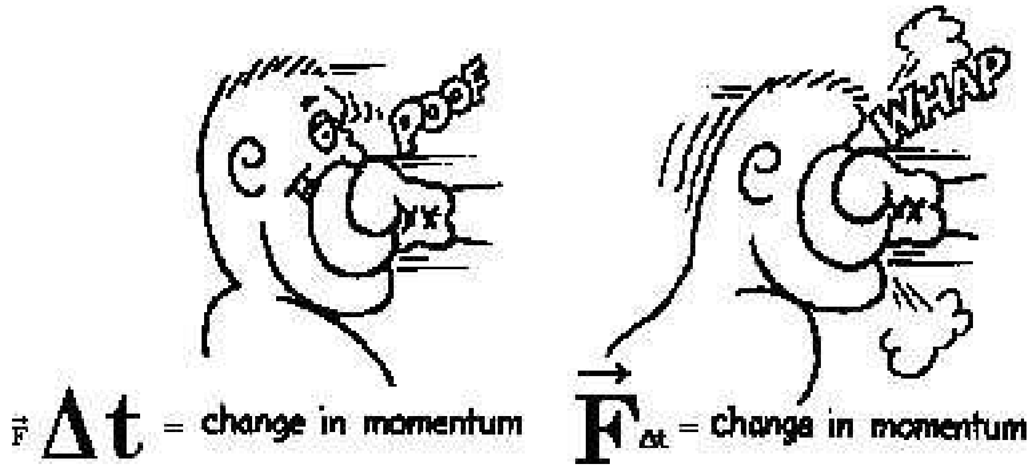
Activity 6: Momentum Changes and Average Forces on an Egg: What's Your Intuition?

(a) If you catch an egg of mass m that is heading toward your hand at speed v, what is the magnitude of the momentum change that it undergoes?

(b) Does the total momentum change differ if you catch the egg more slowly or is it the same?

(c) Suppose the time you take to bring the egg to a stop is Δt. Would you rather catch the egg in such a way that Δt is small or large? Why?

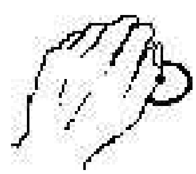
(d) What do you suspect might happen to the average force you exert on the egg while catching it when Δt is small?



You can use Newton's second law to derive a mathematical relationship between momentum change, force, and collision times for objects. This derivation leads to the impulse-momentum theorem. Let's apply Newton's second law to the egg catching scenario.

Activity 7: Force and Momentum Change

(a) Sketch an arrow representing the magnitude and direction of the force exerted by your hand on the egg as you catch it.



(b) Write the mathematical expression for Newton's second law in terms of the net force and the time rate of change of momentum. (See Activity 2(e) for details.)

(c) Explain why, if $\mathbf{F}$ is a constant during the collision lasting a time Δt, then $\frac{d\mathbf{p}}{dt} = \frac{\Delta \mathbf{p}}{\Delta t}$.

(d) Show that for a constant force $\mathbf{F}$ the change in momentum is given by $\Delta \mathbf{p} = \mathbf{F}\,\Delta t$. Note that for a constant force, the term $\mathbf{F}\,\Delta t$ is known as the impulse given to one body by another.

38 Impulse, Momentum, and Interactions[29]

Name ______________________________ Section _______________ Date _______________

Objectives

- To verify the relationship between impulse and momentum experimentally.
- To study the forces between objects that undergo collisions and other types of interactions in a short time period.

Apparatus

- Dynamics cart with flag and track
- Force transducer
- Photogate
- *Science Workshop 750 Interface*
- *DataStudio* software (Impulse-Momentum application)

The Impulse-Momentum Theorem

Real collisions, like those between eggs and hands, a Nerfball and a wall, or a falling ball and a table top are tricky to study because Δt is so small and the collision forces are not really constant over the time the colliding objects are in contact. Thus, we cannot calculate the impulse as $F\,\Delta t$. Before we study more realistic collision processes, let's redo the theory for a variable force. In a collision, according to Newton's second law, the force exerted on a falling ball by the table top at any infinitesimally small instant in time is given by

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad [Eq.\,1]$$

To describe a general collision that takes place between an initial time t_i and a final time t_f , we must take the integral of both sides of the equation with respect to time. This gives

$$\int_{t_i}^{t_f} \mathbf{F} dt = \int_{t_i}^{t_f} \frac{d\mathbf{p}}{dt} dt = (\mathbf{p}_f - \mathbf{p}_i) = \Delta\mathbf{p} \qquad [Eq.\,2]$$

Impulse is a vector quantity defined by the equation

$$\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F}\,dt \qquad [Eq.\,3]$$

By combining equations [2] and [3] we can formulate the impulse-momentum theorem in which

$$\mathbf{I} = \Delta\mathbf{p} \qquad [Eq.\,4]$$

If you are not used to mathematical integrals and how to solve them yet, don't panic. If you have a fairly smooth graph of how the force F varies as a function of time, the impulse integral can be calculated as the area under the F-t curve.

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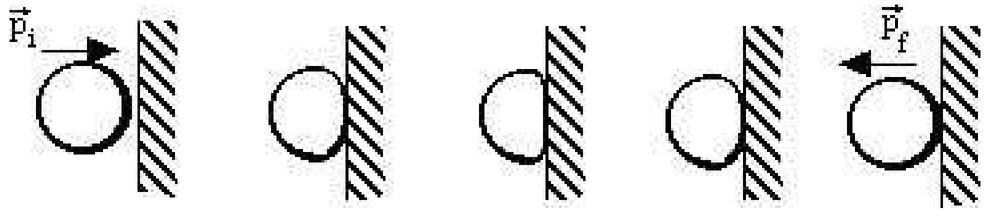
Let's see qualitatively what an impulse curve might look like in a real collision in which the forces change over time during the collision. In particular, let's consider the collision of a Nerfball with a wall as shown below.



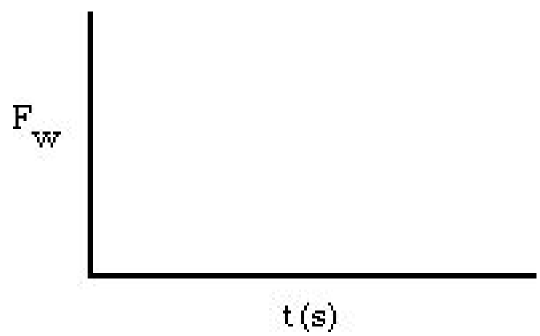
Activity 1: Predicting Collision Forces That Change

(a) Suppose a Nerfball is barreling toward a wall and collides with it. If friction is neglected, what is the net force exerted on the object just before it starts to collide?

(b) When will the magnitude of the force on the ball be a maximum?

(c) Roughly how long does the collision process take? Half a second? Less? Several seconds?

(d) Attempt a rough sketch of the shape of the force the wall exerts on a moving object during a collision.

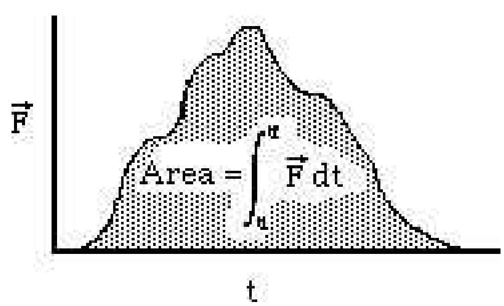


Verification of the Impulse-Momentum Theorem

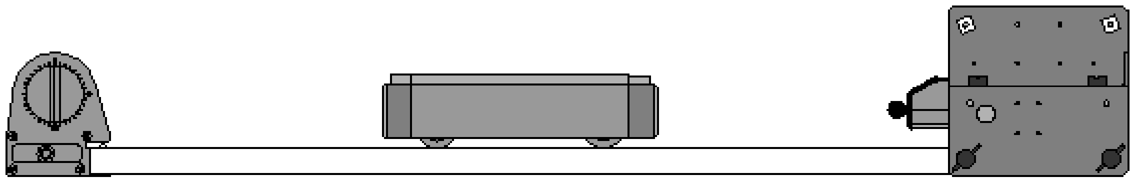
To verify the impulse-momentum theorem experimentally we must show that for an actual collision involving a single force on an object the equation

$$\int_{t_i}^{t_f} \mathbf{F}\, dt = \Delta \mathbf{p}$$

holds, where the impulse integral can be calculated by finding the area under the curve of a graph of F vs. t.



In this experiment you will investigate this theorem by measuring the impulse and the change in momentum of a cart undergoing a one-dimensional collision. The experimental setup is shown in the figure below. The end of the track with the motion detector should be raised about 1.5 cm so that, when released, the cart will collide with the force probe. The force probe will measure the force as a function of time during the collision. The motion detector is used to measure the velocity of the glider before and after the collision. You will use the Impulse-Momentum application to make these measurements.



Activity 2: Verification of the Impulse-Momentum Theorem

(a) Measure and record the mass of the cart, m.

(b) Construct a data table in the space below with the column headings Trial #, Area, v_i, and v_f. Make enough room to record 10 trials.

(c) Open the Impulse-Momentum application. Hold the cart about half-way up the track and press the TARE button on the force sensor. Start recording data and release the cart. Stop recording data after the cart collides with the force transducer and bounces back. The computer will then display graphs of velocity and force versus time.

(d) Determine the area under the force vs. time graph and record the value in your data table. See **Appendix B Introduction to DataStudio** for instructions on how to determine the area under a curve.

(e) Use the smart tool to find the velocity just before the collision and the velocity just after the collision from the velocity versus time graph. Record these values in your data table.

(f) Repeat parts (c) through (e) nine times. Be sure to press the TARE button on the force sensor before each run. Print the graphs for one of your trials.

(g) Construct another data table on the next page with the column headings Trial #, I, Δp, and % Diff. For each trial, calculate and record the impulse, I, and the change in momentum, Δp, in kg m/s. Also, determine the % difference between the two for each trial. Also, show a sample calculation of I, Δp, and % Diff for one of your trials.

(h) Do your results verify the impulse-momentum theorem within experimental uncertainty? Explain.

(i) Is there any indication of a systematic uncertainty? What are the possible sources of error?

39 Newton's Laws and Momentum Conservation[30]

Name ______________________________ Section ______________ Date ______________

Objectives

- To study the forces between objects that undergo collisions and other types of interactions in a short time period.
- To formulate the Law of Conservation of Momentum as a theoretical consequence of Newton's laws and to test it experimentally.

Apparatus

- A tennis ball.
- A video analysis system (*VideoPoint*).
- Graphing and curve fitting software (*Excel*).
- 2 force probes
- 2 dynamics carts
- A track, a pendulum bob, and a level.
- *Science Workshop 750 Interface*
- *DataStudio* software (Two Force Probes application)

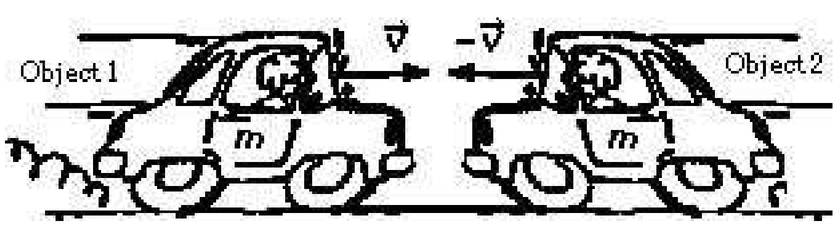
Predicting Interaction Forces between Objects

We recently focused our attention on the change in momentum that an object undergoes when it experiences a force that is extended over time (even if that time is very short!). Since interactions like collisions and explosions never involve just one object, we would like to turn our attention to the mutual forces of interaction between two or more objects. As usual, you will be asked to make some predictions about interaction forces and then be given the opportunity to test these predictions.

Activity 1: Predicting Interaction Forces

(a) Suppose the masses of two objects are the same and that the objects are moving toward each other at the same speed so that

$$m_1 = m_2 \quad \text{and} \quad \mathbf{v}_1 = -\mathbf{v}_2$$



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Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction!

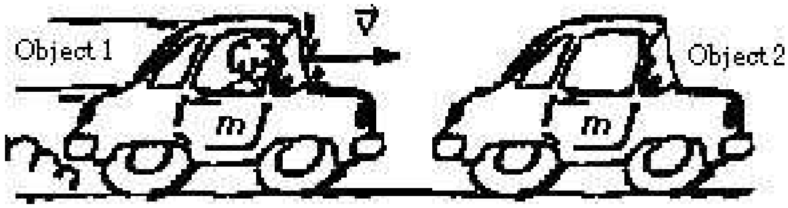
________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(b) Suppose the masses of two objects are the same and that object 1 is moving toward object 2, but object 2 is at rest.

$$m_1 = m_2 \quad \text{and} \quad \mathbf{v}_1 > \mathbf{v}_2$$



Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction!

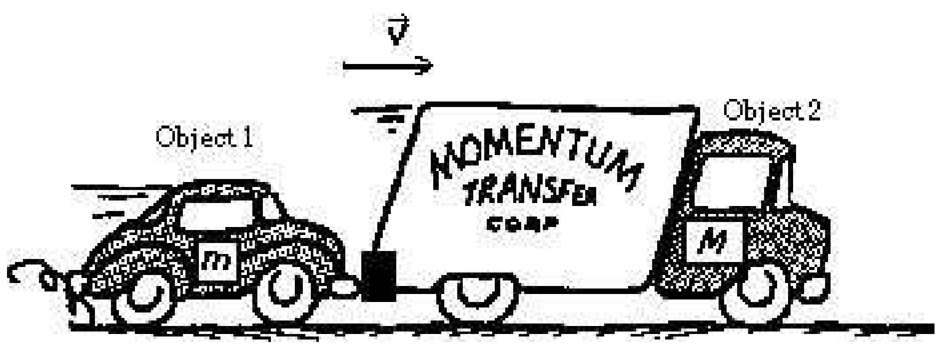
________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(c) Suppose the mass of object 1 is much less than that of object 2 and that it is pushing object 2 that has a dead motor so that both objects move in the same direction at speed v.

$$m_1 \ll m_2 \quad \text{and} \quad \mathbf{v}_1 = \mathbf{v}_2$$



Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction.

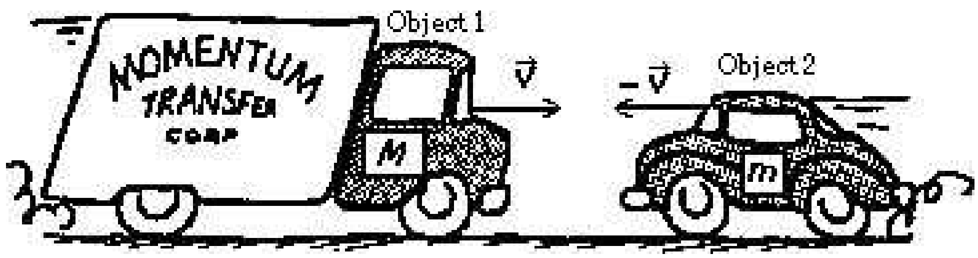
________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(d) Suppose the mass of object 1 is greater than that of object 2 and that the objects are moving toward each other at the same speed so that

$$m_1 > m_2 \quad \text{and} \quad \mathbf{v}_1 = -\mathbf{v}_2$$



Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction.

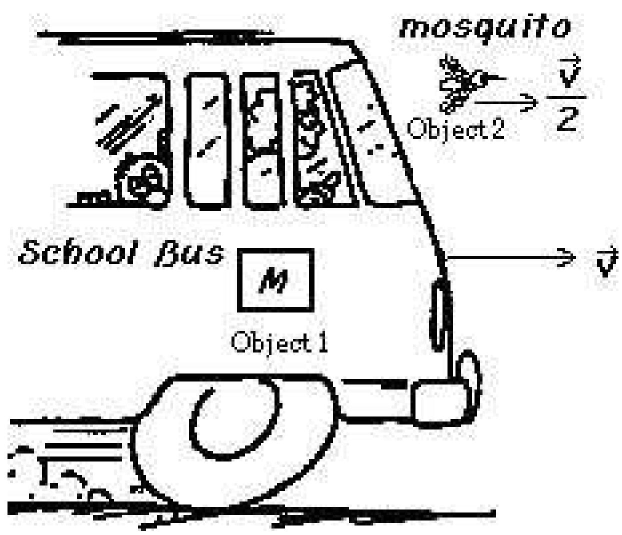
_______ Object 1 exerts more force on object 2.

_______ The objects exert the same force on each other.

_______ Object 2 exerts more force on object 1.

(e) Suppose the mass of object 1 is greater than that of object 2 and that object 2 is moving in the same direction as object 1 but not quite as fast so that

$$m_1 > m_2 \quad \text{and} \quad \mathbf{v}_1 > \mathbf{v}_2$$



Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction.

_______ Object 1 exerts more force on object 2.

_______ The objects exert the same force on each other.

_______ Object 2 exerts more force on object 1.

(f) Suppose the mass of object 1 is greater than that of object 2 and that both objects are at rest until an explosion occurs, so that

$$m_1 > m_2 \quad \text{and} \quad \mathbf{v}_1 = \mathbf{v}_2 = 0$$

Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction.

_______ Object 1 exerts more force on object 2.

_______ The objects exert the same force on each other.

_______ Object 2 exerts more force on object 1.

(g) Provide a summary of your predictions. What are the circumstances under which you predict that one object will exert more force on another object?

Measuring Mutual Forces of Interaction

In order to test the predictions you made in the last activity you can study gentle collisions between two force probes attached to carts. You can strap additional masses to one of the carts to increase its total mass so it has significantly more mass than the other. If a compression spring is available you can set up an "explosion" between the two carts by compressing the spring between the force probes on each cart and letting it go. You can make and display the force measurements with the Two Force Probes application. You can also determine the areas under the force vs. time graphs to find the impulses experienced by the carts during the collisions. See **Appendix B Introduction to DataStudio** for instructions on finding the area under a curve.

Activity 2: Measuring Slow Forces

(a) Play a gentle tug-of-war in which you push the ends of the two force probes back and forth for about 10 seconds with your partner using properly calibrated force probes. What do you observe about the mutual forces?

(b) Play a gentle tug-of-war in which you pull the ends of two force probes back and forth for about 10 seconds with your partner using properly calibrated force probes. What do you observe about the mutual forces?

Now that you're warmed up to this two force measurement technique go ahead and try some different types of gentle collisions between two carts of different masses and initial velocities.

Activity 3: Measuring Collision Forces

(a) Use the two carts to explore various situations that correspond to the predictions you made about mutual forces. Your goal is to find out under what circumstances one object exerts more force on another object. Describe what you did in the space below and attach a printout of at least one of your graphs of force 1 vs. time and force 2 vs. time.

(b) What can you conclude about forces of interactions during collisions? Under what circumstances does one object experience a different magnitude of force than another during a collision? How do the magnitudes and directions of the forces compare on a moment by moment basis in each case?

(c) Do your conclusions have anything to do with Newton's third law?

(d) How does the vector impulse due to object 1 acting on object 2 compare to the impulse of object 2 acting on object 1 in each case? Are they the same in magnitude or different? Do they have the same sign or a different sign? Remember $\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F}\, dt$.

Newton's Laws and Momentum Conservation

In your investigations of interaction forces, you should have found that the forces between two objects are equal in magnitude and opposite in sign on a moment by moment basis for all the interactions you studied. This is of course a testimonial to the seemingly universal applicability of Newton's third law to interactions between ordinary masses. You can combine the findings of the impulse-momentum theorem (which is really another form of Newton's second law since we derived it mathematically from the second law) to derive the Law of Conservation of Momentum shown below.

Law of Conservation of Momentum

$$\sum \mathbf{p} = \mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f} = \text{constant in time}$$

where 1 refers to object 1 and 2 refers to object 2 and i refers to the initial momenta and f to the final momenta.

Activity 4: Deriving Momentum Conservation

(a) What did you conclude in the last activity about the magnitude and sign of the impulse on object 1 due to object 2 and vice versa when two objects interact? In other words, how does $\mathbf{I}_1$ compare to $\mathbf{I}_2$?

(b) Since you have already verified experimentally that the impulse-momentum theorem holds, what can you conclude about how the change in momentum of object 1, $\Delta\mathbf{p}_1$, as a result of the interaction compares to the change in momentum of object 2, $\Delta\mathbf{p}_2$, as a result of the interaction? Remember $\mathbf{I} = \Delta\mathbf{p}$.

(c) Use the result of part (b) to show that the Law of Conservation of Momentum holds for a collision, i.e. $\sum \mathbf{p} = \mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f} =$ constant in time.

In the next few units you will continue to study one- and two-dimensional collisions using momentum conservation. Right now you will attempt to test the Law of Conservation of Momentum for a simple situation by using video analysis. To do this you will make and analyze a video movie in which two carts of different masses undergo a one-dimensional elastic collision. You may not be able to finish this in class, but you can complete the project for homework.

Testing Momentum Conservation

You just used theoretical grounds to derive momentum conservation. This idea still must be tested against experiment. You will make this test by colliding two carts on a track and recording and analyzing their motion before and after they hit.

Activity 5: Colliding Carts

(a) Make a movie of two carts colliding by following these steps.

1. Turn the camera on and center the track in the field of view. The camera should be about 1 m above the center of the track where one cart (the target) will sit. The target cart should be located straight down below the camera. Align the track so that it is parallel to the border of the movie image. Make sure the track is flat by using the small level available at each station. Place a ruler somewhere in the field of view where it won't interfere with the collision. This ruler will be used later to determine the scale.

2. Make a movie of one cart (the projectile) rolling into the other, stationary cart (the target). See **Appendix D: Video Analysis** for details on making the movie. When you save the movie file give it the name Collision.

(b) Determine the position of both carts (the target and the projectile) during the motion. To do this task follow the instructions in **Appendix D: Video Analysis** for recording and calibrating the video data. Mark two objects on each frame; click once on the projectile cart and once on the target cart. The data table should contain five columns with the values of time, x and y positions of the projectile cart, and x and y positions of the target cart. Note: Since this is a horizontal 1D collision the y-coordinates are of no interest. They should be constant for each frame. If they are not, consult your instructor.

(c) Create graphs of position versus time for both carts. See **Appendix C: Introduction to Excel** for more details. Print the graphs and attach them to your write-up.

(d) Use your data to calculate the momenta of carts 1 and 2 before the collision.

(e) Use the data to calculate the momenta of carts 1 and 2 after the collision.

(f) Within the limits of experimental uncertainty, does momentum seem to be conserved (i.e., is the total momentum of the two cart system the same before and after the collision)?

40 Momentum Conservation and Center of Mass[31]

Name ______________________ Section ____________ Date ____________

Objectives

To explore the applicability of conservation of momentum to the mutual interactions among objects that experience no external forces (so that the system of objects is isolated). You will calculate momentum changes for an isolated system consisting of two very unequal masses and to observe momentum changes for a system consisting of two equal masses.

Apparatus

- Two Pasco dynamics carts with equal masses (and springs, magnets, and velcro).
- A track for the carts.
- A video analysis system (*VideoPoint*).

Overview

You have now tested Newton's third law under different conditions and it always seems to hold. The implications of that are profound, because whenever an object experiences a force, another entity must also be experiencing a force of the same magnitude. A single force is only half of an interaction. Whenever there are interactions between two or more objects, it is often possible to draw a boundary around a system of objects and say there is no net external force on it. A closed system with no external forces on it is known as an isolated system. Some examples of isolated systems are shown in the figure below.

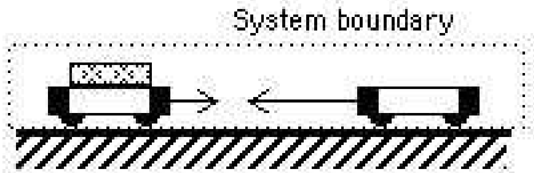
As a consequence of Newton's laws, momentum is believed to be conserved in isolated systems. This means that, no matter how many internal interactions occur, the total momentum of each of the systems pictured below should remain constant. When one of the objects gains some momentum another part of the system must lose the same amount of momentum. If momentum doesn't seem to be conserved then we believe that there is an outside force acting on the system. Thus, by extending the boundary of the system to include the source of that force we can save our Law of Momentum Conservation. The ultimate isolated system is the whole universe. Most astrophysicists believe that momentum is conserved in the universe!

You will begin this unit by examining a situation in which it appears that momentum is not conserved and then seeing how the Law of Conservation of Momentum can hold when the whole isolated system is considered. In the next activity you will make qualitative observations using two carts of equal mass moving toward each other at the same speed. You will observe momentum changes for several types of interactions, including an elastic and inelastic collision and an explosion.

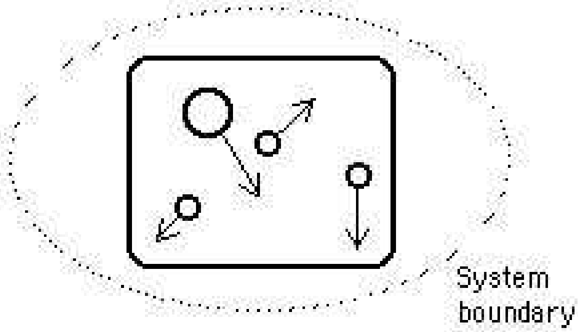
Next, a new quantity, called the center of mass of a system, will be introduced as an alternate way to keep track of the momentum associated with a system or an extended body. In the next unit, you will use this concept to demonstrate that the Law of Conservation of Momentum holds for both one-dimensional and two-dimensional interactions in isolated systems. Several other attributes of the center of mass of a system will be studied.

Examples of isolated systems in which the influence of outside forces is negligible are shown below.

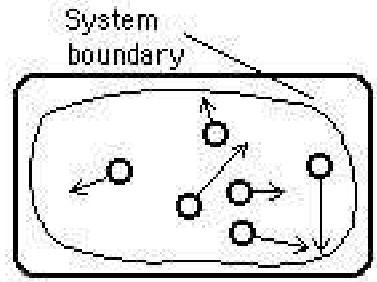
[31] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.



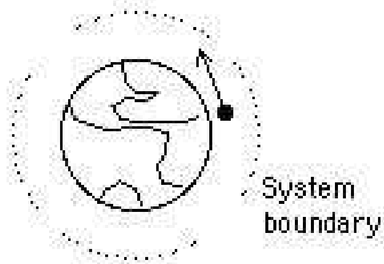
Carts with almost frictionless bearings interact but experience only negligible forces from the track.



Gas molecules interact with each other and with the walls of their container. (Other forces, such as those of the table holding up the container and the gravitational force, are considered to have a negligible effect on the motions of the molecules and container.)



Pucks riding on a cushion of air on an air table interact with each other before hitting the walls of the air table. (Friction forces with the surface of the table are negligible.)



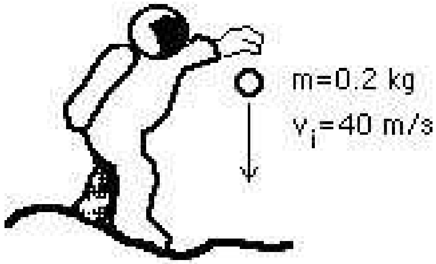
An orbiting satellite and the earth interact. (Forces between these objects and others such as the sun and moon are considered to have a negligible effect on their motions.)

When an Irresistible Force Meets an Immovable Object

Let's assume that a superball and the moon (with an astronaut on it) are the objects in a closed system. (The pull of the earth doesn't affect the falling ball, the astronaut, or the moon nearly as much as they affect each other.) Suppose that the astronaut drops the superball and it falls toward the moon so that it rebounds at the same speed it had just before it hit. If momentum is conserved in the interaction between the ball and the moon, can we notice the moon recoil?

Activity 1: Whapping the Moon with a Superball

(a) Suppose a ball of mass 0.20 kg is dropped and falls toward the surface of the moon so that it hits the ground with a speed of 40 m/s and rebounds with the same speed. According to the Law of Conservation of Momentum, what is the velocity of recoil of the moon?



Mass of Moon: 7.4×10^{22} kg

(b) Will the astronaut notice the jerk as the moon recoils from him? Why or why not?

(c) Consider the ball and the moon as an interacting system with no other outside forces. Why might the astronaut (who hasn't taken physics yet!) have the illusion that momentum isn't conserved in the interaction between the ball and the moon?

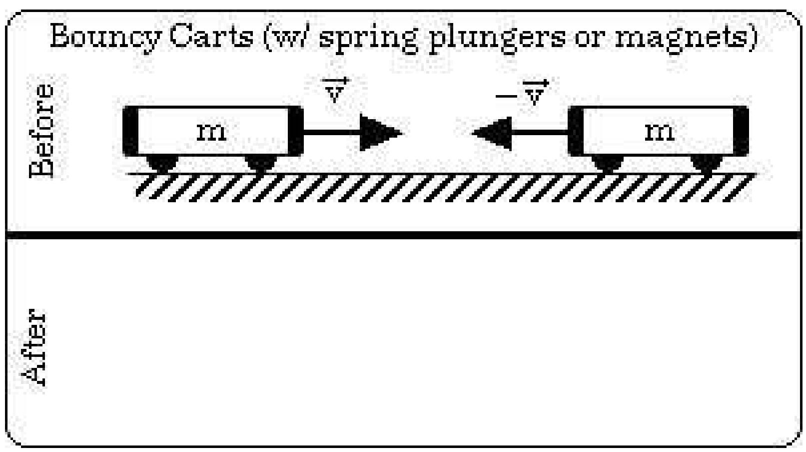
(d) Why might an introductory physics student here on earth have the impression when throwing a ball against the floor or a wall that momentum isn't conserved?

Collisions with Equal Masses: What Do You Know?

Let's use momentum conservation to predict the results of some simple collisions. The diagrams below show objects of equal mass moving toward each other. If the track exerts negligible friction on them then the two cart system is isolated. Assume that the carts have opposite velocities so that $\mathbf{v}_{1i} = -\mathbf{v}_{2i}$. To observe what actually happens, you can use relatively frictionless carts with springs, magnets, and Velcro.

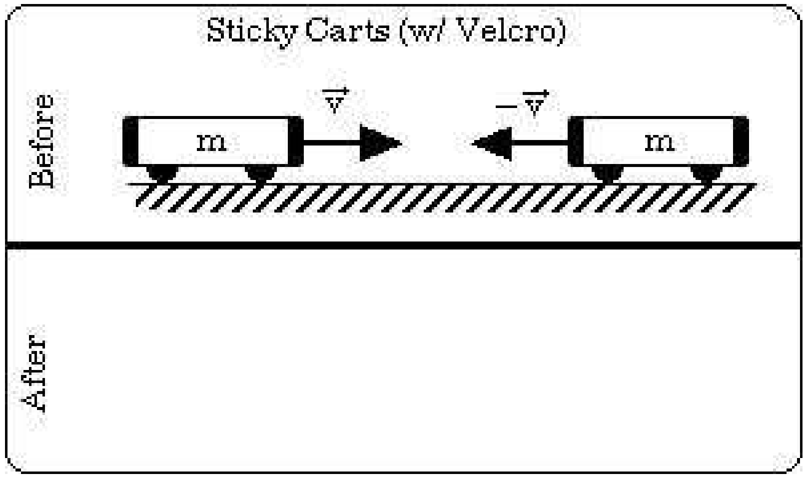
Activity 2: Predictions of the Outcome of Collisions

(a) Sketch a predicted result of the interaction between two carts that bounce off each other so their speeds remain unchanged as a result of the collision. Use arrows to indicate the direction and magnitude of the velocity of each object after the collision.



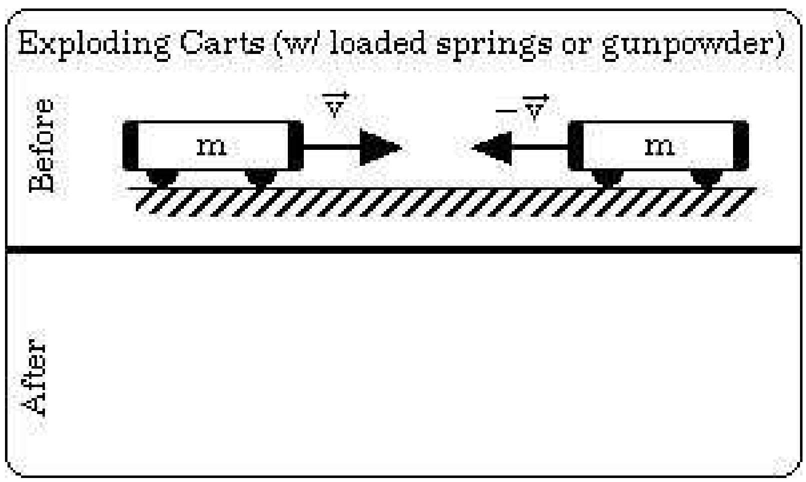
(b) Observe a bouncy collision (also known as an elastic collision) and discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$? How do their magnitudes compare to those of the initial velocities?

(c) Sketch the predicted result of the interaction between two objects that stick to other. Use arrows to indicate the direction and magnitude of the velocity of each object after the collision.



(d) Observe a sticky collision (also known as an inelastic collision) and discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$? How do their magnitudes compare to those of the initial velocities?

(e) Sketch a predicted result of the interaction between two objects that collide and then explode. Use arrows to indicate the direction and magnitude of the velocity of each object after the collision.



(f) Observe an exploding or "superelastic" collision and discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$? How do their magnitudes compare to those of the initial velocities?

(g) What is the total momentum (i.e., the vector sum of the initial momenta) before the collision or explosion in all three situations?

(h) Does momentum appear to be conserved in each case? Is the final total momentum the same as the initial total momentum of the two cart system?

Defining a Center for a Two Particle System

What happens to the average position of a system in which two moving carts having the same mass interact with each other? That is, what happens to $\langle x \rangle = (x_1 + x_2)/2$ as time goes by? What might the motion of the average position have to do with the total momentum of the system? To study this situation you will need a video movie-making and analysis system. In making these observations you'll need to look at the pattern of data points that you place over the frames. You will not need to create graphs or work with numbers.

Activity 3: Motion of the Average Position

(a) Imagine interactions between identical carts moving toward each other at the same speed as described in Activity 2. Do you expect the average position of the carts to move before, during, or after the collision or explosion in each case? Might this have anything to do with the fact that the total momentum of such a system is zero?

(b) Let's use video analysis to study a real situation in which the total momentum of the system is not zero. Do the following:

1. Turn the video camera on and center the track in the field of view. The camera should be about 1 m above the center of the track. Align the track so that it is parallel to the border of the movie image. Make sure the track is flat by using the small level available at each station. Place a ruler somewhere in the field of view where it won't interfere with the collision and parallel to one side of the field of view. This ruler will be used later to determine the scale.
2. Use two carts that have small magnets placed at one end. Make a movie of the collision of two equal mass carts moving in the same direction with different speeds. Orient the carts so they collide on the sides that hold the magnets. See **Appendix D: Video Analysis** for details on making the movie. When you save the movie file give it the name *Collision*.

(c) Determine the average position of both carts during the motion. To do this task follow the instructions in **Appendix D: Video Analysis** for creating and calibrating a data table in VideoPoint. On each frame click once on the point halfway between the centers of the two carts. When you are finished go to the **Edit** menu and highlight **Leave/Hide Trails**.

How does the position average appear to move? Might this motion have anything to do with the fact that the total momentum of the system is directed in one direction? What is your evidence?

(d) You should have found that if the momentum of the carts is constant then the average position moves at a constant rate also. Suppose the masses of the carts are unequal? How does the average position of the two objects move then? Lets have a look at a collision between unequal masses. Make and analyze a new movie as you did before, but add a significant amount of mass to one of the carts. Once again track the motion of the average position by clicking halfway between the centers of the two carts. Is the motion of this average position uniform?

(e) You should have found the average position of a system of two unequal masses does not move at a constant velocity. We need to define a new quantity called the center of mass that is at the center of two equal masses but somewhere else when one of the masses is larger. Use the video analysis system and your movie to find a "center-of-mass location." The center of mass is a location in the isolated system that moves at a constant velocity before, during, and after the collision. Note: You should be able to make some intelligent guesses. Describe what you tried and the outcomes in the space below. In the future you will develop a formal, mathematical definition of the center of mass and learn about some of its important characteristics. Later, you will apply the center-of-mass concept to the analysis of momentum conservation in two-dimensional collisions.

41 Two-Dimensional Collisions[32]

Name ______________ Section ______________ Date ______________

Objectives

To test experimentally that the Law of Conservation of Momentum holds for two-dimensional collisions in isolated systems.

Apparatus

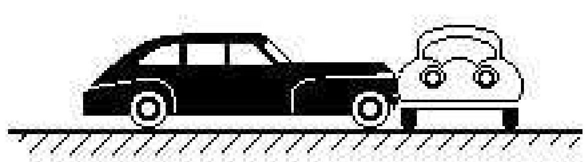
- A set of ball bearings, pendulum bob, a protractor, a level, some carts, and a wooden board.
- A video analysis system (*VideoPoint*).

2-D Collisions: Intelligent Guesses & Observations

Conservation of momentum can be used to solve a variety of collision and explosion problems. So far we have only considered momentum conservation in one dimension, but real collisions lead to motions in two and three dimensions. For example, air molecules are continually colliding in space and bouncing off in different directions.

You probably know more about two-dimensional collisions than you think. Draw on your prior experience with one-dimensional collisions to anticipate the outcome of several two-dimensional collisions. Suppose you were a witness to several accidents in which you closed your eyes at the moment of collision each time two vehicles heading toward each other crashed. Even though you couldn't stand to look, can you predict the outcome of the following accidents?

You see car A enter an intersection at the same time as car B coming from its left enters the intersection. Car B is the same make and model as car A and is traveling at the same speed. The two cars collide and bounce off one another. What happens? Hint: You can use a symmetry argument, your intuition or a quick analysis of 1-D results. For example, pick a coordinate system and think about two separate accidents: the x accident in which car B is moving at speed v_{bx} and car A is standing still, and the y accident in which car A is moving at speed $v_{ay} = v_{bx}$ and car B is standing still.



The diagram below shows an aerial view of several possible two-dimensional accidents that might occur. The first is a collision at right angles of two identical cars.

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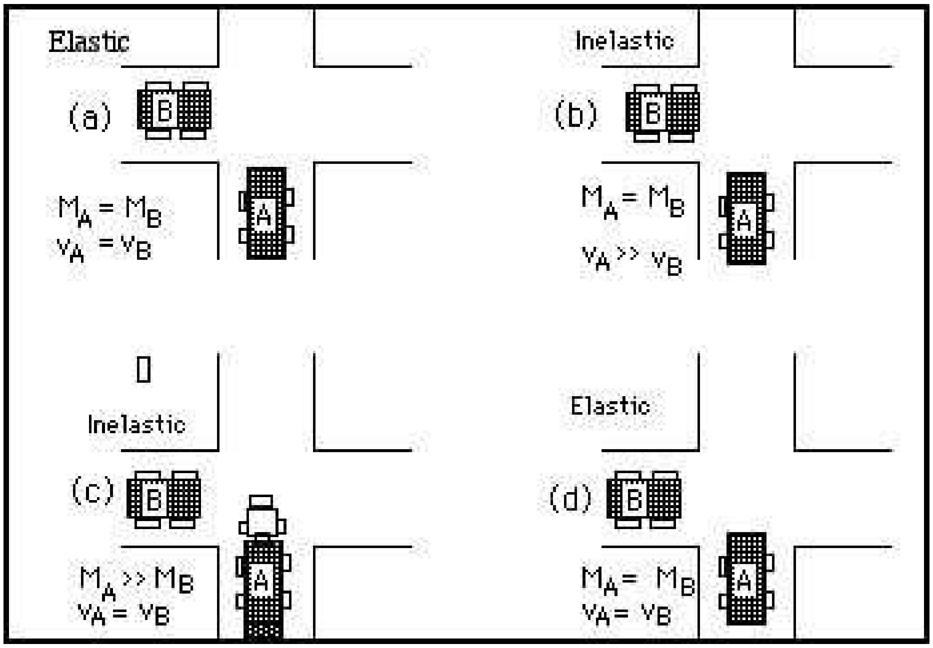


Activity 1: Qualitative 2-D Collisions

(a) Using the diagram above, draw a dotted line in the direction you think your two cars will move after a collision between cars with equal masses and velocities. Explain your reasoning in the space below.

(b) Draw a dotted line for the direction the cars might move if car A were traveling at a speed much greater than that of car B. Explain your reasoning in the space below.

(c) If instead of a car, the vehicle A were a large truck traveling at the same speed as car B, in what direction will the vehicles move? Draw the dotted lines. Explain your reasoning in the space below.

(d) Now suppose that the two vehicles are traveling at the same speed. If the two vehicles were to stick together; in what direction would they move after the collision, if they undergo a perfectly inelastic collision? Explain your reasoning in the space below.

(e) Finally, set up these types of collisions. Observe each type of collision several times. Draw solid lines in the diagram above for the results. How good were your predictions? Explain your reasoning in the space below.

(f) What rules have you devised to predict more or less what is going to happen as the result of a two-dimensional collision?

Theory of 2D Momentum Conservation

Since momentum is a vector, the Law of Conservation of Momentum in two dimensions requires that if the vector conservation equation is broken into components then the conservation law must also hold for each of the vector components. Thus, if we consider the interaction of several objects, and if

$$\sum \mathbf{p} = \mathbf{p}_{1i} + \mathbf{p}_{2i} + \mathbf{p}_{3i} + \ldots = \mathbf{p}_{1f} + \mathbf{p}_{2f} + \mathbf{p}_{3f} + \ldots = \text{a constant}$$

then

$$\sum p_x = p_{1ix} + p_{2ix} + p_{3ix} + \ldots = p_{1fx} + p_{2fx} + p_{3fx} + \ldots = \text{a constant}$$

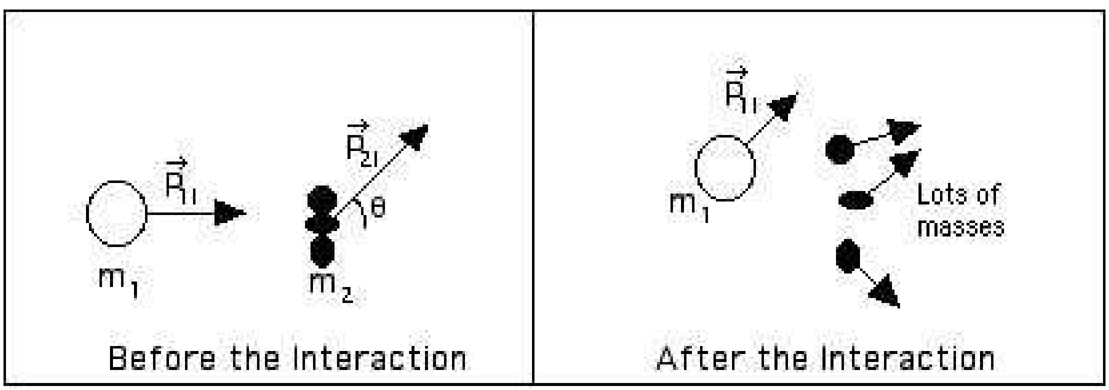
and

$$\sum p_y = p_{1iy} + p_{2iy} + p_{3iy} + \ldots = p_{1fy} + p_{2fy} + p_{3fy} + \ldots = \text{a constant}$$

If a coordinate system is chosen and a given momentum vector makes an angle θ with respect to the designated x-axis then the momentum vector can be broken into components in the usual way:

$$\mathbf{p} = p_x\widehat{\mathbf{i}} + p_y\widehat{\mathbf{j}} = p\cos\theta\widehat{\mathbf{i}} + p\sin\theta\widehat{\mathbf{j}}$$

Let's consider an interaction in which a large mass collides with two smaller hard masses connected by a blob of clay. Assume that this interaction causes the bundle of masses to divide into three fragments as shown in the figure below.



Activity 2: Taking Components

(a) Consider mass #1. Suppose $m_1 = 2.0$kg and the speed $v_{1i} = 1.5$ m/s. What is the initial x-component of momentum? The initial y-component of momentum? Show your calculations.

$p_{1ix} =$

$p_{1iy} =$

(b) Consider mass #2. Suppose $m_2 = 1.8$ kg and the speed $v_{2i} = 2.3$ m/s. If $\theta = 40°$, what is the initial x-component of momentum? The initial y-component of momentum? Show your calculations.

Is Momentum Conserved in Two Dimensions?

During the last few sessions we have placed a lot of faith in the power of Newton's second and third laws to predict that momentum is always conserved in collisions. We have tested the conservation of momentum for one-dimensional collisions and have shown mathematically and experimentally for one-dimensional collisions that if momentum is conserved the center- of-mass of a system will move at a constant velocity regardless of how many internal interactions take place. Now, let's test whether a collision in an isolated two body system will conserve momentum within the limits of experimental uncertainty.

Consider two ball bearings that are free to move in two dimensions on a table. We will record and analyze a video of the two bearings colliding. You can then find the x- and y-components of the momentum for each bearing and test the conservation of momentum in a closed system.

Activity 3: Testing the Conservation of Momentum

(a) Make a movie of two ball bearings colliding by following these steps.

1. Turn the camera on and center the wooden board in the field of view. The camera should be about 1 m above the center of the board where one bearing(the target) will sit. The target bearing should be located straight down below the camera. Use the pendulum bob to position the camera and bearing. Make sure the board is flat by using the small level available at each station. Place a ruler somewhere in the field of view where it won't interfere with the collision and parallel to one edge of the field of view. This ruler will be user later to determine the scale.

2. Make a movie of one bearing (the projectile) rolling into the other, stationary bearing (the target). See **Appendix D: Video Analysis** for details on making the movie. Make the collision a glancing one so that the projectile is scattered to some large angle (otherwise, you will only test momentum conservation in one dimension).

(b) Determine the position of both bearings (the target and the projectile) during the motion. To do this task follow the instructions in **Appendix D: Video Analysis** for creating, calibrating, and analyzing movie data. On each frame click once on the projectile bearing and once on the target. The data table should contain five columns with the values of time, x and y positions of the projectile, and x and y positions of the target. When you calibrate the movie data, note the number of frames per second and record the time interval between successive frames.

$\Delta t =$

(c) Record the masses of the two ball bearings.

$m_1 =$ $m_2 =$

(d) Create a graph of vertical position versus horizontal position for the projectile, and plot the position of the target on the same graph. See **Appendix C: Introduction to Excel** for more details. Make sure the x and y axes cover intervals of the same size so the plot is not distorted. You can adjust the range of an axis by double-clicking anywhere along the axis and modifying the parameters in the pop-up window that appears. Print out the graph.

(e) Draw by eye a best-fit line through the points corresponding to the trajectory of the projectile before the collision. This line will become a new x-axis when you analyze the momentum components of the system. Draw best-fit lines through the points for both ball bearings after the collision. What are the angles of the paths of the target and projectile after the collision with respect to trajectory of the incoming projectile before the collision?

(f) Use the graph to measure the distance each bearing covered before and after the collision. What is the average velocity for each ball bearing before and after the collision? (Hint: Use the value of Δt that you found earlier).

$v_{1i} =$ $v_{1f} =$

$v_{2i} =$ v_{2f}

(g) What are the momentum components for each ball bearing before and after the collision? What is the momentum of the system before the collision along the path of the incoming projectile? What is the momentum of the entire system after the collision? Within the limits of experimental uncertainty is momentum conserved? What is your evidence? What are possible sources of uncertainty in your data?

$p_{1ix} =$ $p_{1iy} =$

$p_{2ix} =$ $p_{2iy} =$

$p_{1fx} =$ $p_{1fy} =$

$p_{2fx} =$ $p_{2fy} =$

$p_{ix} =$ $p_{iy} =$

$p_{fx} =$ $p_{fy} =$

42 Connecting Angular Displacement with Linear Displacement along the Arc

Name ______________________ Section ____________ Date ____________

Objectives:

- Discover the relationship between arc length, radius, and angle
- Understand radian measure

Apparatus

- Wooden disks
- Meter stick
- String

Activity

1. With the string and meter stick, determine the circumference of the disk in cm.

   circumference = ____________

2. Calculate the disk's diameter and radius.

   diameter = ____________ radius = ____________

3. Check that the meter stick is inserted in the base with the scale visible.
4. Be sure the string is attached at the 0-mark on the disk. Wind the string around the disk one full rotation so that the 0-mark and the beginning of the string are at the bottom of the disk and the string is wound around once in such a way that when you unwind it, you can pull it along the meter stick and the disk will rotate counter-clockwise.
5. In this configuration, note the position of the tag on the string with reference to the meter stick.
6. While keeping the string taut, pull it gently to displace the tag five centimeters. Record in the table below the angular position of the bottom of the disk.
7. Repeat step 6 in five centimeter increments until the string is completely unwound.
8. Calculate the angular displacements and plot linear displacement versus angular displacement.

Δs (cm)	θ (rad)	$\Delta\theta$ (rad)
5.0		
10.0		
15.0		
20.0		
25.0		
30.0		
35.0		
40.0		
45.0		

Questions:

1. Do your data points appear to fall on a line? Did you expect them to? Explain.

2. If you were to fit a line through these points, would that line go through the origin? Explain.

3. Fit a line through the data. Express in words the meaning of the slope. What is its physical interpretation?

4. If a bigger disk were used, what would your result be? For a given angular displacement of the bigger disk, how would the linear displacement differ?

5. What is the ratio of some arc length to the radius of the disk?

6. The unit given for the angular displacement is rad, which is an abbreviation for radians. Express, in words, the meaning of the term.

7. How many radians are in half the circumference of the disk? In the whole circumference?

43 Introduction to Rotation[33]

Name ______________________ Section ____________ Date ____________

Objectives

To understand the definitions of angular velocity and angular acceleration and the kinematic equations for rotational motion on the basis of observations. We will discover the relationship between linear velocity and angular velocity and between linear acceleration and angular acceleration.

Apparatus

- A rotator consisting of an axle, a metal disk, and a fixture to hold the disk.
- A stopwatch.
- A meter stick, drawing compass, flexible ruler, a protractor, and some string.

Overview

Earlier in the course, we studied centripetal force and acceleration, which characterize circular motion. In general, however, we have focused on studying motion along a straight line as well as the motion of projectiles. We have defined several measurable quantities to help us describe linear and parabolic motion, including position, velocity, acceleration, force, and mass. In the real world, many objects undergo circular motion and/or rotate while they move. The electron orbiting a proton in a hydrogen atom, an ice skater spinning, and a hammer that tumbles about while its center of mass moves along a parabolic path are just three of many rotating objects.

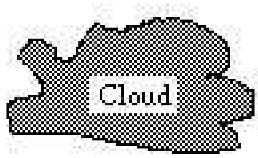
Since many objects undergo rotational motion it is useful to be able to describe their motions mathematically. The study of rotational motion is also very useful in obtaining a deeper understanding of the nature of linear and parabolic motion.

We are going to try to define several new quantities and relationships to help us describe the rotational motion of rigid objects, i.e., objects that do not change shape. These quantities will include angular velocity, angular acceleration, rotational inertia and torque. We will then use these new concepts to develop an extension of Newton's second law to describe rotational motion for masses more or less concentrated at a single point in space (e.g., the electron in the hydrogen atom) and for extended objects (like the tumbling hammer).

Rigid vs. Non-rigid Objects

We will begin our study of rotational motion with a consideration of some characteristics of the rotation of rigid objects about a fixed axis of rotation. The motions of objects, such as clouds, that change size and shape as time passes are hard to analyze mathematically. In this unit we will focus primarily on the study of the rotation of particles and rigid objects around an axis that is not moving. A rigid object is defined as an object that can move along a line or can rotate without the relative distances between its parts changing.

Shown in the figure below are examples of a non-rigid object in the form of a cloud that can change shape and of a rigid object in the form of an empty coffee cup that does not change shape.



[33] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

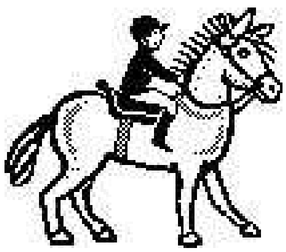
A hammer tossed end over end and an empty coffee cup are examples of rigid objects. A ball of clay that deforms permanently in a collision and a cloud that grows are examples of non-rigid objects.

A Puzzler

Use your imagination to solve the rotational puzzler outlined below. It's one that might stump someone who hasn't taken physics.

Activity 1: Horses of a Different Speed

You are on a white horse, riding off at sunset with your beau on a chestnut mare riding at your side. Your horse has a speed of 4.0 m/s and your beau's horse has a speed of 3.5 m/s, yet he/she constantly remains at your side. Where are your horses? Make a sketch to explain your answer.

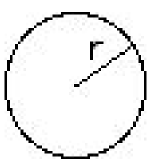


Review of the Geometry of Circles

Remember way back before you came to college when you studied equations for the circumference and the area of a circle? Let's review those equations now, since you'll need them a lot from here on in.

Activity 2: Circular Geometry

(a) What is the equation for the circumference, C, of a circle of radius r? What are the units of C?

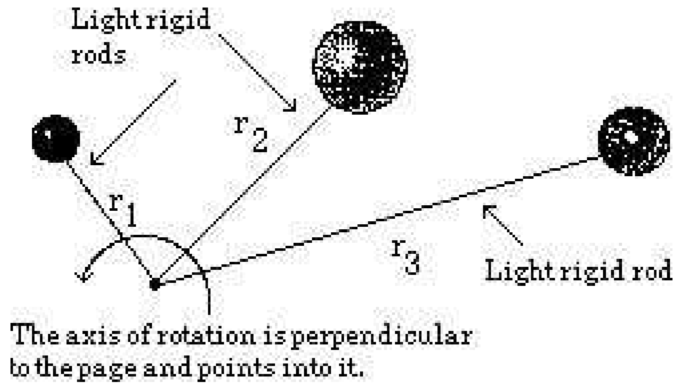


(b) What is the equation for the area, A, of a circle of radius r? What are the units of A?

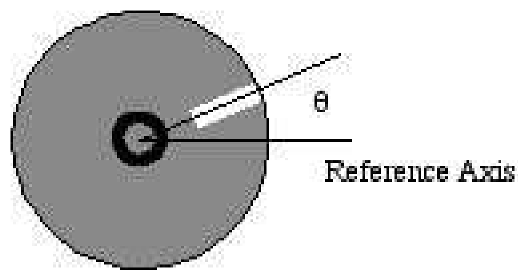
(c) If someone told you that the area of a circle was $A = r$, how could you refute them immediately? What's wrong with the idea of area being proportional to r?

Distance from an Axis of Rotation and Speed

Let's begin our study by examining the rotation of objects about a common axis that is fixed. What happens to the speeds of different parts of a rigid object that rotates about a common axis? How does the speed of the object depend on its distance from an axis? You should be able to answer this question by observing the rotational speed of the rotator at each experimental station.



Place the disk in the fixture and slowly rotate it a constant speed. The figure below shows the rotator and the definition of angular displacement.



Activity 3: Spinning the Rotator: Speed vs. Radius

(a) Measure how long it takes the white marker to sweep through a known angle. Record the time and the angle in the space below.

(b) Calculate the distance of the paths traced out by the outer edge of the white marker and the inner edge as it rotated through the angle you just recorded. (Note: What do you need to measure to perform this calculation?) Record your data below.

(c) Calculate the average speed of the outer edge of the white marker and the average speed of the inner edge of the marker. How do they compare?

(d) Do the speeds seem to be related in any way to the distances of inner and outer edges of the white marker from the axis of rotation? If so, describe the relationship mathematically.

(e) As you rotate, does the distance from the axis of rotation to the outer edge of the white marker change?

(f) As you rotate, does the distance from the axis of rotation to the inner edge of the white marker change?

(g) At any given time during your rotation, is the angle between the reference axis and the inner edge of the white marker the same as the angle between the axis and the outer edge of the white marker, or do the angles differ?

(h)At any given time during your rotation, is the rate of change of the angle between the reference axis and the inner edge of the white marker the same as the rate of change of the angle between the axis and the outer edge, or do the rates differ?

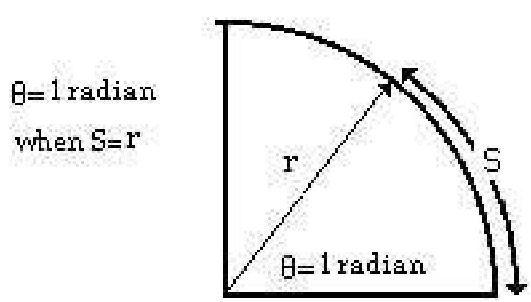
(i) What happens to the linear velocity, **v**, of the outer edge of the marker as it rotates at a constant rate? Hint: What happens to the magnitude of the velocity, i.e., its speed? What happens to its direction?

(j) Is the outer edge of the white marker accelerating? Why or why not?

Radians, Radii, and Arc Lengths

An understanding of the relationship between angles in radians, angles in degrees, and arc lengths is critical in the study of rotational motion. There are two common units used to measure angles: degrees and radians.

1. A degree is defined as 1/360th of a rotation in a complete circle.
2. A radian is defined as the angle for which the arc along the circle is equal to its radius as shown in the figure below.



In the next series of activities you will be relating angles, arc lengths, and radii for a circle.

Activity 4: Relating Arcs, Radii, and Angles

(a) Let's warm up with a review of some very basic mathematics. What should the constant of proportionality be between the circumference of a circle and its radius? Write the appropriate equation.

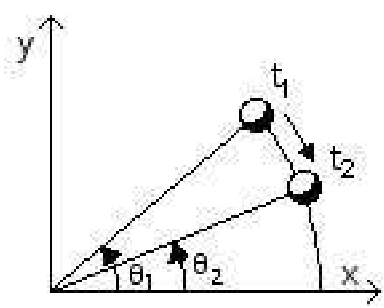
(b) Approximately how many degrees are in one radian? Let's do this experimentally. Using the compass draw a circle and measure its radius. Then, use the flexible ruler to trace out a length of arc, s, that has the same length as the radius. Next measure the angle in degrees that is subtended by the arc.

(c) Theoretically, how many degrees are in one radian? Please calculate your result to three significant figures. Using the equation for the circumference of a circle as a function of its radius and the constant $\pi = 3.1415927...$, figure out a general equation to find degrees from radians. **Hint:** How many times does a radius fit onto the circumference of a circle? How many degrees fit in the circumference of a circle?

(d) If an object moves 30 degrees on the circumference of a circle of radius 1.5 m, what is the length of its path?

(e) If an object moves 0.42 radians on the circumference of a circle of radius 1.5 m, what is the length of its path?

(f) Remembering the relationship between the speed of the outer edge of the rotator and the distance, r, from the rotator's axis the outer edge, what equations would you use to define the magnitude of the average "angular" velocity, $\langle\omega\rangle$? **Hint:** In words, $\langle\omega\rangle$ is defined as the amount of angle swept out by the object per unit time. Note that the answer is not simply θ/t!



(g) How many radians are there in a full circle consisting of 360 degrees?

(h) When an object moves in a complete circle in a fixed amount of time, what quantity (other than time) remains unchanged for circles of several different radii?

Relating Linear and Angular Quantities

It's very useful to know the relationship between the variables s, v, and a, which describe linear motion and the corresponding variables θ, ω, and α, which describe rotational motion. You now know enough to define these relationships.

Activity 5: Linear and Angular Variables

(a) Using the definition of the radian, what is the general relationship between a length of arc, s, on a circle and the variables r and θ in radians.

(b) Assume that an object is moving in a circle of constant radius, r. Using the relationship you found in part (a) above, take the derivative of s with respect to time to find the velocity of the object. Show that the magnitude of the linear velocity, v, is related to the magnitude of the angular velocity, ω, by the equation $v = \omega r$.

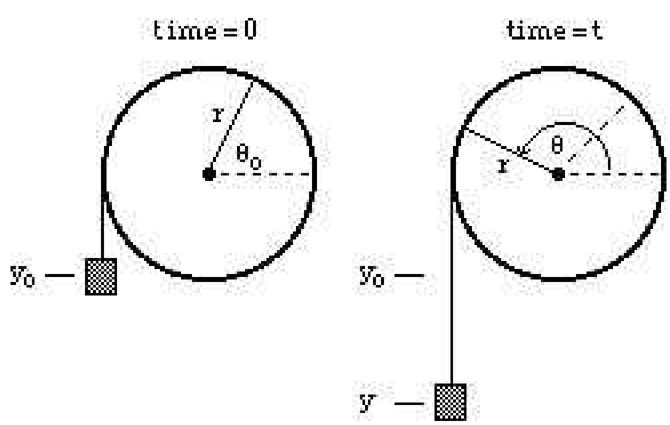
(c) Assume that an object is accelerating in a circle of constant radius, r. Using the relationship you found in part (b) above, take the derivative of v with respect to time to find the tangential acceleration of the object. Show that the linear acceleration, a_t, tangent to the circle is related to the angular acceleration, α, by the equation $a_t = \alpha r$.

The Rotational Kinematic Equations for Constant α

The set of definitions of angular variables are the basis of the physicist's description of rotational motion. We can use them to derive a set of kinematic equations for rotational motion with constant angular acceleration that are similar to the equations for linear motion.

Activity 6: The Rotational Kinematic Equations

The figure below shows a massless string wound around a spool of radius r. The mass falls with a constant acceleration, a. Refer to this figure and the results of Activity 5 to answer the following questions.



(a) What is the equation for θ in terms of y and r?

(b) What is the equation for ω in terms of v and r?

(c) What is the equation for α in terms of a and r?

(d) Consider the falling mass in the figure above. Suppose you are standing on your head so that the positive y-axis is pointing down. Using the relationships between the linear and angular variables in parts (a), (b), and (c), derive the rotational kinematic equations for constant accelerations for each to the linear kinematic equations listed below. **Warning:** Don't just write the analogous equations! Show the substitutions needed to derive the equations on the right from those on the left.

1. $v = v_0 + at$ $\qquad\qquad$ $\omega =$

2. $y = y_0 + v_0 t + \frac{1}{2}at^2$ $\qquad\qquad$ $\theta =$

3. $v^2 = v_0^2 + 2ay$ $\qquad\qquad$ $\omega^2 =$

44 Newton's Second Law for Rotation[34]

Name ______________________ Section ____________ Date ____________

Objectives

To understand torque and its relation to angular acceleration and rotational inertia on the basis of both observations and theory.

Apparatus

- A Rotating Disk System
- A hanging mass of 300-400 g (for applying torque)
- String and pulley
- A meter stick and a ruler
- A video analysis system (*VideoPoint*).

Overview

We have used the definition of rotational inertia, I, to determine a theoretical equation for the rotational inertia of a disk. This equation was given by

$$I = \frac{1}{2}Mr^2.$$

Does this equation adequately describe the rotational inertia of a rotating disk system? If so, then we should find that, if we apply a known torque, τ, to the disk system, its resulting angular acceleration, α, is actually related to the system's rotational inertia, I, by the equation

$$\tau = I\alpha \quad \text{or} \quad \alpha = \tau/I$$

The purpose of this experiment is to determine if, within the limits of experimental uncertainty, the measured angular acceleration of a rotating disk system is the same as its theoretical value. The theoretical value of angular acceleration can be calculated using theoretically determined values for the torque on the system and its rotational inertia.

Theoretical Calculations

You'll need to take some basic measurements on the rotating disk system to determine theoretical values for I and τ. Values of rotational inertia calculated from the dimensions of a rotating object are theoretical because they purport to describe the resistance of an object to rotation. An experimental value is obtained by applying a known torque to the object and measuring the resultant angular acceleration.

Activity 1: Theoretical Calculations

(a) Calculate the theoretical value of the rotational inertia of the metal disk using basic measurements of its radius and mass. Ignore the small hole in the middle in your calculation. Be sure to state units!

$r_d =$ $M_d =$

$I_d =$

[34] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b)The rotating fixture that holds the disk has a complex shape. We have determined its moment of inertia without the disk and recorded the result on the fixture. Record that value here. Be sure to state units.

$I_f =$

(c) Calculate the theoretical value of the rotational inertia, I, of the whole system. Don't forget to include the units.

$I =$

(d) In preparation for calculating the torque on your system, summarize the measurements for the falling mass, m, and the radius of the spool that has the string wrapped around it in the space below. Don't forget the units!

$m =$ $\qquad$ $r_s =$

(e) Calculate the theoretical value for the torque on the rotating system as a function of the magnitude of the hanging mass and the radius, r_s, of the spool, assuming the tension in the string is equal to the weight of the falling mass (this introduces an error of less than 1 percent). Be sure to include units.

$\tau_{th} =$

(f) Based on the values of torque and rotational inertia of the system, what is the theoretical value of the angular acceleration of the disk? What are the units?

$\alpha_{th} =$

Activity 2: Experimental Measurement of Angular Acceleration

(a) Place the video camera about 1 m above the rotator, and center the rotator in the field of view of the camera by viewing the rotator with the *VideoPoint Capture* software. Use the small level to ensure that the surface of the rotator is level. Place a ruler of known length in the field of view of the camera and parallel to one side of the frame.

(b) Place the rotator so the string will pass smoothly over the pulley and put 300-500 g of mass on the end of the string. Release the rotator and use the video camera to record the motion of the disk for at least two full turns. See **Appendix D: Video Analysis** for details.

(c) Determine the angular displacement of the rotator as a function of time. Be careful to place the origin of your coordinate system on the axle of the rotator so the angular displacement you measure will be the desired one. To do this task follow the instructions in **Appendix D: Video Analysis** for recording, calibrating, and analyzing a movie data file. The file should contain three columns with the values of time, x-position, and y-position for one complete revolution.

(d) What is the expression for the angular displacement of the disk in terms of the x and y positions of the marker that you recorded above? Note that these positions should be relative to an origin placed on the axle of the rotator.

$\theta =$

(e) We want to graph the angular displacement of the disk as a function of time. To do this:

1. Export your data to an *Excel* file and launch *Excel*.
2. Calculate the angular displacement θ in radians for the first row in the spreadsheet. Record the result here. You will use this result later to check the calculations you make with *Excel*.

   $\theta =$
3. Calculating the angular position for all the data as we just did would be horribly tedious. Instead, use an *Excel* formula to figure out the angular positions. (See Appendix C for details.) You may find it helpful to know that there is an *Excel* function ATAN2 that takes the inverse tangent of the ratio of two numbers. For instance, if you put "=ATAN2(C3,D3)" into a cell, *Excel* will calculate the inverse tangent of the ratio of the number in cell D3 to the number in cell C3. (Note that the ratio that is taken has the second argument on top: in this case, it's D3/C3, not C3/D3.)
4. Does the value of the first row agree with the calculation you made in part (2) above? If it does not check both calculations again. If that fails consult your instructor.
5. Graph the angular displacement (column 4) as a function of time (column 1). You will see discontinuous jumps in your data because the function you used in part (3) always calculates angles in the range $-\pi$ to π. You must add different increments of 2π, 4π, etc. to adjust the scale of the angular displacement. You should create another column of data in your data table containing the angular positions with appropriate multiples of 2π added to them.

(f) We now want to extract the angular acceleration from the data.

1. To describe the time dependence of the angular displacement what type of polynomial should we use to fit the data? How are the coefficients of the polynomial related to the angular acceleration?
2. Fit the data with a polynomial and write the resulting equation for the time dependence of the angular position in the space below. Be sure to include the proper units with the coefficients. Determine the experimental value for the angular acceleration from the fit and record it below. Also, print a copy of your plot with the fitted curve and the equation and put it in your notebook.

Compare your experimental results for α to your theoretical calculation of α for the rotating system. Present this comparison with a summary of your data and calculated results.

Activity 3: Comparing Theory with Experiment

(a) Summarize the theoretical and experimental values of angular acceleration.

$\alpha_{th} =$

$\alpha_{exp} =$

(b) Do theory and experiment agree within the limits of experimental uncertainty? What is the percent deviation?

45 Rolling

Name ____________________ Section __________ Date __________

Objectives

To apply our understanding of rotation to the case of rolling without skidding and discover its fundamental properties.

Apparatus

- A spool for rolling.
- A meter stick and a ruler
- A video analysis system (*VideoPoint*).

Overview

We have extensively studied the linear motion of objects and we recently began the study of rotational motion. When something rolls it is combining both types of motion in one. In addition, the motion requires a unique application of forces and it has some surprising characteristics that are often poorly understood. In this unit we will investigate these unique features.

Activity 1: Rolling — Guesses and Predictions

To begin our investigation consider the following questions. You might find it useful to test some of your answers by simply rolling the spool along the table and observing its motion.

1. In your own words, what is rolling?

2. How is the rotational motion related to the linear motion?

3. Make a sketch of the spool rolling and the forces acting on the spool. What is the effect, if any, of friction? How would the motion change if there was no friction in the system at all?

Activity 2: Investigating Rolling

(a) Make a movie of the spool rolling along your laboratory table by following these steps.

1. Turn the camera on and point it along the long axis of your table. The camera should be around 1 m from where you will roll the spool along the surface of the table. Place a ruler somewhere in the field of view where it won't interfere with the motion and parallel to one edge of the field of view. This ruler will be user later to determine the scale.
2. Practice rolling the spool in front of the camera a few times to make sure the camera is pointing so there is no change in the vertical position of the center of the spool as it moves across the camera's field of view. Make a movie of the spool rolling at a constant speed along the table. See **Appendix D: Video Analysis** for details on making the movie.

(b) Determine the position of a point on the rim of the spool and the center of the spool during the motion. To do this task follow the instructions in **Appendix D: Video Analysis** for creating and calibrating movie data. On each frame click first on the point on the rim and then on the center. Make sure the vertical position of the center did not change much during the movie. If it did, check the orientation of your camera and retake the movie. The file should contain five columns with the values of time, x and y positions of the point on the rim, and x and y positions of the center of the spool.

(c) Create a plot of the horizontal position of the center of the spool versus time. See **Appendix C: Introduction to Excel** for more details. Fit your data and extract the speed of the center of the spool. Print your plot and attach it to your write-up. Measure the radius of the spool.

$v_{\text{spool}} =$ $\qquad\qquad\qquad\qquad$ $r_{\text{spool}} =$

(d) What is the general expression for the angular displacement of the point on the rim relative to the center of the spool in terms of the x and y positions that you recorded above?

$\theta_{\text{rim}} =$

(e) We want to graph the angular displacement of the point on the rim relative to the center as a function of time. To do this:

1. Calculate the angular displacement θ_{rim} in radians for the first row in your data table. Record the result here.

   $\theta_1 =$
2. Calculating the angular displacement for all the data as we just did would be horribly tedious. Instead, use an *Excel* formula to figure out the angular displacements. (See Appendix C for details.) You may find it helpful to know that there is an *Excel* function ATAN2 that takes the inverse tangent of the ratio of two numbers. For instance, if you put "=ATAN2(C3,D3)" into a cell, *Excel* will calculate the inverse tangent of the ratio of the number in cell D3 to the number in cell C3. (Note that the ratio that is taken has the second argument on top: in this case, it's D3/C3, not C3/D3.)
3. Graph the angular displacement as a function of time. You will see discontinuous jumps in your data because the function you used in part (3) always calculates angles in the range $-\pi$ to π. You must add different increments of 2π, 4π, etc. to adjust the scale of the angular displacement. Do this, print your plot and attach it to your write-up.

(f) Fit your graph of the angular displacement versus time and extract the angular speed. Calculate the ratio of the speed of the center of the spool and the angular speed. What are the units of this ratio? Is it close in value to any property of the spool? Is it related to any property of the spool? Why?

$\omega_{\text{rim}} =$ $\qquad\qquad\qquad\qquad$ $v_{\text{spool}}/\omega_{\text{rim}} =$

(g) Make a plot of the vertical and horizontal components of the position of the point on the rim as a function of time. Attach a copy of your plot to the unit. What is the speed of the point on the rim when it is in contact with the table? Explain.

(h) How would you define rolling now? What is the role of friction?

46 Moment of Inertia

Name ______________________ Section ____________ Date ____________

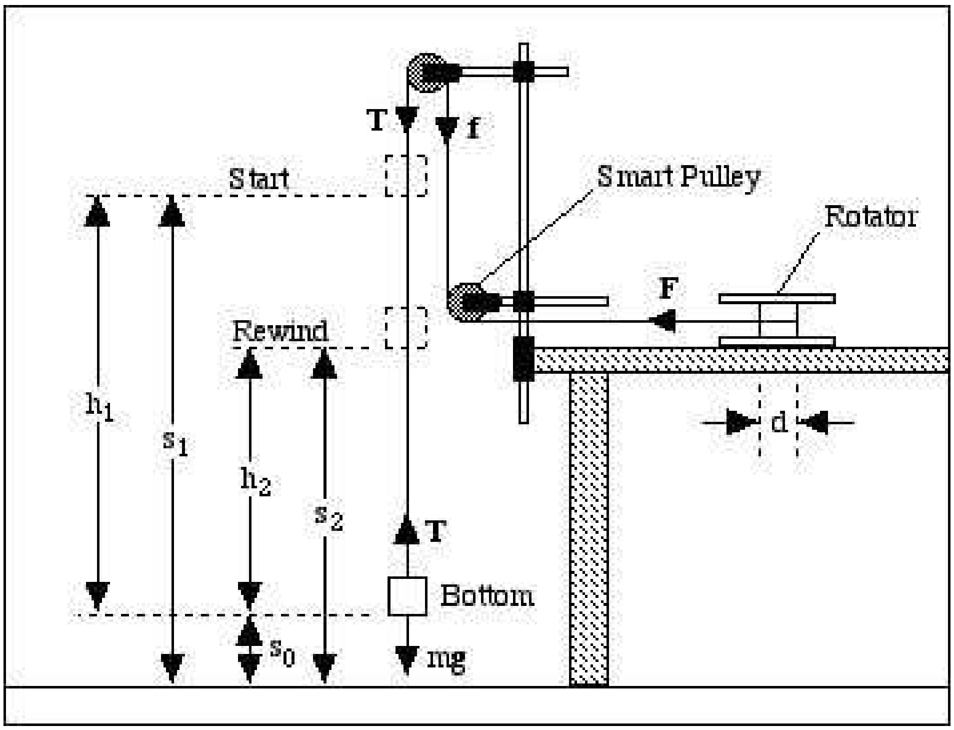
Objective

To investigate Newton's second law of motion for rotating bodies by applying it to determine the moment of inertia of a disk.

Moment of Inertia

In this experiment you will determine the moment of inertia of a body by applying Newton's second law of motion for rotating bodies. The moment of inertia of a body is a measure of the tendency of the body to resist a change in its state of rotational motion. When a net external torque, τ, acts on a rigid body free to rotate about some axis, an angular acceleration, α, is produced that is proportional to the torque. The proportionality constant is the moment of inertia, I, of the body. This is the rotational analog of Newton's second law of motion.

The experimental setup is shown in the figure below. The apparatus consists of a mass, m, connected by a string passing over two pulleys to the drum of a rotator.



Apparatus

- Rotator
- Smart pulley
- Regular pulley
- Variety of masses
- String
- Vernier caliper
- 2-meter stick

- *Science Workshop 750 Interface*
- *DataStudio* software (Atwood's Machine application)

Activity 1: Prediction for the Moment of Inertia

(a) Write the symbolic expression for Newton's second law for rotating bodies in the space below.

(b) Write an expression for the net torque on the rotator in terms of the radius of the rotator drum, r, and the magnitude of the net force supplied by the string, F.

(c) Write an expression for the angular acceleration of the rotator drum in terms of the radius of the drum, r, and the acceleration of the hanging mass, a.

(d) Notice that the net force on the drum, F, is equal to the tension in the string, T, minus the frictional force, f. Write an expression for F in terms of T and f.

(e) Apply Newton's second law to the hanging mass to obtain an expression for the tension, T, in terms of m, g, and a.

(f) The frictional force, f, can be determined by application of the principle of conservation of energy. If the mass, m, falls a height, h_1, and then rises a height, h_2, on the rewind, the loss in its potential energy between the two positions is equal to the work done by the frictional force. Using this, find an expression for f in terms of m, g, h_1, and h_2.

(g) Combine the equations abov, to get the following expression for the moment of inertia of the rotator (with or without the disk):

$$I = \frac{md^2}{4a}\left(g - a - g\frac{h_1 - h_2}{h_1 + h_2}\right)$$

where d is the diameter of the drum of the rotator.

Activity 2: Determining the Moment of Inertia

(a) Use the vernier caliper to measure the diameter of the drum of the rotator. Each member of the group should make an independent measurement and record the average value as d. If you have questions about how to read the vernier, consult your instructor.

(b) Launch the Atwood's Machine application.

(c) Remove the disk from the rotator and construct a data table below with the column headings m, s_0, s_1, s_2, h_1, h_2, a, and I. Don't forget to include appropriate units in the column headings.

(d) Hang 50 grams on the string and lower the mass slowly until all the string is unwound from the drum. Measure and record the distance from the bottom of the mass to the floor as s_0.

(e) Rewind the string on the drum until the mass is a few centimeters from the upper pulley. Measure and record the distance, s_1, from the floor to the mass.

(f) Release the mass, start recording data, and stop it at its highest position on the rewind. Measure and record the distance, s_2, from the floor to this position of the mass.

(g) The computer will display a graph of the velocity versus time for the fall of the mass. Fit the data to determine the acceleration and record this value in your data table.

(h) Repeat steps (d)-(g) two more times for a total of three trials.

(i) Replace the 50 gram mass with a 70 gram mass and repeat steps (d)-(h).

(j) Place the disk on the rotator, construct a new data table, and repeat steps (d)-(i).

(k) Calculate and record the moment of inertia for each trial. Show a sample calculation for one trial in the space below.

(l) Determine the average value for the moment of inertia of the rotator without the disk and record his value as I_R below.

(m) Determine the average value for the moment of inertia of the rotator with the disk and record this value as I_{R+D}.

(n) Calculate and record the experimental value for the moment of inertia of the disk, $I_{exp} = I_{R+D} - I_R$.

(o) Calculate the theoretical moment of inertia of the disk, I_{th}, assuming that it is uniform, and determine the % difference between I_{exp} and I_{th}.

(p) Does the experimental value for the moment of inertia of the disk agree with the theoretical value within experimental uncertainties? Do the results verify Newton's second law of motion for rotating bodies?

47 Angular Momentum and Torque as Vectors[35]

Name ______________________ Section ____________ Date ____________

Objectives

To understand the definitions of torque and angular momentum as vector quantities and to understand the mathematical properties and some applications of the vector cross product. We will also learn the relationship between torque and angular momentum.

Apparatus

- A horizontal pivot
- Two identical spring scales
- A ruler
- A protractor
- Rods and connectors (Styrofoam ball and bamboo skewers)
- Two small masses (100g & 200g)
- A Rotating Disk System
- String

Overview

This unit presents us with a consolidation and extension of the concepts in rotational motion that you have studied so far. You studied the analogy and relationships between rotational and linear quantities (i.e., position and angle, linear velocity and angular velocity, linear acceleration and angular acceleration, and force and torque) without taking into account, in any formal way, the fact that these quantities actually behave like the mathematical entities we call vectors. We will discuss the vector nature of rotational quantities and, in addition, define a new vector quantity called angular momentum that is the rotational analog of linear momentum.

Angular momentum and torque are special vectors because they are the product of two other vectors a position vector and a force or linear momentum vector. To describe them we need to introduce a new type of vector product known as the vector cross product. We will explore the definition and unique nature of the vector cross product used to define torque and angular momentum. We will study the relationship between torque and angular momentum as well as the theoretical basis of the Law of Conservation of Angular Momentum.

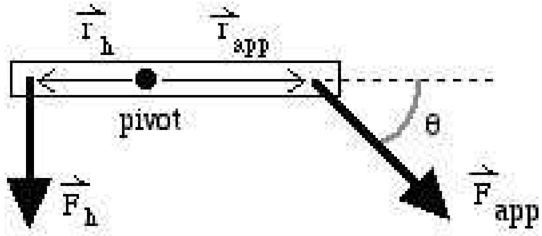
Observation of Torque when F and r are not Perpendicular

Recently, you "discovered" that if we define torque as the product of a lever arm and perpendicular force, an object does not rotate when the sum of the torques acting on it add up to zero. However, we didn't consider cases where **F** and **r** are not perpendicular, and we didn't figure out a way to tell the direction of the rotation resulting from a torque. Let's consider these complications by generating torques with spring balances and a lever arm once more.

Activity 1: Torque as a Function of Angle

(a) Suppose you were to hold one of the scales at an angle of 90° with respect to the lever arm, r_h, and pull on it with a steady force. Meanwhile you can pull on the other scale at several angles other than 90° from its lever arm, r_{app}, as shown below. Would the magnitude of the balancing force be less than, greater than, or equal to the force needed at 90°? What do you predict? Explain.

[35] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.



(b) You should determine exactly how the forces compare to that needed at a 90° angle. Determine this force for at least four different angles and figure out a mathematical relationship between F, r, and θ. Set up a spreadsheet to do the calculations shown in the table below. Hint: Should you multiply the product of the measured values of r and F by $\sin\theta$ or by $\cos\theta$ to get a torque that is equal in magnitude to the holding torque?

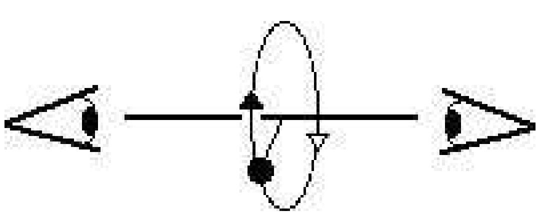
r_h (m)	F_h (N)	τ_h (N m)	r_{app} (m)	F_{app} (N)	θ (deg)	θ (rad)	$\cos\theta$	$\sin\theta$	$r_{app}F_{app}\cos\theta$ (N m)	$r_{app}F_{app}\sin\theta$ (N m)

(c) Within the limits of uncertainty, what is the most plausible mathematical relationship between τ and r, F, and θ?

The activity you just completed should give you a sense of what happens to the magnitude of the torque when the pulling force, **F**, is not perpendicular to the vector, **r**, from the axis of rotation. But how do we define the direction of the rotation that results when the torque is applied to an object that is initially at rest and not balanced by another torque? Let's consider the directions we might associate with angular velocity and torque in this situation.

Activity 2: Angular Rotation, Torque, and Direction

(a) Suppose a particle is moving around in a circle with an angular velocity that has a magnitude of ω associated with it. According to observer #1, does the particle appear to be moving clockwise or counter clockwise? How about the direction of the particle's motion according to observer #2?



(b) Is the clockwise vs. counterclockwise designation a good way to determine the direction associated with ω in an unambiguous way? Why or why not?

(c) Can you devise a better way to assign a minus or plus sign to an angular velocity?

(d) Similar consideration needs to be given to torque as a vector. Can you devise a rule to assign a minus or plus sign to a torque? Describe the rule.

Discussion of the Vector Cross Product

An alternative to describing positive and negative changes in angle is to associate a positive or negative vector with the axis of rotation using an arbitrary but well accepted rule called the right-hand rule. By using vectors we can describe separate rotations of many body systems all rotating in different planes about different axes.

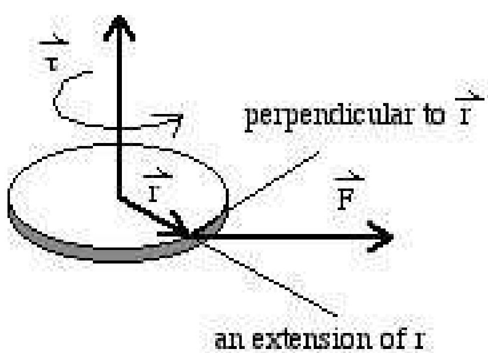
By using this vector assignment for direction, angular momentum and torque can be described mathematically as "vector cross products." The vector cross product is a very strange type of vector multiplication worked out many years ago by mathematicians who had never even heard of angular momentum or torque. The peculiar properties of the vector cross product and its relationship to angular momentum and torque is explained in most introductory physics textbooks. The key properties of the vector that is the cross product of two vectors **r** and **F** are that:

1. The magnitude of the cross product is $rF\sin\theta$ where θ is the angle between the two vectors;

   $$|\mathbf{r}\times\mathbf{F}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta.$$

   The term $\sin\theta$ picks out the component of **F** along a line perpendicular to **r**.

2. The cross product of two vectors **r** and **F** is a vector that lies in a direction perpendicular to both **r** and **F** and is up if **F** causes a counter-clockwise rotation, and is down if **F** causes a clockwise rotation. These properties of the cross product are pictured below.



The spatial relationships between **r**, **F**, and $\boldsymbol{\tau}$ are very difficult to visualize. In the next activity you can connect some thin rods of various sizes to each other at angles of your own choosing and make some "vector cross products."

Activity 3: Making Models of Vector Cross Products

(a) Pick out rods of two different lengths and connect them at some angle you choose. Consider one of the rods to be the **r** vector and the other to be the **F** vector. Measure the angle θ and the lengths of **r** and **F** in meters.

Then compute the magnitude of the cross product as $rF \sin\theta$ in newton meters (N m). Show your units! Note: You should assume that the magnitude of the force in newtons is represented by the length of the rod in meters.

(b) Attach a "cross product" rod perpendicular to the plane determined by $\mathbf{r}$ and $\mathbf{F}$ with a length of $rF \sin\theta$. Sketch the location of $\mathbf{F}$ relative to $\mathbf{r}$ in the space below. Show the direction and magnitude of the resultant torque $\boldsymbol{\tau}$. *Finally, show your cross product model to an instructor or fellow student for confirmation of its validity.*

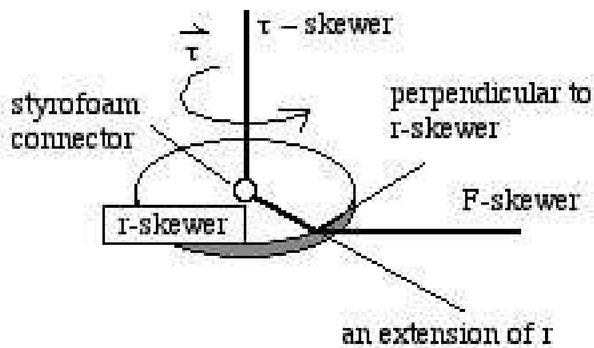
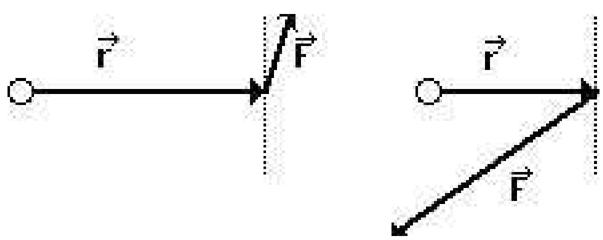


Figure 2: A Model of the Vector Cross Product

(c) In the diagrams below the vectors $\mathbf{r}$ and $\mathbf{F}$ lie in the plane of the paper. Calculate the torques for the following two sets of $\mathbf{r}$ and $\mathbf{F}$ vectors. In each case measure the length of the $\mathbf{r}$ vector in meters and assume that the length of the $\mathbf{F}$ vector in cm represents the force in newtons. Use a protractor to measure the angle, θ, between the extension of the $\mathbf{r}$-vector and the $\mathbf{F}$-vector. Calculate the magnitude of the torques. Place the appropriate symbol to indicate the direction of the torque in the circle as follows: a circle with an x in it indicates a vector into the page, while a circle with a dot in it indicates a vector out of the page.



$r =$ ______ m $\quad F =$ ______ N $\quad \theta =$ ______ $\quad \tau =$ ______ N·m

$r =$ ______ m $\quad F =$ ______ N $\quad \theta =$ ______ $\quad \tau =$ ______ N·m

Momentum and its Rotational Analog

Once we have defined the properties of the vector cross product, another important rotational vector is easily obtained, that of angular momentum relative to an axis of rotation.

Activity 4: Angular and Linear Momentum

(a) Write the rotational analogs of the linear quantities shown. Note: Include the formal definition (which is different than the analog) in spaces marked with an asterisk (*). For example the rotational analog for velocity is angular velocity $\boldsymbol{\omega}$ and the definition of its magnitude is $|\boldsymbol{\omega}| = d\theta/dt$ rather than v/r.

Linear Quantity	Rotational Quantity	Definition
x (position)		*
v (velocity)		*
a (acceleration)		*
F (Force)		*
m (mass)		
$F = ma$		

(b) What do you think will be the rotational definition of angular momentum in terms of the vectors **r** and **p**? **Hint:** This is similar mathematically to the definition of torque and also involves a vector cross product. Note that torque is to angular momentum as force is to momentum.

(c) What is the rotational analog in terms of the quantities I and $\boldsymbol{\omega}$? Do you expect the angular momentum to be a vector? Explain.

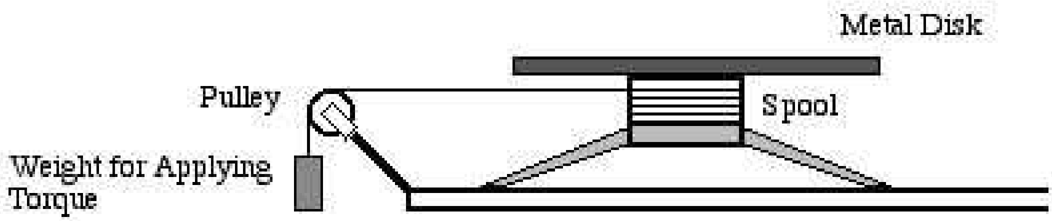
(d) Summarize your guesses in the table below.

Linear Equation	Rotational Equation
$\mathbf{p} = m\mathbf{v}$ [definition in terms of **r** & **p**]	$\mathbf{L} =$
$\mathbf{p} = m\mathbf{v}$ [definition in terms of I & $\boldsymbol{\omega}$]	$\mathbf{L} =$

Torque and Change of Angular Momentum

Earlier in this course you applied a very brief force along a line through the center of mass of a rolling cart. Do you remember how it moved? What happened when you applied a gentle but steady force along a line through the center of mass of the cart? Let's do analogous things to a disk that is free to rotate on a relatively frictionless bearing, with the idea of formulating laws for rotational motion that are analogous to Newton's laws for linear motion.

Figure out how to use a system like that shown in the figure below to observe the motion of the disk under the influence of a brief torque and a steady torque. In describing the Laws of Rotational Motion be sure to consider vector properties and take both the magnitudes and directions of the relevant quantities into account in your wordings.



Activity 5: Applied Torques and Resultant Motion

(a) What happens to the angular velocity and hence the angular momentum of the disk before, during, and after the application of a brief torque? State a First Law of Rotational Motion (named after yourself, of course) in terms of torques and angular momenta. **Hint:** Newton's first law states that the center of mass of a system of particles or a rigid object that experiences no net external force will continue to move at constant velocity.

The Rotational First Law in words:

The Rotational First Law as a mathematical expression:

(b) What happens to the magnitude and direction of the angular velocity (and hence the angular momentum) of the disk during the application of a steady torque? How do they change relative to the magnitude and direction of the torque? If possible, give a precise statement of a Second Law of Rotational Motion relating the net torque on an object to its change in angular momentum. **Note:** Take both magnitudes and directions of the relevant vectors into account in your statement. Hint: Newton's second law of motion states that the center of mass of a system of particles or rigid object that experiences a net external force will undergo an acceleration inversely proportional to its mass.

The Rotational Second Law in Words:

The Rotational Second Law as a Vector Equation:

Angular Momentum Conservation

Now you can use the vector expression for Newton's second law of Rotational Motion to show that, in theory, we expect angular momentum on a system to be conserved if the net torque on that system is zero.

48 Conservation of Angular Momentum

Name ______________________ Section ____________ Date ____________

Objectives

To test the Law of Conservation of Angular Momentum and to explore the applicability of angular momentum conservation among objects that experience no external torques.

Apparatus

- A Rotating Disk System
- A mass of 1 kg
- A meter stick and a ruler
- A video analysis system (*VideoPoint*)

Overview

As a consequence of Newton's laws, angular momentum like linear momentum is believed to be conserved in isolated systems. This means that, no matter how many internal interactions occur, the total angular momentum of any system should remain constant. When one of the objects gains some angular momentum another part of the system must lose the same amount. If angular momentum isn't conserved, then we believe that there is some outside torque acting on the system. By expanding the boundary of the system to include the source of that torque we can always preserve the Law of Angular Momentum Conservation.

In this unit you will test the notion of the conservation of angular momentum. As in the test of the conservation of linear momentum, we will investigate what happens when two bodies undergo a "rotational" collision. You will drop a large weight onto a rotating disk and determine the moment of inertia, the angular speed, and finally, the angular momentum of the rotator-disk-weight system before and after this perfectly inelastic collision.

Activity 1: The Moment of Inertia Before and After the Collision

(a) Calculate the theoretical value of the rotational inertia of the metal disk using basic measurements of its radius and mass. Be sure to state units and show the expression you used!

$r_d =$ $M_d =$

$I_d =$

(b)The rotating fixture that holds the disk has a complex shape. We have determined its moment of inertia without the disk and recorded the result. Record that value here. Be sure to state units.

$I_f =$

(c) After dropping the weight on the rotating disk, the system will have a new moment of inertia. Derive a formula for the moment of inertia of a disk-shaped weight of mass m_w and radius r_w revolving about the origin at a distance, r_r. (You will have to use the parallel axis theorem to do this.) Measure the mass of the weight and use a vernier caliper to measure its diameter.

$I_w =$

$m_w =$ $r_w =$

(d) Come up with a formula for the rotational inertia, I, of the whole system before and after the collision and calculate the moment of inertia before the collision only. (The moment of inertia after the collision will be determined after you do the experiment.) Don't forget to include the units.

$I_{\text{before}} =$

$I_{\text{after}} =$

Activity 2: Measurement of Angular Acceleration

(a) Place the video camera about 1 m above the rotator, align the camera with the center of the rotator using the pendulum, and center the rotator in the field of view of the camera by viewing it with the *VideoPoint Capture* software. Place a ruler of known length in the field of view of the camera and parallel to one side of the frame. Check that the rotator is flat with the small water-bubble level.

(b) Give the rotator a push and begin recording its motion with the video camera. See **Appendix D: Video Analysis** for details. While the rotator is moving hold the 1 kg weight near the rim of the metal disk and close to, but not quite touching, the surface of the moving metal disk. After at least one revolution of the metal disk drop the 1-kg mass onto the disk and record the motion of the disk for at least one revolution afterward.

(c) Measure the distance of the center of the weight you dropped from the center of the rotator r_r. To do this, measure the distance from the center of the rotator to the edge of the weight and use the result from Activity 1 part (c) for the diameter of the weight. Calculate the distance from the origin to the center of the weight. Use these results and those from Activity 1 part (d) to calculate the final moment of inertia.

$r_{\text{edge}} =$ $r_w =$ $r_r =$

$I_{\text{after}} =$

(d) Determine the angular speed before and after the collision. To do this task see the instructions in **Appendix D: Video Analysis** for creating and analyzing a movie file.

1. Find the last frame before you dropped the weight on the rotator and click on the position of the white marker on the metal disk. Under the **Edit** menu highlight **Leave/Hide Trails**. Now go backward through the film until the rotator has gone through one full rotation. Estimate to the nearest fraction of a frame how many frames there are in one revolution. You also need to know the time between frames Δt_{frame}, which you can get from the data table in *VideoPoint*.

   $N_{\text{before}} =$ $\Delta t_{\text{frame}} =$

   Calculate the time for one revolution before the collision and the angular speed.

   $t_{\text{before}} =$ $\omega_{\text{before}} =$

2. We now follow a similar procedure to determine the angular speed after the collision. Under the **Edit** Menu highlight **Clear All...** to get rid of your previous results. Find the first frame after you dropped the weight on the rotator and click on the position of the white marker. Now click forward and estimate to the nearest fraction of a frame the number of frames in one full revolution.

   $N_{\text{after}} =$ $\Delta t_{\text{frame}} =$

   Calculate the time for one revolution and the angular speed after the collision.

   $t_{\text{after}} =$ $\omega_{\text{after}} =$

(e) Calculate the angular momentum before and after the collision. Calculate the percent difference between the two results. Is angular momentum conserved?

$L_{\text{before}} =$

$L_{\text{after}} =$

(f) Would the procedure you followed above change if the weight was moving horizontally at a constant velocity when you dropped it? If it changed, what would be different?

49 Hooke's Law

Name ______________________ Section __________ Date __________

Objectives:

- To explore the nature of elastic deformation and restoring forces

Introduction

There is no such thing as a perfectly rigid body. The stiffest of metal bars can be twisted, bent, stretched, and compressed. Delicate measurements show that even small forces cause these distortions. Under certain circumstances (typically, when the forces are not too large), a body deformed by forces acting upon it will return to its original size and shape when the forces are removed, a capacity known as elasticity. Permanent distortion from large forces is referred to as plastic deformation. In this lab, you will stay within the elastic limit.

Apparatus:

- two springs and supports
- collection of masses
- 2-meter stick

Activity:

1. Suspend one of the springs from the support. Using the meter stick, observe the position of the lower end of the spring and record the value in the table below.
2. Hang 100 grams from the lower end of the spring and again record the position of this end.
3. Repeat 3 with loads of 200, 300, 400, and 500 grams hung from the spring.
4. Repeat 2, 3 and 4 with the second spring.

Mass suspended from spring 1 (g)	Position reading (m)	Elongation (m)	Mass suspended from spring 2 (g)	Position reading (m)	Elongation (m)
0		0	0		0
100			100		
200			200		
300			300		
400			400		
500			500		

5. Determine the elongation produced by each load.
6. Plot a curve using the values of the elongation as the abscissas and the forces due to the corresponding loads as ordinates for each spring. Make sure you use compatible units.

Questions:

1. What does your curve show about the dependence of each spring's elongation upon the applied force?

2. What are the units of the proportionality constant?

3. The slope of the lines is known as the force constant, k. Which spring has the larger force constant? Express in your own words the physical implications of different size force constants.

50 Periodic Motion

Name ______________________ Section ____________ Date ____________

Objectives

- To learn directly about some of the characteristics of periodic motion period, frequency, and amplitude.
- To investigate the relationships between position, velocity, acceleration, and force in simple harmonic motion.
- To investigate energy in simple harmonic motion.

Introduction

Periodic motion is motion that repeats itself. You can see the repetition in the position-, velocity-, or acceleration-time graphs. The length of time to go through one cycle and begin to repeat the motion is called the period. The number of cycles in each second is called the frequency. The unit of frequency, cycles per second, is given a special name — Hertz.

Motion of a Mass Hanging from a Spring

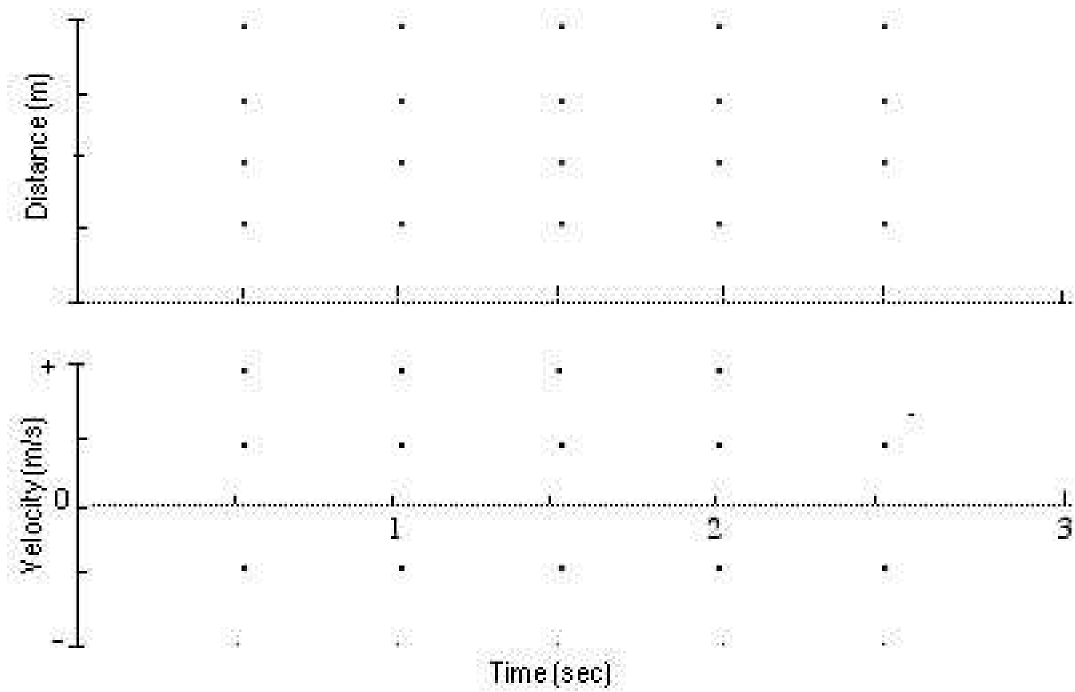
In this unit you will investigate the motion of a mass hanging from a spring.

Apparatus

- Large spring
- Force probe
- Motion detector
- Variety of masses
- *Science Workshop 750 Interface*
- *DataStudio* software (Position & Velocity Graphs application and SHM application)

Activity 1: Periodic Motion of a Mass-Spring System

(a) Open the Position & Velocity Graphs application. Hang the large spring from the force probe hook with the large diameter coils down and hang a 200-g mass from the spring. Place the motion detector facing up directly below the spring. Push the mass straight upward 15 cm, and let go. Adjust the height of the support so that the mass comes no closer than 0.15 m to the detector. Record data for a few seconds to display position-time and velocity-time graphs of the motion. Sketch the graphs on the axes below.



Comment: Note that when an object returns to the same position, it does not necessarily mean that a cycle is ending. It must return to the same position, and the velocity and acceleration must also return to the same values in both magnitude and direction for this to be the start of a new cycle.

(b) Label the graphs above with: "B" at the Beginning of a cycle and "E" at the End of the same complete cycle. "A" on each spot where the mass is moving Away from the detector fastest. "T" on each spot where the mass is moving Toward the detector fastest. "S" on each spot where the mass is standing Still. "F" where the mass is Farthest from the motion detector. "C" where the mass is Closest to the motion detector.

(c) Do the position and velocity graphs appear to have the same period? Do their peaks occur at the same times? If not, how are the peaks related in time?

(d) Use the Smart Tool to measure the period and frequency of the motion. (For better accuracy, measure the total time over as many cycles as possible and divide by the number of cycles.)

(e) Using the Smart Tool, determine and record the maximum displacement. Then record data with the mass at rest to find the equilibrium position. Draw a straight line on your position graph indicating the equilibrium position in terms of the distance from the motion detector. Calculate and record the amplitude of the motion (the difference between the maximum displacement and the equilibrium position).

Simple Harmonic Motion

The motion of a mass hanging from a spring that you looked at in Activity 1 is a close approximation to a kind of periodic motion called simple harmonic motion (sometimes abbreviated SHM).

Activity 2: What Factors Determine the Period of the Mass-Spring System?

What can you do to change the period of the SHM of the mass-spring system? What will happen to the period if you increase the amplitude? Increase the mass? Increase the spring constant (use a stiffer spring)?

Activity 3: The Period of SHM and the Amplitude

(a) Repeat the procedure of Activity 1, but with a different starting position (other than 15 cm). (Warning: Do not make the amplitude so large that the mass comes closer than 0.15 m from the motion detector.) When you have good graphs, find and record the period and the amplitude using the methods described in Activity 1.

(b) Take ratios of the period and the amplitude of Activity 1 to those determined here.

(c) Is there evidence that the period depends on amplitude? (Did the change in amplitude result in a comparable change in period?) Explain. How does this compare with your prediction?

Activity 4: The Dependence of the Period of a SHM on the Mass

(a) Carefully measure the period for two other masses. Record the masses and the measured periods in a table in the space below along with the mass and period from Activity 1.

(b) Does the period depend on the mass? Does it increase or decrease as mass is increased?

(c) Determine the mathematical relationship between the period T and the mass m by finding a function that fits the data. Write the equation that provides the best fit to the data in the space below.

Comment: You should have found that T is independent of amplitude and proportional to $\sqrt{m}$. The actual expression for the period is

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Velocity, Acceleration, Force and Energy

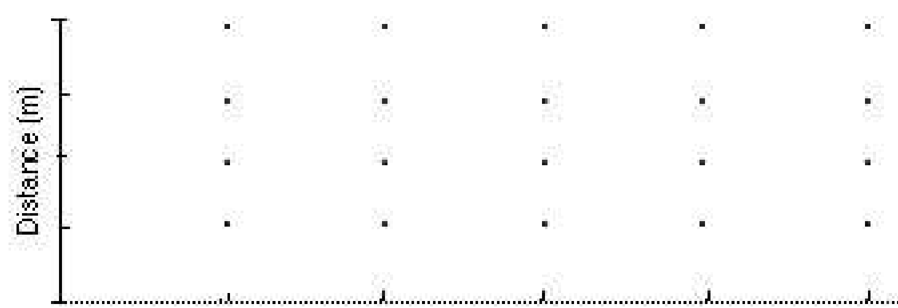
In this investigation you will look more carefully at the distance, velocity and acceleration graphs for simple harmonic motion. You will also look at the force graph, and will examine the energy associated with simple harmonic motion.

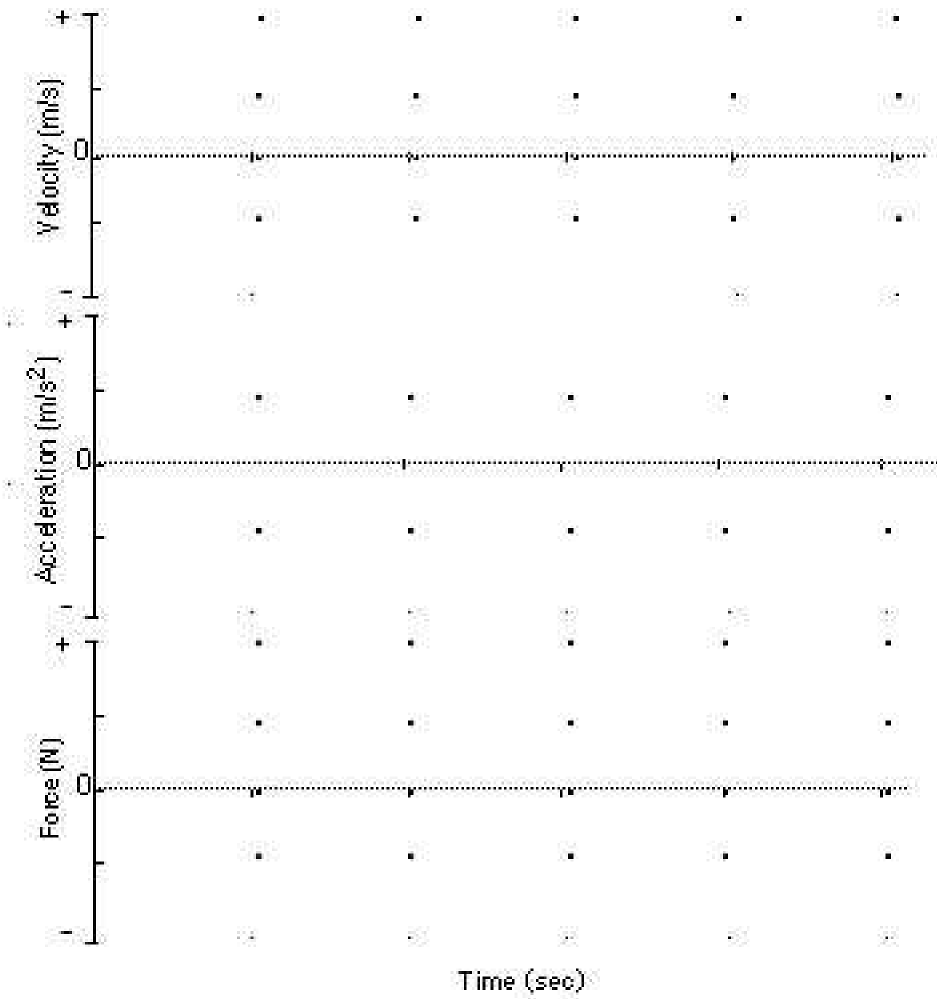
Activity 5: Determination of the Spring Constant

Measure the distance the spring stretches for four different masses and use these data to determine the spring constant, k. Record your data and the result for k in the space below.

Activity 6: Velocity, Acceleration, and Force for SHM

(a) Consider the motion you looked at in Activity 1 when the mass was 200 g and the initial position was 15 cm. Sketch the position and velocity graphs that you observed on the axes below using dashed lines.





(b) Based on what you know about the relationships between velocity, acceleration, and force, use dashed lines to sketch your predictions for the acceleration and force graphs.

(c) Suspend the 200-g mass from the spring and open the SHM application. Start the mass oscillating with an amplitude of 15 cm and record data for a few seconds. When you have obtained good graphs, sketch the results on the above axes using solid lines.

(d) When the mass is at its maximum distance from the detector, is the velocity maximum, minimum or some other value according to your graphs? Does this agree with your predictions? Does this agree with your observations of the oscillating mass? Explain.

(e) When the mass has its maximum positive velocity, is its distance from the detector maximum, minimum, the equilibrium value or some other value according to your graphs? What about when it reaches maximum negative velocity? Does this agree with your predictions? Does this agree with your observations of the oscillating mass? Explain.

(f) According to your graphs, for what distances from the detector is the acceleration maximum? For what distances is the acceleration zero? What is the velocity in each of these cases?

(g) Compare the force and acceleration-time graphs. Describe any similarities. Does the force graph agree with your prediction?

(h) From your graphs, what would you say is the relationship between force and acceleration?

(i) Compare the force and distance(position)-time graphs. What would you say is the relationship between force and position?

Activity 7: Energy of a Mass Undergoing SHM

Now use the graphs from the last activity to examine the energy relationships in simple harmonic motion.

(a) At what points is the kinetic energy of the mass zero? Label these points on your distance and velocity graphs above with a K.

(b) Calculate the elastic potential energy due to the spring at one of these points. Label the point you use on your velocity and distance graphs with a 1. Use $U = \frac{1}{2}kx^2$, where x is the distance from the equilibrium position and k is the force constant of the spring, which you have already measured. Use the Smart Tool to measure x. Show your data and calculations in the space below.

(c) At what points is the potential energy zero? Label these points with a P on your distance and velocity graphs.

(d) If you measured the kinetic energy at one of these points, what would you expect its value to be? Explain.

(e) Check your prediction. Calculate the kinetic energy at one of these points. Label the point you use on your velocity graph with a 2. Use $K = \frac{1}{2}mv^2$. Use the Smart Tool to determine v. Show your data and calculations in the space below.

(f) Did your calculated kinetic energy agree with your prediction?

(g) If you calculated the potential and kinetic energies at a point where neither of these was zero, what would you expect the total energy to be? Explain.

(h) Check your prediction. Pick a point where the mass has both kinetic and potential energy and calculate them both. Label this point on your distance and velocity graphs with a 3. Show your calculations.

(i) Does your result agree with your prediction? Does it appear that energy is conserved? Explain.

51 The Period of a Pendulum

Name ______________________ Section ____________ Date ____________

Objectives:

- Study the influences on the motion of a simple pendulum
- Calculate the acceleration due to gravity from measurements of the period and length of a simple pendulum

Introduction:

The period of a simple pendulum is related to its length and the acceleration due to gravity according to the relationship:

$$T = 2\pi\sqrt{\frac{L}{g}}. \qquad [Eq.\,1]$$

T is the period, L is the length, and g is the gravitational acceleration. This assumes the oscillations are small. Let's check this prediction experimentally.

Apparatus:

- string attached to stand
- collection of masses
- stop watch
- meter stick

Activity:

1. With L = 1.0 m, place a 1-kg mass at the end of your pendulum. Time twenty-five (25) oscillations of amplitude not greater than 10 degrees. The period is the total time divided by the number of oscillations. Calculate the period and period squared, and enter the relevant data into the table below.
2. Repeat 1 for masses of 500 g, 200 g, 100 g, 50 g, and 20 g.

Trial No.	Mass (kg)	Length[L] (m)	No. of Oscillations	Total Time (s)	Period[T] (s)
1					
2					
3					
4					
5					
6					

3. With the 200-g mass, fix the length L to be 1.5 m. Time twenty-five (25) oscillations of amplitude not more than 10 degrees. Calculate the period and period squared, and enter the relevant data into the table below.
4. Repeat 3 for pendulum lengths of 1.0, 0.7, 0.4, 0.25, and 0.15 meters.

Trial No.	Mass (kg)	Length[L] (m)	No. of Oscillations	Total Time (s)	Period[T] (s)	T^2 (s^2)
7						
8						
9						
10						
11						
12						

5. Plot T versus mass and T^2 versus L on SEPARATE graphs. NOTE: Be sure the T versus mass graph contains the origin. (If you don't know how to do this, consult your instructor.) Fit the data and determine the slopes of the lines of each graph. Be sure to include UNITS with each slope.

slope: period versus mass ______________________

slope: period^2 versus length ______________________

Questions:

1. Interpret the slope of the period versus mass line: What is the relationship between mass and period? How does the period depend on the mass?

2. Interpret the slope of the period^2 versus length line: What is the relationship between length and period? How does period depend on pendulum length?

3. If the length of the pendulum were $\frac{1}{16}$ its original length, by how much would its period change?

4. Using the relationship between length and period (equation 1) and the slope you measured for the T^2 vs L graph, determine the acceleration due to gravity g. Calculate the percent difference between your value and the accepted value of [9.8 $\frac{\mathrm{m}}{\mathrm{s}^2}$].

52 Resonance in Tubes

Name ______________________ Section ____________ Date ____________

Objectives:

- Determine the resonant frequency for a tube open at one end.
- Determine tube lengths at resonance for a tube of variable length.
- Determine the velocity of sound in air in the laboratory (two ways).

Introduction

The Economy Resonance Tube is designed for the study of resonance in columns of air. The tube set includes a movable inner tube with a closed end and an outer tube which is open at both ends. The inner tube also includes a measuring tape to easily find the length of the air column. To adjust the length of the closed tube, simply slide the inner tube until the desired length appears on the measuring tape. Open tube experiments can also be performed by removing the inner tube.

In order that the tube resonate, the frequency of the vibrating air must coincide with the natural frequency of the tube (which may be its fundamental or one of its overtones). For the Economy Resonance Tube, which is closed at one end, this requirement is met if the tube length is an odd number of quarter wavelengths of the sound waves produced by the source ($L = \lambda/4, 3\lambda/4, 5\lambda/4$, etc., where L is the length of the tube and λ is the wave length of the sound). Note that if the length of the tube is gradually increased while the source is vibrating, the distance between successive resonance positions is $\lambda/2$.

Note: Due to edge effects at the open end of a tube, the effective length of the tube depends on the radius of the opening. Thus, $L_{eff} = L + 0.6r$, where L_{eff} is the *effective* length, L is the length measured, and r is the tube radius.

Apparatus:

- Economy Resonance Tube
- Open speaker
- Sine wave generator
- 2 banana plug leads
- Sound sensor
- Meter stick
- Data Studio 750 Interface
- Thermometer

Room Temperature (°C) ______________________ Tube radius (m) ______________________

Activity 1: Fixed tube length

1. Connect the open speaker to the sine wave generator using standard banana plug leads.
2. Adjust the length of the tube to 50 cm (check with meter stick).
3. Place the tube in front of the speaker in such a way that the tube is open at one end (the speaker can be set at an angle relative to the tube length).
4. Set the sound sensor inside the tube and connect it to the Data Studio interface.
5. To activate the sound sensor, perform the following sequence: Start up *DataStudio* by going to *Start* → *Programs* → *Physics Applications* → *DataStudio*. Click on *Create Experiment*, then *Setup*, then *Add Sensor or Instrument*. Scroll down to *Sound level sensor* and select, then click *OK*. Double click *Graph* at left. Click *Start* to begin taking data.

6. Start at a frequency of 50 Hz and increase until you find the frequency of the largest resonance (indicated by a peak on the sound level graph). This is the fundamental frequency. Record the result here:

7. The resonant frequencies for a tube open at one end are given by $f = nv/4L$ where n is an odd integer, v is the velocity of sound and L is the effective tube length. From the fundamental frequency you just found, calculate the velocity of sound in air (using $n = 1$) and record it here:

Activity 2: Fixed frequency

1. Adjust the tube length to 20 cm.
2. Set the speaker inside the open end of the tube so that it is closed at both ends.
3. Set the sine wave generator frequency to 600 Hz (with low amplitude).
4. Slowly move the inner tube to increase the effective length of the tube. Record the length of the tube when resonance is achieved:

5. Increase the length of the tube until three more resonance lengths are found for the constant frequency and record them here:

6. The maxima you have determined are spaced a distance $\lambda/2$ apart, where λ is the wavelength. Find the differences between adjacent resonance lengths and calculate the average of the three values:

7. Find λ from your average value of $\lambda/2$ and calculate the velocity of sound in air from $v = f\lambda$.

8. The velocity of sound in air at 0°C is 331.4 m/s. The temperature dependence of sound velocity in air is given by $v(T) = 331.4 + 0.6T$, where T is in °C and v is in m/s. Calculate an “accepted” value of the velocity of sound in air from this formula.

9. What is the percent difference between your experimental result and the “accepted” value?

53 Galilean Relativity

Name ______________________ Section ____________ Date ____________

Objective

To investigate the relationship between different inertial reference frames and develop the equations of Galilean relativity.

Overview

Before we begin our study of Einstein's special theory of relativity we will first investigate the effects of observing a phenomenon in two different inertial frames of reference. A spring-launched projectile will be "fired" from a moving platform and we will discover how the description of this phenomenon changes(or doesn't change) in a frame that moves along with the platform. You will use the video analysis software and mathematical modeling tools to find the equations that describe the horizontal motion (x vs. t) and the vertical motion (y vs. t) of the projectile in a stationary frame and then in an inertial frame moving at constant speed with the launcher. The set of relationships between the inertial reference frames forms Galilean relativity. These relationships provide an intuitive, "common sense" picture of the world that works well at low velocities, but fails in many surprising ways at high velocities. At high velocities we must resort to Einstein's special theory of relativity that we will discuss later. To do the activities in this unit you will need:

- A video analysis system (*VideoPoint*).
- The film Moving Launcher.
- Graphing and curve fitting software (*Excel*).

Activity 1: Observing Projectile Motion From a Moving Launcher

(a) Use the *VideoPoint* package to analyze the film *Moving Launcher* and determine the position of the projectile in each frame with a fixed origin. To do this task follow the instructions of **Appendix D: Video Analysis** for calibrating the film and analyzing the data. When you are analyzing the movie, place the fixed origin in the first frame of the movie at the dark spot on the cart the launcher rides. This makes the comparison with the activities below easier. Change the origin by performing the following steps.

1. Click on the arrow icon near the top of the menu bar to the left. The cursor will be shaped like an arrow when you place it on the movie frame.
2. Click at the origin (where the axes cross) and drag the origin to the desired location.
3. Click on the circle at the top of the menu bar to the left to return to the standard cursor for marking points on the film.

(b) Collect the data for analysis by following the instructions in **Appendix D**. The data table should contain three columns with the values of time, x-position, and y-position. Print the data table and attach it to this unit.

(c) Determine the position of the launcher at the first and last frames of the movie. Using these results, what is the horizontal and vertical speed of the launcher during the movie?

$x_0 =$ $\qquad$ $x_1 =$ $\qquad$ $\Delta t =$ $\qquad$ $v_{x\ launcher} =$

$y_0 =$ $\qquad$ $y_1 =$ $\qquad$ $v_{y\ launcher} =$

(d) Plot and fit the position versus time data for the horizontal and vertical positions of the projectile. See **Appendix C: Introduction to *Excel*** for details. Print each plot and attach it to this unit. Record the equation of each fit here. Be sure to properly label the units of each coefficient.

$x(t) =$

$y(t) =$

What is the horizontal speed of the projectile? How did you determine this?

Activity 2: Changing Reference Frames

(a) We now want to consider how the phenomenon we just observed would appear to an observer that was riding along on the launcher at a constant speed. Assume the moving observer places her origin at the same place you put your origin on the first frame of the movie. Predict how each graph will change for the moving observer.

Horizontal position versus time:

Vertical position versus time:

(b) Use the *VideoPoint* package to analyze the film *Moving Launcher* again and determine the position of the projectile in each frame. This time, though, use a moving origin that is placed at the same point on the cart on each frame. Use the same point on the launcher that you used to define the origin in the first frame during the previous activity. To change the origin from frame to frame follow these instructions.

1. Open the movie as usual and enter one object to record. First you must select the existing origin and change it from a fixed one to a moving one. Click on the arrow near to top of the menu bar to the left. The cursor will have the shape of an arrow when you place it on the movie frame. Click on the existing origin (where the axes cross) and it will be highlighted.
2. Under the **Edit** menu drag down and highlight **Edit Selected Series**. A dialog box will appear. Click on the box labelled **Data Type** and change the selection to **Frame-by-Frame**. Click OK.
3. Click on the circle at the top of the menu bar to the left to change the cursor back to the usual one for marking points. Go to the first frame of interest. When the cursor is placed in the movie frame it will be labelled with "Point S1". Click on the object of interest. The film will NOT advance and the label on the cursor will now be "Origin 1". Click on the desired location of the origin in that frame. The film will advance as usual. Repeat this procedure to accumulate the x- and y-positions relative to the origin you've defined in each frame.

(c) Collect the data for analysis. The data table should contain four columns with the values of time, x-position, and y-position and the position of the origin. Print the data table and attach it to this unit.

(d) Plot and fit the position versus time data for the horizontal and vertical positions. See **Appendix C: Introduction to *Excel*** for details. When you have found a good fit to the data, record your result below, print the graph, and attach a copy to this unit. Be sure to properly label the units of each coefficient in your equation.

$x(t) =$

$y(t) =$

What is the horizontal speed of the projectile? How did you determine this?

Activity 3: Relating Different Reference Frames

(a) Compare the two plots for the vertical position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected "prediction" here.

(b) Compare the two plots for the horizontal position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected "prediction" here.

(c) What is the difference between the horizontal velocities in the two reference frames? How does this difference compare with the horizontal velocity of the launcher? How are the horizontal velocities of the projectile in each inertial reference frame and the velocity of the launcher that you determined above related to one another? Does this relationship make sense? Why or why not?

(d) Consider a point $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ on the ball's trajectory in the stationary observer's reference frame. If the moving observer's time frame is moving at the speed $v_{x\ launcher}$ then what would the moving observer measure for x? Call this horizontal position of the moving observer x'.

(e) What would the moving observer measure for y? Call this vertical position of the moving observer y'.

(f) The relationships you found above are from Galilean relativity. You should have obtained the following results.

$$x' = x - v_{x\ launcher}t$$

$$y' = y$$

$$v_x = v'_x + v_{x\ launcher}$$

The primes refer to measurements made in the moving frame of reference in this case. If you did not get these expressions consult your instructor.

Activity 4: Testing Galilean Relativity

You can test your mathematical relationships with *Excel*. You will use your data for the stationary observer and the relationship you derived to calculate what the observer moving with the launcher would measure.

In your data table for the stationary observer, you have columns giving x, the horizontal position, as a function of time, t. Using an *Excel* formula, apply the Galilean transformation above to these numbers to determine x', the horizontal position as measured by the moving observer. Make a plot showing x' as a function of t.

Your "transformed" data for the stationary observer should closely resemble the results for the moving observer. Is this what you observe? If not, consult your instructor. Print and attach a copy of your plot to this unit.

54 The Twins Paradox

Name ____________________ Section __________ Date __________

Objective

To investigate some of the unusual implications of Einstein's special theory of relativity.

Overview

Einstein's theory of special relativity leads to a variety of apparent paradoxes that depart radically from our everyday expectations. One of the most celebrated is the twins paradox, in which an identical twin makes a long interstellar journey while the other twin remains on the (roughly) stationary Earth. When the space-faring twin returns she finds her partner has aged considerably more than she has. In this unit you will explore the quantitative aspects of the paradox and some of the surprising consequences.

Activity 1: Setting Things Up

Problems in special relativity are often very counterintuitive, so it is instructive to consider the situation non-relativistically first. Investigate this problem without applying any of the new ideas you have learned about the theory of special relativity.

One member of a pair of identical twins has decided to embark on a long space voyage. The two twins have lived their lives in close proximity to one another and are very similar in appearance. The adventurous twin boards a fast spacecraft and leaves the Earth behind at a speed of 0.99c or 99% of the speed of light. The space-faring twin's itinerary is rather monotonous, and she simply travels at this constant speed for a time, turns around, and returns to the Earth at the same speed. She measures the time of her trip to be Δt_0. In the meantime the Earth-bound twin has seen twenty years pass by. We will refer to this time as Δt.

(a) In mathematical terms, what is the relationship between the times Δt_0 and Δt?

(b) Which time is associated with which twin?

(c) When the twins are reunited will their appearances differ?

Activity 2: Applying Special Relativity

(a) Now we want to apply the lessons of special relativity. Time dilation implies that moving clocks run more slowly when observed by someone in a different inertial frame. For the twins paradox what does this imply about the time interval the space-faring twin measures during her trip? Will it be less than, equal to, or greater than the interval measured by the Earth-bound twin? Will the space-faring twin age more, less, or the same amount as the Earth-bound twin?

(b) What is the mathematical relationship between Δt_0 and Δt according to the special theory of relativity?

(c) How much time has passed on the Earth-bound twin's clock?

(d) How much time has passed on the space-faring twin's clock?

(e) If this result is inconsistent with your prediction above how should you resolve the contradiction?

(f) How will the two twins' appearances differ, if at all? Is the difference only in the measurement of the time intervals or are there real physiological differences between the twins after the trip?

(g) If the average speed of the space-faring twin was more like the typical orbital speed of the space shuttle (about 7.4 km/s) what would the time difference between the twins' clocks be?

Activity 3: Graphical Analysis

(a) Find a mathematical relationship for the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. Show your work and record your result here.

(b) You will now use *Excel* to make a plot showing how this ratio behaves as a function of β (the speed expressed as a fraction of the speed of light). To do this, start up *Excel*, and create a column headed "beta." This column should contain the series of numbers 0,0.05,0.10,0.15,. . .,1. To create such a column of numbers, enter the first two rows and highlight them. Then grab the lower-right corner of the second cell with the mouse and drag down (in the same way as if you were dragging down a formula).

After you have created the β column, create a second column containing the ratio of time intervals (i.e., the relationship you found in part (a)). Use an *Excel* formula.

(c) Make a plot of the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. At what speed does the effect of time dilation become significant? Is there a limit to the ratio? Is there any reason to restrict the range of β to 0-1? Clearly state your reasoning. Print your plot and attach a copy to this unit.

(d) Consider the following scenario. As the space-faring twin's craft recedes from the Earth it is moving at a constant speed. Since no inertial frame can be considered "better" than any other there is nothing physically inconsistent with the view that the space-faring twin is observing the Earth recede from her at a constant velocity. Hence, the space-faring twin will observe clocks on the Earth to move slowly and the Earth-bound twin will age at a slower rate than the space-faring one. Is this reasoning flawed? How?

(e) If the scenario is not flawed how can it be that the space-faring twin was found to have aged less in the original problem?

A Treatment of Experimental Data

Recording Data

When performing an experiment, record all required original observations as soon as they are made. By "original observations" is meant what you actually see, not quantities found by calculation. For example, suppose you want to know the stretch of a coiled spring as caused by an added weight. You must read a scale both before and after the weight is added and then subtract one reading from the other to get the desired result. The proper scientific procedure is to record both readings as seen. Errors in calculations can be checked only if the original readings are on record.

All data should be recorded with units. If several measurements are made of the same physical quantity, the data should be recorded in a table with the units reported in the column heading.

Significant Figures

A laboratory worker must learn to determine how many figures in any measurement or calculation are reliable, or "significant" (that is, have physical meaning), and should avoid making long calculations using figures which he/she could not possibly claim to know. *All sure figures plus one estimated figure are considered significant.*

The measured diameter of a circle, for example, might be recorded to four significant figures, the fourth figure being in doubt, since it is an estimated fraction of the smallest division on the measuring apparatus. How this doubtful fourth figure affects the accuracy of the computed area can be seen from the following example.

Assume for example that the diameter of the circle has been measured as $.526\underline{4}$ cm, with the last digit being in doubt as indicated by the line under it. When this number is squared the result will contain eight digits, of which the last five are doubtful. Only one of the five doubtful digits should be retained, yielding a four-digit number as the final result.

In the sample calculation shown below, each doubtful figure has a short line under it. Of course, each figure obtained from the use of a doubtful figure will itself be doubtful. The result of this calculation should be recorded as 0.2771 cm^2, including the doubtful fourth figure. (The zero to the left of the decimal point is often used to emphasize that no significant figures precede the decimal point. This zero is not itself a significant figure.)

$$(.526\underline{4}\ \text{cm})^2 = .277\underline{09696}\ \text{cm}^2 = 0.277\underline{1}\ \text{cm}^2$$

In multiplication and division, the rule is that a calculated result should contain the same number of significant figures as the least that were used in the calculation.

In addition and subtraction, do not carry a result beyond the first column that contains a doubtful figure.

Statistical Analysis

Any measurement is an intelligent estimation of the true value of the quantity being measured. To arrive at a "best value" we usually make several measurements of the same quantity and then analyze these measurements statistically. The results of such an analysis can be represented in several ways. Those in which we are most interested in this course are the following:

<u>Mean</u> - The mean is the sum of a number of measurements of a quantity divided by the number of such measurements. (In other words, the mean is the same thing as what people generally call the "average.") It generally represents the best estimate of true value of the measured quantity.

<u>Standard Deviation</u> - The standard deviation (σ) is a measure of the range on either side of the mean within which approximately two-thirds of the measured values fall. For example, if the mean is 9.75 m/s^2 and the standard deviation is 0.10 m/s^2, then approximately two-thirds of the measured values lie within the range 9.65 m/s^2 to 9.85 m/s^2. A customary way of expressing an experimentally determined value is: Mean$\pm\sigma$, or (9.75$\pm$ 0.10) m/s^2. Thus, the standard deviation is an indicator of the spread in the individual measurements, and a small σ implies high precision. Also, it means that the probability of any future measurement falling in this range is approximately two to one. The equation for calculating the standard deviation is

$$\sigma = \sqrt{\frac{\sum (x_i - \langle x \rangle)^2}{N-1}}$$

where x_i are the individual measurements, $\langle x \rangle$ is the mean, and N is the total number of measurements.

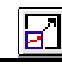
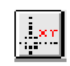
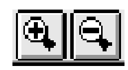
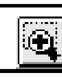
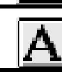
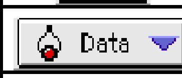
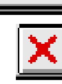
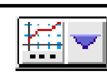
% Difference - Often one wishes to compare the value of a quantity determined in the laboratory with the best known or "accepted value" of the quantity obtained through repeated determinations by a number of investigators. *The % difference is calculated by subtracting the accepted value from your value, dividing by the accepted value, and multiplying by 100.* If your value is greater than the accepted value, the % difference will be positive. If your value is less than the accepted value, the % difference will be negative. The % difference between two values in a case where neither is an accepted value can be calculated by choosing either one as the accepted value.

B Introduction to DataStudio

Quick Reference Guide

Shown below is the quick reference guide for DataStudio.

What You Want To Do	How You Do It	Button
Start recording data	Click the 'Start' button or select 'Start Data' on the Experiment menu (or on the keyboard press CTRL - R (Windows) or Command - R (Mac))	
Stop recording (or monitoring) data	Click the 'Stop' button or select 'Stop Data' on the Experiment menu (or on the keyboard press CTRL - . (period) (Win) or Command - . (Mac))	
Start monitoring data	Select 'Monitor Data'on the Experiment menu (or on the keyboard press CTRL - M (Win) or Command - M (Mac))	**none**

On the Graph Display	In the Graph Toolbar	Button
Re-scale the data so it fills the Graph display window	Click the 'Scale to Fit' button.	
Pinpoint the x- and y-coordinate values on the Graph display	Click the 'Smart Tool' button. The coordinates appear next to the 'Smart Tool'.	
'Zoom In' or 'Zoom Out'	Click the 'Zoom In' or 'Zoom Out' buttons.	
Magnify a selected portion of the plotted data	Click the 'Zoom Select' button and drag across the data section be to magnified.	
Create a Calculation	Click the 'Calculate' button	
Add a text note to the Graph	Click the 'Note' button.	
Select from the Statistics menu	Click the Statistics menu button	
Add or remove a data run	Click the 'Add/Remove Data' menu button	
Delete something	Click the 'Delete' button	
Select Graph settings	Click the 'Settings' menu button	

Selecting a Section of Data

1. To select a data section, hold the mouse button down and move the cursor to draw a rectangle around the data of interest. The data in the region of interest will be highlighted.

2. To unselect the data, click anywhere in the graph window.

Fitting a Section of Data

1. Select the section of data to be fitted.

2. Click on the **Fit** button on the Graph Toolbar and select a mathematical model. The results of the fit will be displayed on the graph.
3. To remove the fit, click the **Fit** button and select the checked function type.

Finding the Area Under a Curve

1. Use the **Zoom Select** button on the Graph Toolbar to zoom in around the region of interest in the graph. See the quick reference guide above for instructions.
2. Select the section of data that you want to integrate under.
3. Click the **Statistics** button on the Graph Toolbar and select **Area**. The results of the integration will be displayed on the graph.
4. To undo the integration, click on the **Statistics** button and select **Area**.

C Introduction to Excel

Microsoft Excel is the spreadsheet program we will use for much of our data analysis and graphing. It is a powerful and easy-to-use application for graphing, fitting, and manipulating data. In this appendix, we will briefly describe how to use Excel to do some useful tasks.

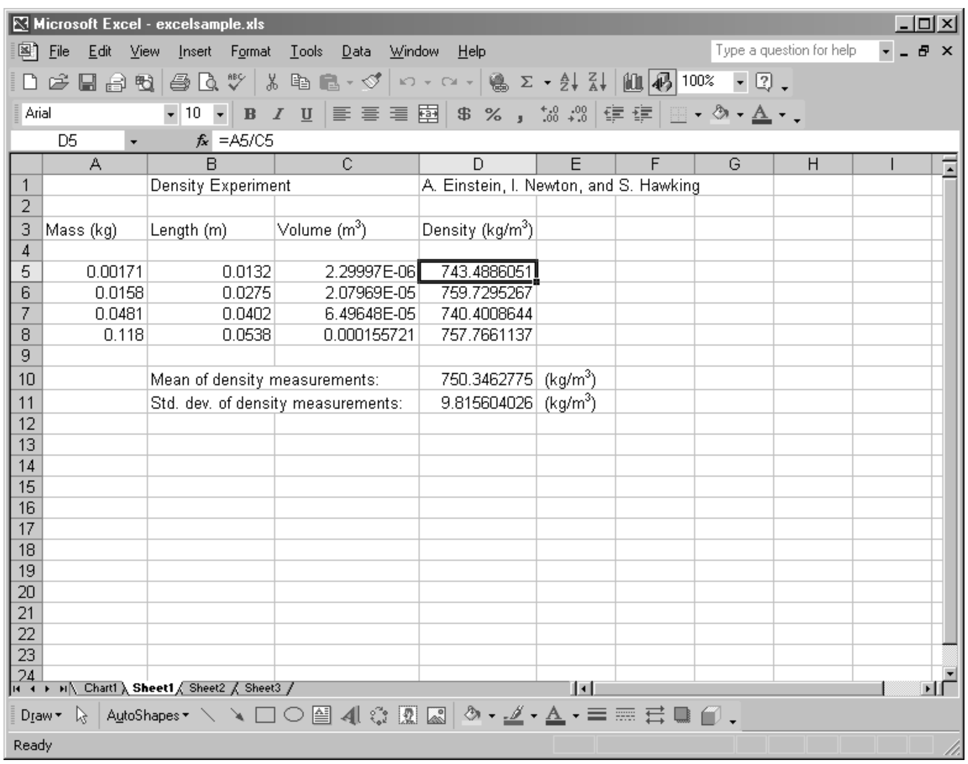
C.1 Data and formulae

The figure below shows a sample Excel spreadsheet containing data from a made-up experiment. The experimenter was trying to measure the density of a certain material by taking a set of cubes made of the material and measuring their masses and the lengths of the sides of the cubes. The first two columns contain her measured results. **Note that the top of each column contains both a description of the quantity contained in that column and its units.** You should make sure that all of the columns of your data tables do as well. You should also make sure that the whole spreadsheet has a descriptive title and your names at the top.

In the third column, the experimenter has figured out the volume of each of the cubes, by taking the cube of the length of a side. To avoid repetitious calculations, she had Excel do this automatically. She entered the formula "=B3∧3" (without the quotes) into cell C3. Note the equals sign, which indicates to Excel that a formula is coming. The ∧ sign stands for raising to a power. After entering a formula into a cell, you can grab the square in the lower right corner of the cell with the mouse and drag it down the column. This will copy the cell, making the appropriate changes, into the rest of the column. For instance, in this case, cell C4 contains the formula "=B4∧3," and so forth.

Column D was similarly produced with a formula that divides the mass in column A by the volume in column C.

At the bottom of the spreadsheet we find the mean and standard deviation of the calculated densities (that is, of the numbers in cells D3 through D6). Those are computed using the formulae "=average(D3:D6)" and "=stdev(D3:D6)".



C.2 Graphs

Making graphs in Excel is relatively easy. First, use the mouse to select the columns of numbers you want to graph. (If the two columns aren't next to each other, select the first one, then hold down the control key while selecting the second one.) Then click on the "chart wizard" button (which looks like this ).

There are a number of different styles of graphs that the chart wizard can generate. In nearly every case, you will want to select an "XY (scatter)" graph. Click "Next" to proceed to customize your graph. The most useful customization options come in step 3 of the process. Under "Title," you can put appropriate labels on the x and y axes of your graphs and give the overall graph a descriptive title. **All graphs must have correctly labeled axes (including units).** If the graph contains only one set of data points, you may wish to uncheck the box that says "Show legend": the information in the legend is probably already contained in the title and axis labels, so the legend just takes up space.

Sometimes, you may want to make a graph in Excel where the x column is to the right of the y column in your worksheet. In these cases, Excel will make the graph with the x and y axes reversed. There are at least two ways to fix this problem. The simplest way is to make a copy of the y column in the worksheet and paste it so that it's to the right of the x column. If you don't want to do that, here's another way. In step 2 of the chart wizard, look under the "Series" tab. Click on this icon  next to the place where it says "X values." You can now select which column of data you want to go on the x axis. Do the same thing to select the correct y column.

C.3 Fitting lines and curves

After you've made a graph, you can have Excel draw a straight line or curve that is a best fit to the data. Under the "chart" menu, select "Add trendline." The most common sort of trendline you will add is a linear fit, but you can also have Excel draw other sorts of best-fit curves. Under the "options" tab, you can check a box that causes Excel to display the equation for the line or curve it has drawn. **Excel will not put the correct units on the numbers in this equation, but you should.**

D Video Analysis

Making a Movie

To make a movie, perform the following steps:

1. Start up *Videopoint Capture* by going to *Start* → *Programs* → *Physics Applications* → *VideoPoint* → *VP Capture.*
2. The program will first ask you to choose a file name and location for the video you are going to make. You should choose to put the file on the **Desktop**.
3. Click on the *Capture rate* box and set the capture rate to 30 frames per second.
4. Go to the *Size & Colors* under the *Capture Options* menu, and choose the largest available size for the video.
5. Go to *Preferences* under the *Edit* menu. Check the box that says *Convert Captured File Before Editing.* A dialog box will pop up; select the *Intel Indeo Video 4.5* option and click OK. Click OK **again** to close the Preferences box. (**If you forget to do this step, you won't be able to analyze the video.**)
6. Be sure the camera is at least 1 meter from the object you will be viewing. This constraint is required to reduce the effect of perspective for objects viewed near the edge of the field of view. Set up the camera so that its field of view is centered on the expected region where you will perform the experiment.
7. Focus the camera by rotating the barrel on the outside of the lens until you have a clear picture.
8. Place a ruler or some object of known size in the field of view where it won't interfere with the experiment. The ruler should be the same distance away from the camera as the motion you are analyzing so the horizontal and vertical scales will be accurately determined. It should also be parallel to one of the sides of the movie frame.
9. One member of your group should perform the computer tasks while the others do the experiment.
10. To start recording your video, click *Record.* When you're done, click *Stop.*
11. The next step is to decide how much of the movie to save. Use the slider to step through the movie frame by frame. When you find the first frame you want to save, click *First.* When you find the last frame you want to save, click *Last.* (You may want to save the entire movie, in which case *First* and *Last* really will be the first and last frames. Often, though, there will be "dead" time either at the beginning or the end of the movie, which you might as well cut out before saving.)
12. After you've selected the range of frames you want to save, the button at the lower right should say *Keep.* Make sure that the box next to this button says *All.* (If it says *Double*, change it to *All.*) Then click *Keep* and *Save.* You will see a quick replay of the movie as Videopoint converts and saves it.
13. Click *Open in Video Point.*

Analyzing the Movie

To determine the position of an object at different times during the motion, perform the following steps:

1. *VideoPoint* will request the number of objects you want to track in the movie. Carefully read the instructions for the unit you are working on to find this number. Enter it in the space provided. You will now see several windows. (Note: You may have to move the movie window out of the way to see the other windows.) One contains the movie and has control buttons and a slider along the bottom of the frame to control the motion of the film. Experiment with these controls to learn their function. Another window below the movie frame (labeled Table) contains position and time data and a third window to the right of the frame (labeled Coordinate Systems) describes the coordinate system in use.

2. This is a good time to calibrate the scale. Go to a frame where an object of known size is clearly visible (see item 6 in the previous section). Under the **Movie** menu highlight **Scale Movie**. A dialog box will appear. Enter the length of the object and set **Scale Type** to **Fixed**. Click **Continue**. Move the cursor over the frame and click on the ends of scaling object.

3. You are now ready to record the position and time data. Go to the first frame of interest. Move the cursor over the frame and it will change into a small circle with an attached label. Place the circle over the object of interest in the frame and click. The x and y positions will be stored and the film advanced one frame. Move the circle over the position of the object in the frame and repeat. Continue this process until you have mapped out the motion of the object. If you entered more than one object to keep track of when you opened the movie, then you will click on all those objects in each frame before the film advances.

4. When you have entered all the points you want, go to the **File** menu and select **Export data**. This will allow you to save your data table as an Excel file. Save this file on the desktop (by clicking on the "Open" button, which actually doesn't open anything), and double-click on the saved file to start up Excel. You will now be able to continue your data analysis in Excel.

5. Once you have looked at your data in Excel and made sure everything looks OK, you can quit Videopoint Analysis. If you are sure you have exported your data correctly to Excel, there is no need to save in Videopoint.

Changing the Origin

To change the position of the origin take the following steps.

1. Click on the arrow icon near the top of the menu bar to the left. The cursor will be shaped like an arrow when you place it on the movie frame.

2. Click at the origin (where the axes cross) and drag the origin to the desired location.

3. Click on the circle at the top of the menu bar to the left to return to the standard cursor for marking points on the film.

Using a Moving Coordinate System

To record the position of an object and to change the coordinate system from frame to frame take the following steps.

1. Open the movie as usual and enter one object to record. First we have to select the existing origin and change it from a fixed one to a moving one. Click on the arrow near the top of the menu bar to the left. The cursor will have the shape of an arrow when you place it on the movie frame. Click on the existing origin (where the axes cross) and it will be highlighted.

2. Under the **Edit** menu drag down and highlight **Edit Selected Series**. A dialog box will appear. Click on the box labeled **Data Type** and highlight the selection **Frame-by-Frame**. Click OK.

3. Click on the circle at the top of the menu bar to the left to change the cursor back to the usual one for marking points. Go to the first frame of interest. When the cursor is placed in the movie frame it will be labeled with "Point S1." Click on the object of interest. The film will NOT advance and the label on the cursor will change to "Origin 1." Click on the desired location of the origin in that frame. The film will advance as usual. Repeat the procedure to accumulate the x- and y-positions relative to the origin you've defined in each frame.

E Instrumentation

Introduction

Being both quantitative and experimental, physics is basically a science of measurement. A great deal of effort has been expended over the centuries improving the accuracy with which the fundamental quantities of length, mass, time, and charge can be measured.

It is important that the appropriate instrument be used when measuring. Ordinarily, a rough comparison with a numerical scale, taken at a glance and given in round numbers, is adequate. Increasing precision, though, requires a more accurate scale read to a fraction of its smallest division. The “least count” of an instrument is the smallest division that is marked on the scale. This is the smallest quantity that can be read directly without estimating fractions of a division.

Even at the limit of an instrument’s precision, however, accidental errors — which cannot be eliminated — still occur. These errors result in a distribution of results when a series of seemingly identical measurements are made. The best estimate of the true value of the measured quantity is generally the arithmetic mean or average of the measurements.

Other errors, characteristic of all instruments, are known as systematic errors. These can be minimized by improving the equipment and by taking precautions when using it.

Length Measurement

Three instruments will be available in this class for length measurements: a ruler (one- or two-meter sticks, for example), the vernier caliper, and the micrometer caliper.

The Meter Stick

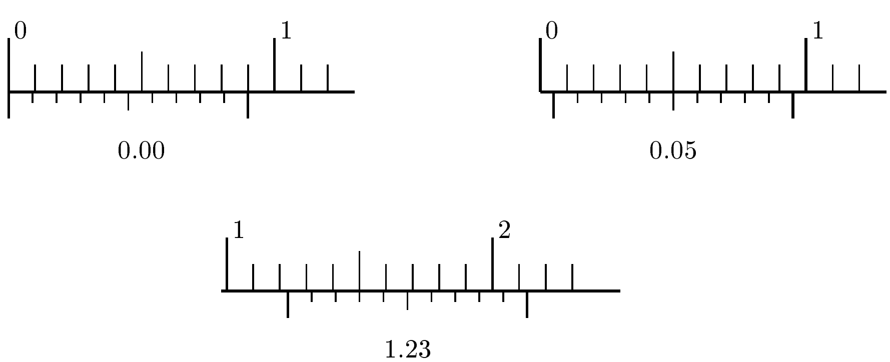
A meter stick, by definition, is 1 meter (m) long. Its scaled is divided, and numbered, into 100 centimeters (cm). Each centimeter, in turn, is divided into 10 millimeters. Thus 1 cm $= 10^{-2}$ m, and 1 mm $= 10^{-1}$ cm $= 10^{-3}$ m.

When measuring a length with a meter stick, different regions along the scale should be used for the series of measurements resulting in an average value. This way, non-uniformities resulting from the meter stick manufacturing process will tend to cancel out and so reduce systematic errors. The ends of the stick, too, should be avoided, because these may be worn down and not give a true reading. Another error which arises in the reading of the scale is introduced by the positioning of the eyes, an effect known as parallax. Uncertainty due to this effect can be reduced by arranging the scale on the stick as close to the object being measured as possible.

The Vernier Caliper

A vernier is a small auxiliary scale that slides along the main scale. It allows more accurate estimates of fractional parts of the smallest division on the main scale.

On a vernier caliper, the main scale, divided into centimeters and millimeters, is engraved on the fixed part of the instrument. The vernier scale, engraved on the movable jaw, has ten divisions that cover the same spatial interval as nine divisions on the main scale: each vernier division is $\frac{9}{10}$ the length of a main scale division. In the case of a vernier caliper, the vernier division length is 0.9 mm. [See figures below.]



Examples of vernier caliper readings

To measure length with a vernier caliper, close the jaws on the object and read the main scale at the position indicated by the zero-line of the vernier. The fractional part of a main-scale division is obtained from the first vernier division to coincide with a main scale line. [See examples above.]

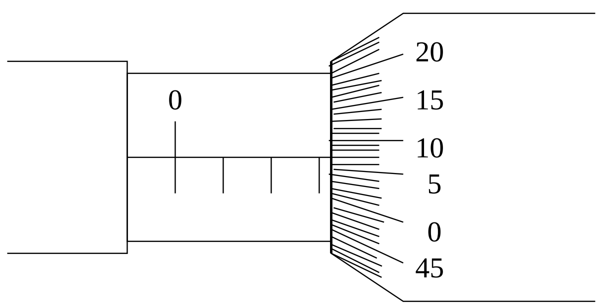
If the zero-lines of the main and vernier scales do not coincide when the jaws are closed, all measurements will be systematically shifted. The magnitude of this shift, called the zero reading or zero correction, should be noted and recorded, so that length measurements made with the vernier caliper can be corrected, thereby removing the systematic error.

The Micrometer Caliper

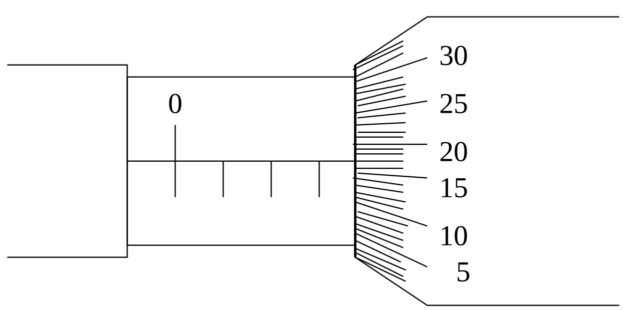
A micrometer caliper is an instrument that allows direct readings to one hundredth of a millimeter and estimations to one thousandth of a millimeter or one millionth of a meter (and, hence, its name). It is essentially a carefully machined screw housed in a strong frame. To measure objects, place them between the end of the screw and the projecting end of the frame (the anvil). The screw is advanced or retracting by rotating a thimble on which is engraved a circular scale. The thimble thus moves along the barrel of the frame which contains the screw and on which is engraved a longitudinal scale divided in millimeters. The pitch of the screw is 0.5 mm, so that a complete revolution of the thimble moves the screw 0.5 mm. The scale on the thimble has 50 divisions, so that a turn of one division is $\frac{1}{50}$ of 0.5 mm, or 0.01 mm.

Advance the screw until the object is gripped gently. Do not force the screw. A micrometer caliper is a delicate instrument.

To read a micrometer caliper, note the position of the edge of the thimble along the longitudinal scale and the position of the axial line on the circular scale. The first scale gives the measurement to the nearest whole division; the second scale gives the fractional part. It takes two revolutions to advance one full millimeter, so note carefully whether you are on the first or second half of a millimeter. The result is the sum of the two scales. (See examples below).



0.307 cm



0.367 cm

As with the vernier caliper, the zero reading may not be exactly zero. A zero error should be checked for and recorded, and measurements should be appropriately corrected.

Mass Measurement

Three kinds of instruments will be available to determine mass: a digital scale and two types of balances. The operation of the first instrument is trivial, and so will not be explained here.

Please understand that with each of these instruments we are really comparing weights, not masses, but the proportionality of weight and mass allows the instruments to be calibrated for mass.

The Equal-Arm Balance

The equal-arm balance has two trays on opposite sides of a pivot. The total mass placed on one tray required to balance the object on the other gives the mass of the object. Most equal-arm balances have a slider, as well, that can move along a scale and allow for greater precision than the smallest calibrated mass available. Typically, this scale has 0.5 g divisions.

The Triple-Beam Balance

The triple-beam balance, so-called because of its three slider scales, can be read to 0.1 g and estimated to half that. With an object on the tray, the masses of the different scales are slid to notches until balanced. Get close with the larger masses first and then fine-adjust with the smallest slider.

Time Measurement

Time measurements in this course will be made either with a computer or with a stop watch. This first is out of your control.

The Stop Watch

The stop watches you will use in class have a time range of from hours to hundredths of a second. There are two buttons at the top: a stop/start button and a reset button. The operation of these should be evident, although once the watch is reset, the reset button also starts the watch (but doesn't stop it). Please be aware of this feature.

Charge Measurements

The magnitude of charge is among the most difficult measurements to make. Instead a number of indirect measurements are undertaken to understand electric phenomena. These measurements are most often carried out with a digital multimeter

The Digital Multimeter

The digital multimeters available for laboratory exercises have pushbutton control to select five ac and dc voltage ranges, five ac and dc current ranges, and six resistance ranges. The ranges of accuracy are 100 microvolts to 1200 volts ac and dc, 100 nanoamperes to 1.999 amperes ac and dc, and 100 milliohms to 19.99 megaohms.

To perform a DC voltage measurement, select the DCV function and choose a range maximum from one of 200 millivolts or 2, 20, 200, or 1200 volts. Be sure the input connections used are V-Ω and COMMON. The same is true for AC voltage, regarding range and inputs, but the ACV function button should be selected.

For DC current choose DC MA (for DC milliamperes), while for AC current choose AC MA. Your choices for largest current are 200 microamperes or 2, 20, 200, or 2000 milliamperes. Check that the input are connected to MA and COMMON.

There are two choices for resistance measurement: Kilohms (kΩ) and Megohms (20MΩ). The input connectors are the same as when measuring voltage, namely V-Ω and COMMON. The range switches do not function with the Megohm function, but one of the range buttons must be set. The maximum settings for Kilohm readings are 200Ω or 2, 20, 200, or 2000kΩ.