Physics 309 Test 2

I pledge that I have given nor received unauthorized assistance during the completion of this work.

Name ______________________ Signature ______________________

Questions (6 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided. For multiple-choice questions circle the correct answer.

1. What is the power source of the Sun?

2. In solving the Schroedinger equation for the harmonic oscillator we required one of the coefficients in the series solution of the differential equation be equal to zero. Why?

3. The energy states for some molecule are the following: 0.0 eV, 0.4 eV, 0.8 eV, 2.0 eV, 3.0 eV, 4.0 eV, and 5.0 eV. Does this molecule behave like a harmonic oscillator? Explain.

4. What is the areal or surface target density? How is it related to the cross-sectional area of a particle beam?

Do not write below this line.

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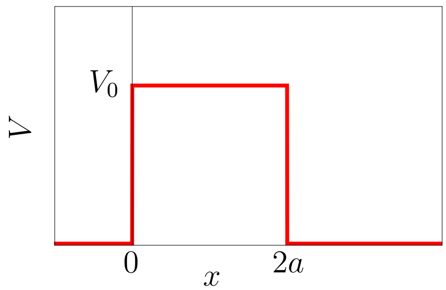
5. In solving the rectangular barrier problem when we studied solar fusion we required the wave wave function and its first derivative to be equal when the potential changed value (at $x = 0$ and $x = 2a$ in the figure). Why?



Problems. Write your solutions on a separate sheet and clearly show all work for full credit.

1. (15 pts.) In solving the Schroedinger equation for the harmonic oscillator potential we rewrote the Schroedinger equation in the form

$$\frac{d^2\phi}{d\xi^2} + \left(\frac{\alpha}{\beta^2} - \xi^2\right)\phi = 0$$

where $\xi = \beta x$, $\alpha = 2mE/\hbar^2$ and $\beta = \sqrt{m\omega_0/\hbar}$. What is the asymptotic form of this differential equation? In other words, what does it look like for large ξ? The general asymptotic solution is

$$\phi_{asymp} = A_{asymp}e^{-\xi^2/2} + B_{asymp}e^{\xi^2/2}$$

but we require $B_{asymp} = 0$ to keep the probability density finite at large ξ (or large x). Show the remaining term in ϕ_{asymp} is a solution of the asymptotic form of the differential equation in the limit as $\xi \to \infty$.

Do not write below this line.

Problems(continued). Write your solutions on a separate sheet and clearly show all work for full credit.

2. (25 pts.) An electron beam is incident on a barrier of height $V_0 = 8\ eV$. At $E = 8.10\ eV$, the transmission coefficient is $T = 5.18 \times 10^{-2}$. What is the width a of the barrier? The expression for T is

$$
\begin{aligned}
T &= \frac{1}{1 + \frac{1}{4}\frac{V_0^2}{E(E-V_0)}\sin^2(2k_2 a)} \qquad E > V_0 \\
&= \frac{1}{1 + \frac{1}{4}\frac{V_0^2}{E(V_0-E)}\sinh^2(2\kappa a)} \qquad E < V_0
\end{aligned}
$$

where

$$k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} = 0.162\ \text{Å}^{-1} \quad \text{and} \quad \kappa = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} = 0.162\ \text{Å}^{-1}$$

at $E = 8.10\ eV$ and 1.0 Å $= 1$ angstrom $= 10^{-10}$ m.

3. (30 pts.) In class we found the general solution to the rectangular barrier problem shown in the figure is the following

$$\phi_1 = Ae^{ik_1x} + Be^{-ik_1x} \qquad \phi_2 = Ce^{ik_2x} + De^{-ik_2x} \qquad \phi_3 = Fe^{ik_1x} + Ge^{-ik_1x}$$

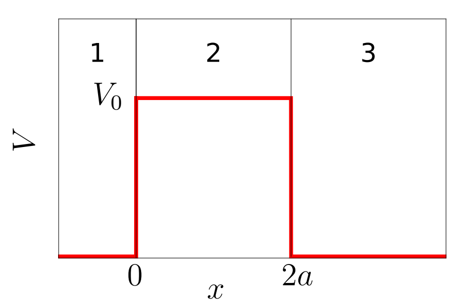
where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

and the subscripts refer to the different regions labeled in the plot. Note the potential is the same in regions 1 and 3. We also used a shifted coordinate system where $x' = x - 2a$ so x' would be zero at the interface between regions 2 and 3. The general solution in the x' coordinates becomes

$$\phi_1' = A'e^{ik_1x'} + B'e^{-ik_1x'} \qquad \phi_2' = C'e^{ik_2x'} + D'e^{-ik_2x'} \qquad \phi_3' = F'e^{ik_1x'} + G'e^{-ik_1x'}$$

where k_1 and k_2 retain their previous meaning. Starting with ϕ_2' and ϕ_3' above apply the boundary conditions and generate equations for C' and D' in the shifted coordinate system that depend only on F', G', k_1, and k_2. Don't set $G' = 0$.



Physics 309 Equations and Constants

$$E = h\nu = \hbar\omega \qquad v_{wave} = \lambda\nu \qquad I \propto |\vec{E}|^2 \qquad \lambda = \frac{h}{p} \qquad p = \hbar k \qquad K = \frac{p^2}{2m} \quad E = \frac{\hbar^2 k^2}{2m}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)+V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_{\ x} = -i\hbar\frac{\partial}{\partial x} \quad \hat{A}\ |\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\ \rangle = \int_{-\infty}^{\infty}\psi^*\hat{A}\ \psi dx$$

$$\langle\phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty}\phi_n^*\phi\, dx = \delta_{n',n} \quad \langle\phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty}\phi_{k'}^*\phi_k\, dx = \delta(k-k') \quad e^{i\phi} = \cos\phi + i\sin\phi$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle \qquad |\phi\rangle = e^{\pm ikt} \qquad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle$$

$$|\psi(t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega t} \quad |\psi(t)\rangle = \int b(k)|\phi(k)\rangle e^{-i\omega(k)t}dk \quad \Delta p\Delta x \geq \frac{\hbar}{2} \qquad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$$

The wave function, $\Psi(\vec{r},t)$, contains all we know of a system and its square is the probability of finding the system in the region $\vec{r}$ to $\vec{r}+d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued* ($\psi_1(a) = \psi_2(a)$ and $\psi_1'(a) = \psi_2'(a)$) .

$$V_{HO} = \frac{\kappa x^2}{2} \quad \omega = 2\pi\nu = \frac{2\pi}{T} = \sqrt{\frac{\kappa}{m}} \quad E_n = (n+\frac{1}{2})\hbar\omega_0 = \hbar\omega \quad |\phi_n\rangle = e^{-\xi^2/2}H_n(\xi) \quad \xi = \beta x \quad \beta^2 = \frac{m\omega_0}{\hbar}$$

$$\tilde{\psi}_1 = \mathbf{t}\psi_3 = \mathbf{d_{12}p_2d_{21}p_1^{-1}}\tilde{\psi}_3 \quad \mathbf{d_{nm}} = \frac{1}{2}\begin{pmatrix}1+\frac{k_m}{k_n} & 1-\frac{k_m}{k_n}\\ 1-\frac{k_m}{k_n} & 1+\frac{k_m}{k_n}\end{pmatrix} \quad k = \sqrt{\frac{2m(E-V)}{\hbar^2}}$$

$$\mathbf{p_n} = \begin{pmatrix}e^{-ik_n2a} & 0\\ 0 & e^{+ik_n2a}\end{pmatrix} \quad \mathbf{p_n^{-1}} = \begin{pmatrix}e^{ik_n2a} & 0\\ 0 & e^{-ik_n2a}\end{pmatrix} \quad T = \frac{\text{transmitted flux}}{\text{incident flux}} = \frac{1}{|t_{11}|^2}$$

$$R = \frac{\text{reflected flux}}{\text{incident flux}} \quad R+T = 1 \quad \text{flux} = |\psi|^2 v \quad V(r) = \frac{Z_1Z_2e^2}{r} \quad E = \frac{1}{2}\mu v^2 + V(r)$$

$$\frac{dN_s}{dt} = \frac{d\sigma}{d\Omega}\frac{dN_{inc}}{dt}n_{tgt}d\Omega \quad n_{tgt} = \frac{\rho_{tgt}}{A_{tgt}}N_A\frac{V_{hit}}{a_{beam}} = \frac{\rho_{tgt}}{A_{tgt}}N_A L_{tgt} \quad d\Omega = \frac{dA}{r^2}$$

$$\mu = \frac{m_1m_2}{m_1+m_2} \quad \psi(x) = \sum_{n=1}^{\infty}a_nx^n \quad \langle K\rangle = \frac{3}{2}kT \qquad n(v) = 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2}v^2e^{-mv^2/2k_BT}$$

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/k$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax} dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x \quad \int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\int \frac{x}{\sqrt{x^2+a^2}} dx = \sqrt{x^2+a^2} \quad \int \frac{x^2}{\sqrt{x^2+a^2}} dx = \frac{1}{2} x\sqrt{x^2+a^2} - \frac{1}{2} a^2 \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \sinh x = \frac{e^x - e^{-x}}{2} \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

Hermite polynomials ($H_n(\xi)$)

$$\begin{aligned} H_0(\xi) &= \frac{1}{\sqrt{\sqrt{\pi}}} & H_4(\xi) &= \frac{1}{\sqrt{384\sqrt{\pi}}}(16\xi^4 - 48\xi^2 + 12) \\ H_1(\xi) &= \frac{1}{\sqrt{2\sqrt{\pi}}} 2\xi & H_5(\xi) &= \frac{1}{\sqrt{3840\sqrt{\pi}}}(32\xi^5 - 160\xi^3 + 120\xi) \\ H_2(\xi) &= \frac{1}{\sqrt{8\sqrt{\pi}}}(4\xi^2 - 2) & H_6(\xi) &= \frac{1}{\sqrt{46080\sqrt{\pi}}}(64\xi^6 - 480\xi^4 + 720\xi^2 - 120) \\ H_3(\xi) &= \frac{1}{\sqrt{48\sqrt{\pi}}}(8\xi^3 - 12\xi) \end{aligned} \tag{1}$$