Physics 309 Test 2

I pledge that I have given nor received unauthorized assistance during the completion of this work.

Name ______________________ Signature ______________________

Questions (6 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided. For multiple-choice questions circle the correct answer.

1. What is the paradox of solar fusion?

2. Use classical physics to explain solar fusion without resorting to quantum mechanics. Quantitative proof is not required.

3. A known mass m_1 is oscillating freely on a vertical spring with measured period T_1. An unknown mass m_2 on the same spring has a measured period T_2. What is the spring constant k and the unknown mass in terms of the given quantities and any other constants?

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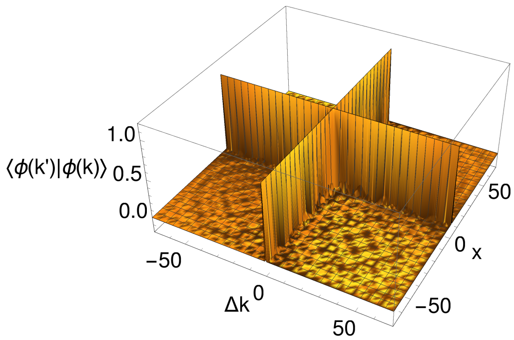
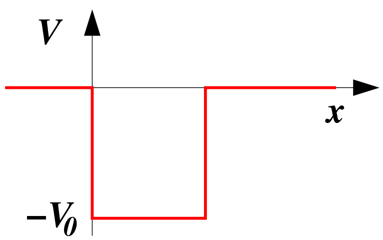
4. The figure below shows a plot of the inner product of two free-particle wave functions $\phi_k(x) = e^{ikx}/\sqrt{a}$ and $\phi_{k'}(x)$ that are non-zero over the range $-a/2 < x < a/2$. We are letting $a \to \infty$. How does this plot establish the orthonormality of these functions?



5. The propagation matrix $\tilde{\mathbf{p}_2}$ we developed to study tunneling through a barrier is shown below.

$$\tilde{\mathbf{p}_2} = \begin{pmatrix} e^{-ik_2 2a} & 0 \\ 0 & e^{+ik_2 2a} \end{pmatrix}$$

How would the matrix or its elements change if the barrier became a hole of depth $-V_0$ as shown in the figure? Explain your reasoning.



Problems. Write your solutions on a separate sheet and clearly show all work for full credit.

1. (10 pts.) Show that the matrix

$$\mathbf{p_1^{-1}} = \begin{pmatrix} e^{ik_1 2a} & 0 \\ 0 & e^{-ik_1 2a} \end{pmatrix}$$

is the inverse of $\mathbf{p_1}$.

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Problems(continued). Write your solutions on a separate sheet and clearly show all work for full credit.

2. (25 pts.) Recall our old friend, Newton's Second Law, $\vec{F} = m\vec{a}$ and perhaps a new one in the drag force equation $F_d = -bv$ which can be combined to form a differential equation in the velocity for an object falling straight down

$$m\frac{dv}{dt} = bv - mg \qquad \text{or} \qquad \frac{dv}{dt} - \frac{b}{m}v + g = 0$$

where b is a parameter describing the drag force, m is the mass and g is the acceleration of gravity. Solve this differential equation using the Method of Frobenius (the power series method) and generate the recursion relationship that relates different coefficients to one another.

3. (35 pts.) One hundred neutrons are in a one-dimensional box with walls at $x = 0$ and $x = a$. At $t = 0$, the state of each neutron is the following.

$$\psi(x, t = 0) = \frac{1}{\sqrt{a}} \quad \text{for} \quad 0 < x < a$$

The eigenfunctions and eigenvalues for this particle in a box are

$$|\phi_n\rangle = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \qquad E_n = n^2\frac{\hbar^2\pi^2}{2ma^2} = n^2 E_1 \quad 0 < x < a$$

and $|\phi_n\rangle$ is zero outside the box.

1. What is $\Psi(x,t)$ in terms of x, t, n, a, and E_1 and any other necessary constants?

2. Approximate $\Psi(x,t)$ using the first two, nonzero terms in the Fourier series and calculate the time-dependent probability density $|\Psi(x,t)|^2$ for this approximate form. This function oscillates between two extremes. What is the frequency of that oscillation? Your answer should be in terms of x, t, n, a, and E_1 and any other necessary constants.

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Physics 309 Equations and Constants

$$E = h\nu = \hbar\omega \qquad v_{wave} = \lambda\nu \qquad I \propto |\vec{E}|^2 \qquad \lambda = \frac{h}{p} \qquad p = \hbar k \qquad K = \frac{p^2}{2m}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)+V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_{\ x} = -i\hbar\frac{\partial}{\partial x} \quad \hat{A}\ |\phi\rangle = a|\phi\rangle \quad \langle\hat{A}\ \rangle = \int_{-\infty}^{\infty}\psi^*\hat{A}\ \psi dx$$

$$\langle\phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty}\phi_n^*\phi\, dx = \delta_{n',n} \quad \langle\phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty}\phi_{k'}^*\phi_k\, dx = \delta(k-k') \quad e^{i\phi} = \cos\phi + i\sin\phi$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle \qquad |\phi\rangle = e^{\pm ikt} \qquad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle$$

$$|\psi(t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega t} \quad |\psi(t)\rangle = \int b(k)|\phi(k)\rangle e^{-i\omega(k)t}dk \quad \Delta p\Delta x \geq \frac{\hbar}{2} \qquad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$$

The wave function, $\Psi(\vec{r},t)$, contains all we know of a system and its square is the probability of finding the system in the region $\vec{r}$ to $\vec{r}+d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued* ($\psi_1(a) = \psi_2(a)$ and $\psi_1'(a) = \psi_2'(a)$) .

$$V_{HO} = \frac{\kappa x^2}{2} \quad \omega = 2\pi\nu = \frac{2\pi}{T} = \sqrt{\frac{\kappa}{m}} \quad E_n = (n+\frac{1}{2})\hbar\omega_0 = \hbar\omega \quad |\phi_n\rangle = e^{-\xi^2/2}H_n(\xi) \quad \xi = \beta x \quad \beta^2 = \frac{m\omega_0}{\hbar}$$

$$\tilde{\psi}_1 = \mathbf{t}\psi_3 = \mathbf{d_{12}p_2d_{21}p_1^{-1}}\tilde{\psi}_3 \quad \mathbf{d_{nm}} = \frac{1}{2}\begin{pmatrix}1+\frac{k_m}{k_n} & 1-\frac{k_m}{k_n}\\ 1-\frac{k_m}{k_n} & 1+\frac{k_m}{k_n}\end{pmatrix} \quad \mathbf{p_n} = \begin{pmatrix}e^{-ik_n2a} & 0\\ 0 & e^{+ik_n2a}\end{pmatrix} \quad T = \frac{1}{|t_{11}|^2}$$

$$E = \frac{\hbar^2k^2}{2m} \quad k = \sqrt{\frac{2m(E-V)}{\hbar^2}} \quad T = \frac{\text{transmitted flux}}{\text{incident flux}} \quad R = \frac{\text{reflected flux}}{\text{incident flux}} \quad R+T = 1 \quad \text{flux} = |\psi|^2v$$

$$V(r) = \frac{Z_1Z_2e^2}{r} \quad E = \frac{1}{2}\mu v^2 + V(r) \quad \vec{R} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

$$\mu = \frac{m_1m_2}{m_1+m_2} \quad \psi(x) = \sum_{n=1}^{\infty}a_nx^n \quad \langle K\rangle = \frac{3}{2}kT \qquad n(v) = 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2}v^2e^{-mv^2/2k_BT}$$

Speed of light (c)	$2.9979 \times 10^{8}\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/k$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^{6}\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^{9}\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-\text{Å}$		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax} dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x \quad \int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\int \frac{x}{\sqrt{x^2+a^2}} dx = \sqrt{x^2+a^2} \quad \int \frac{x^2}{\sqrt{x^2+a^2}} dx = \frac{1}{2} x\sqrt{x^2+a^2} - \frac{1}{2} a^2 \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\int \frac{x^3}{\sqrt{x^2+a^2}} dx = \frac{1}{3}(-2a^2 + x^2)\sqrt{x^2+a^2}$$

Hermite polynomials ($H_n(\xi)$)

$$\begin{aligned}
H_0(\xi) &= \frac{1}{\sqrt{\sqrt{\pi}}} & H_4(\xi) &= \frac{1}{\sqrt{384\sqrt{\pi}}}(16\xi^4 - 48\xi^2 + 12) \\
H_1(\xi) &= \frac{1}{\sqrt{2\sqrt{\pi}}} 2\xi & H_5(\xi) &= \frac{1}{\sqrt{3840\sqrt{\pi}}}(32\xi^5 - 160\xi^3 + 120\xi) \\
H_2(\xi) &= \frac{1}{\sqrt{8\sqrt{\pi}}}(4\xi^2 - 2) & H_6(\xi) &= \frac{1}{\sqrt{46080\sqrt{\pi}}}(64\xi^6 - 480\xi^4 + 720\xi^2 - 120) \\
H_3(\xi) &= \frac{1}{\sqrt{48\sqrt{\pi}}}(8\xi^3 - 12\xi)
\end{aligned} \tag{1}$$