Physics 309 Test 2

I pledge that I have given nor received unauthorized assistance during the completion of this work.

Name ______________________________ Signature ______________________________

Questions (8 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided. For multiple-choice questions circle the correct answer.

1. The average separation between the first twelve energy states of a diatomic molecule is 0.42 ± 0.1 eV. Does it behave like a harmonic oscillator? Be quantitative in your answer.

2. We studied the chemical reaction $\mathrm{H} + \mathrm{H} \rightarrow \mathrm{H}_2$ ($\Delta E = 4.48$ eV/rxn) as a candidate for the source of the power output of the Sun using the Sun's supply of hydrogen. We decided it would not work. Why? Explain.

3. In solving the three-dimensional Schroedinger equation for the CO rotator problem we encountered the following equation

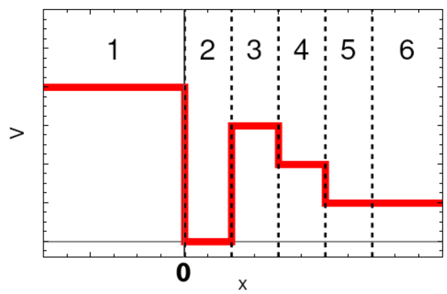
$$-\left\{\frac{\sin^2\theta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{2\mu r^2\sin^2\theta}{\hbar^2}(E-V)\right\}=\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2}$$

where R, Θ, and Φ are solutions to the radial (r), angular (θ), and azimuthal (ϕ) parts. What does this equation equal? Explain your reasoning.

DO NOT WRITE BELOW THIS LINE.

4. Recall your comparison of theory and data in the alpha decay lab. What did the theory get right? What is at least one weakness in the theory?

5. Consider the potential barrier shown below where the potential in each region is V_i where i is the index shown in the figure. How would you use the transfer-matrix approach to connect the wave function $\underset{\sim}{\psi}_1$ in region 1 to the wave function $\underset{\sim}{\psi}_6$ in region 6? Regions 2-5 are each a distance a wide. Give your answer in the appropriate notation used in class for the discontinuity and propagation matrices. What is the form of the wave number k in each region?



Problems. Write your solutions on a separate sheet and clearly show all work for full credit.

1. (15 pts.) Show that the matrix

$$\mathbf{p_1} = \begin{pmatrix} e^{-ik_1 2a} & 0 \\ 0 & e^{ik_1 2a} \end{pmatrix}$$

is the inverse of $\mathbf{p_1^{-1}}$.

2. (20 pts.) An electron beam is sent through a potential barrier 4.0 Å long. The transmission coefficient exhibits a fourth maximum at $E = 90$ eV. What is the height of the barrier?

For a rectangular barrier of width $2a$: $\frac{\hbar^2 k_1^2}{2m} = E \quad \frac{\hbar^2 k_2^2}{2m} = E - V$

$$T = \left[1 + \frac{1}{4}\frac{V^2}{E(E-V)}\sin^2(2k_2 a)\right]^{-1} \quad E > V$$

$$T = \left[1 + \frac{1}{4}\frac{V^2}{E(V-E)}\sinh^2(2\kappa a)\right]^{-1} \quad E < V$$

Problems(continued). Write your solutions on a separate sheet and clearly show all work for full credit.

3. (25 pts.) Recall our old friends, Newton's Second Law, $\vec{F} = m\vec{a}$ and Hooke's Law, $|\vec{F}| = -Kx$ which can be combined to form a differential equation

$$m\frac{d^2x}{dt^2} = -Kx \qquad \text{or} \qquad \frac{d^2x}{dt^2} = -\omega_0^2 x$$

where $\omega_0 = \sqrt{\frac{K}{m}}$. The solutions of this equation have been known to us for some time, but now solve this differential equation using the Method of Frobenius (*i.e.*, the power series method) and obtain the recursion relation between successive coefficients. If the first coefficient in the series is $a_0 = 1$, calculate the next coefficient in the series. Which function is your result - sine or cosine?

Physics 309 Equations and Constants

$$E = h\nu = \hbar\omega \qquad v_{wave} = \lambda\nu \qquad I \propto |\vec{E}|^2 \qquad \lambda = \frac{h}{p} \qquad p = \hbar k \qquad K = \frac{p^2}{2m} \quad E = \frac{\hbar^2 k^2}{2m}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_x = -i\hbar\frac{\partial}{\partial x} \quad \hat{A}\,|\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\,\rangle = \int_{-\infty}^{\infty} \psi^* \hat{A}\,\psi dx$$

$$\langle \phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty} \phi_n^* \phi\, dx = \delta_{n',n} \quad \langle \phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty} \phi_{k'}^* \phi_k\, dx = \delta(k-k') \quad e^{i\phi} = \cos\phi + i\sin\phi$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \rightarrow b_n = \langle\phi_n|\psi\rangle \qquad |\phi\rangle = e^{\pm ikt} \qquad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \rightarrow b(k) = \langle\phi(k)|\psi\rangle$$

$$|\psi(t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega t} \quad |\psi(t)\rangle = \int b(k)|\phi(k)\rangle e^{-i\omega(k)t} dk \quad \Delta p \Delta x \geq \frac{\hbar}{2} \qquad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$$

The wave function, $\Psi(\vec{r}, t)$, contains all we know of a system and its square is the probability of finding the system in the region $\vec{r}$ to $\vec{r} + d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued* ($\psi_1(a) = \psi_2(a)$ and $\psi_1'(a) = \psi_2'(a)$) .

$$V_{HO} = \frac{\kappa x^2}{2} \quad \omega = 2\pi\nu = \frac{2\pi}{T} = \sqrt{\frac{\kappa}{m}} \quad E_n = (n + \frac{1}{2})\hbar\omega_0 = \hbar\omega \quad |\phi_n\rangle = e^{-\xi^2/2}H_n(\xi) \quad \xi = \beta x \quad \beta^2 = \frac{m\omega_0}{\hbar}$$

$$\tilde{\psi}_1 = \mathbf{t}\psi_3 = \mathbf{d_{12}p_2d_{21}p_1^{-1}}\tilde{\psi}_3 \quad \mathbf{d_{nm}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_m}{k_n} & 1-\frac{k_m}{k_n} \\ 1-\frac{k_m}{k_n} & 1+\frac{k_m}{k_n} \end{pmatrix} \quad k = \sqrt{\frac{2m(E-V)}{\hbar^2}}$$

$$\mathbf{p_n} = \begin{pmatrix} e^{-ik_n2a} & 0 \\ 0 & e^{+ik_n2a} \end{pmatrix} \quad \mathbf{p_n^{-1}} = \begin{pmatrix} e^{ik_n2a} & 0 \\ 0 & e^{-ik_n2a} \end{pmatrix} \quad T = \frac{\text{transmitted flux}}{\text{incident flux}} = \frac{1}{|t_{11}|^2}$$

$$R = \frac{\text{reflected flux}}{\text{incident flux}} \quad R+T=1 \quad \text{flux} = |\psi|^2 v \quad V(r) = \frac{Z_1Z_2e^2}{r} \quad E = \frac{1}{2}\mu v^2 + V(r)$$

$$\frac{dN_s}{dt} = \frac{d\sigma}{d\Omega}\frac{dN_{inc}}{dt}n_{tgt}d\Omega \quad n_{tgt} = \frac{\rho_{tgt}}{A_{tgt}}N_A\frac{V_{hit}}{a_{beam}} = \frac{\rho_{tgt}}{A_{tgt}}N_AL_{tgt} \quad d\Omega = \frac{dA}{r^2} \quad dA = r^2\sin\theta d\theta d\phi$$

$$x_{cm} = \frac{\sum m_ix_i}{\sum m_i} \quad \mu = \frac{m_1m_2}{m_1+m_2} \quad \psi(x) = \sum_{n=1}^{\infty} a_nx^n \quad \langle K\rangle = \frac{3}{2}kT \quad n(v) = 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2e^{-mv^2/2k_BT}$$

Speed of light (c)	$2.9979\times10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381\times10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62\times10^{-5}\ eV/k$	electron-volt (eV)	$1.6\times10^{-19}\ J$
Planck constant (h)	$6.621\times10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357\times10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546\times10^{-34}\ J-s$	Electron charge (e)	$1.6\times10^{-19}\ C$
	$6.5821\times10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11\times10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67\times10^{-27}kg$	atomic mass unit (u)	$1.66\times10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68\times10^{-27}\ kg$		
	$939\ MeV/c^2$		

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x} \quad \int x^ndx = \frac{x^{n+1}}{n+1} \quad \int e^{ax}dx = \frac{e^{ax}}{a} \quad \int\frac{1}{x} = \ln x \quad \int\frac{1}{\sqrt{x^2+a^2}}dx = \ln\left[x+\sqrt{x^2+a^2}\right]$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \sinh x = \frac{e^{x} - e^{-x}}{2} \quad \cosh x = \frac{e^{x} + e^{-x}}{2}$$

Hermite polynomials ($H_n(\xi)$)

$$\begin{aligned}
H_0(\xi) &= \frac{1}{\sqrt{\sqrt{\pi}}} & H_4(\xi) &= \frac{1}{\sqrt{384\sqrt{\pi}}}(16\xi^4 - 48\xi^2 + 12) \\
H_1(\xi) &= \frac{1}{\sqrt{2\sqrt{\pi}}}2\xi & H_5(\xi) &= \frac{1}{\sqrt{3840\sqrt{\pi}}}(32\xi^5 - 160\xi^3 + 120\xi) \\
H_2(\xi) &= \frac{1}{\sqrt{8\sqrt{\pi}}}(4\xi^2 - 2) & H_6(\xi) &= \frac{1}{\sqrt{46080\sqrt{\pi}}}(64\xi^6 - 480\xi^4 + 720\xi^2 - 120) \\
H_3(\xi) &= \frac{1}{\sqrt{48\sqrt{\pi}}}(8\xi^3 - 12\xi)
\end{aligned} \tag{1}$$