Physics 309 Test 1

I pledge that I have given nor received unauthorized assistance during the completion of this work.

Name ____________________________ Signature ____________________________

Questions (8 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided.

1. Cite at least one observation or experimental result that violated classical physics and describe how it was violated?

2. Consider an initial wave packet $|\psi(x,0)\rangle$ bound in a non-zero potential $V(x)$ and its associated one-dimensional Schroedinger equation. What is the full, time-dependent solution $|\psi(x,t)\rangle$ in terms of eigenfunctions and eigenvalues? Your answer should be symbolic and be sure to define the components.

3. Do electrons orbit the the atomic nucleus in the same way planets orbit the sum? Explain.

4. The eigenfunctions and eigenvalues of the particle in a box are

$$|\phi\rangle = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \qquad E_n = n^2 \frac{\hbar^2 \pi^2}{2ma^2} \qquad 0 < x < a$$

and zero outside the box. Consider the following sequence of measurements.

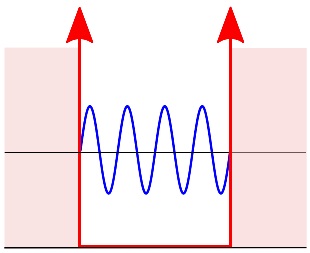
(a) The energy of the particle is measured. A value E_1 is obtained.

(b) The value of the position of the particle is made and a value x_2 is obtained.

(c) The energy of the particle is measured again.

What possible values of the energy can you obtain in step 4.c? Explain.

5. What is the quantum number of the particle in an infinite square well with the wave function shown in the figure below? Explain your answer.



E_n

Problems. Clearly show all work on a separate piece of paper for full credit.

Note: If you encounter integrals that you are unable to solve label them I_1, I_2, ... and so on and carry that notation through to the solution.

1. (15 pts.) Consider a wave function of the form $\Psi(x,t) = A\sin(kx - \omega t)$. Using the definition of the wavelength λ and the period T show

$$k = \frac{2\pi}{\lambda} \qquad\qquad \omega = \frac{2\pi}{T} = 2\pi\nu$$

where $\nu = 1/T$ is the frequency of the wave.

2. (20 pts.) At $t = 0$, N free-particle protons are on a line segment of length L and centered on $x = 0$. It is equally probable to find any proton at any point on the line segment. What are the $b(k)$ for this initial wave packet in terms of L and any other constants?

3. (25 pts.) Consider a case of one dimensional nuclear 'fusion'. A neutron is in the potential well of a nucleus that we will approximate with an infinite square well with walls at $x = 0$ and $x = L$. The eigenfunctions and eigenvalues are

$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2} \qquad \begin{aligned} \phi_n &= \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad && 0 \le x \le L \\ &= 0 && x < 0 \text{ and } x > L \end{aligned}$$

The neutron is in the $n = 4$ state when it fuses with another nucleus that is twice its size, instantly putting the neutron in a new infinite square well with walls at $x = 0$ and $x = 3L$.

   (a) What are the new eigenfunctions and eigenvalues of the fused system?

   (b) What is the probability of finding the fused system to be in the $n' = 5$ state? In the $n' = 6$ state?

Physics 309 Equations

$$R_T(\nu) = \frac{Energy}{time \times area} \quad E = h\nu = \hbar\omega \quad v_{wave} = \lambda\nu \quad I \propto |\vec{E}|^2 \quad K_{max} = h\nu - W \quad K = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\lambda = \frac{h}{p} \quad p = \hbar k \quad \underset{\sim}{p}_x = -i\hbar\frac{\partial}{\partial x} \quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \Psi(x,t) = S(x)T(t)$$

$$\hat{A}\,|\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\,\rangle = \int_{-\infty}^{\infty} \psi^* \hat{A}\,\psi dx \quad |\phi(k)\rangle = \frac{e^{\pm ikx}}{\sqrt{2\pi}} \quad E = \frac{\hbar^2 k^2}{2m} \quad |\phi_n\rangle = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \quad E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$$

$$k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T} = 2\pi\nu \langle\phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty} \phi_{n'}^*\phi_n\,dx = \delta_{n',n} \quad \langle\phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty} \phi_{k'}^*\phi_k\,dx = \delta(k-k')$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle \quad |\psi(x,t)\rangle = \int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle$$

$$\Delta p\Delta x \geq \hbar \quad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2 \quad \langle f(x)\rangle = \frac{\int_{-\infty}^{\infty} f(x)P(x)dx}{\int_{-\infty}^{\infty} P(x)dx} \quad \langle f_n\rangle = \frac{\sum_{n=0}^{\infty} f_n P_n}{\sum_{n=0}^{\infty} P_n}$$

$$|\psi(x,t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega_n t} \quad |\psi(x,t)\rangle = \int_{-\infty}^{\infty} b(k)\phi(k)e^{-i\omega t}dk \quad \text{If } f(x) = \sqrt{\frac{1}{2\pi\sigma^2}}\,e^{-x^2/2\sigma^2}\text{, then } \Delta x = \sigma$$

$$e^{ix} = \cos x + i\sin x \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

The wave function, $\psi(\vec{r},t)$, contains all we know of a system and $|\psi|^2$ is the probability of finding it in the region $\vec{r}$ to $\vec{r} + d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued*.

$$\ln(ab) = \ln a + \ln b \quad \ln\left(\frac{a}{b}\right) = \ln a - \ln b \quad \ln x^n = n\ln x \quad \ln(e^a) = a \quad e^{\ln a} = a$$

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax}dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2}$$

$$\int \frac{x^2}{\sqrt{x^2+a^2}}dx = \frac{1}{2}x\sqrt{x^2+a^2} - \frac{1}{2}a^2\ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x^3}{\sqrt{x^2+a^2}}dx = \frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2}$$

$$\int \cos(Ax)dx = \frac{\sin(Ax)}{A} \quad \int \cos(Ax)x dx = \frac{\cos(Ax)}{A^2} + \frac{x\sin(Ax)}{A}$$

More Physics 309 Equations

$$\int \sin(Ax)dx = -\frac{\cos(Ax)}{A} \qquad \int \sin(Ax)x dx = \frac{\sin(Ax)}{A^2} - \frac{x\cos(Ax)}{A}$$

$$\int \sin(Ax)x^2 dx = \frac{2x\sin(Ax)}{A^2} - \frac{(A^2x^2-2)\cos(Ax)}{A^3} \qquad \int \cos(Ax)x^2 dx = \frac{2x\cos(Ax)}{A^2} + \frac{(A^2x^2-2)\sin(Ax)}{A^3}$$

$$\int_a^b \sin(Ax)dx = \frac{\cos(aA)-\cos(Ab)}{A} \qquad \int_a^b \sin(Ax)x dx = \frac{-\sin(aA)+aA\cos(aA)+\sin(Ab)-Ab\cos(Ab)}{A^2}$$

$$\int_a^b \cos(Ax)dx = \frac{\sin(Ab)-\sin(aA)}{A} \qquad \int_a^b \cos(Ax)x dx = \frac{-aA\sin(aA)-\cos(aA)+Ab\sin(Ab)+\cos(Ab)}{A^2}$$

$$\int \sin(Ax)\sin(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} - \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \cos(Ax)\cos(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} + \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \sin(Ax)\cos(Bx)dx = -\frac{\cos(x(A-B))}{2(A-B)} - \frac{\cos(x(A+B))}{2(A+B)}$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right] \qquad \sin 2A = 2\sin A\cos A \qquad \cos 2A = \cos^2 A - \sin^2 A$$

Physics 309 Conversions, and Constants

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/K$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		