Physics 309 Final

I pledge that I have given nor received unauthorized assistance during the completion of this work.

Name ______________________________ Signature ______________________________

Questions (4 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided.

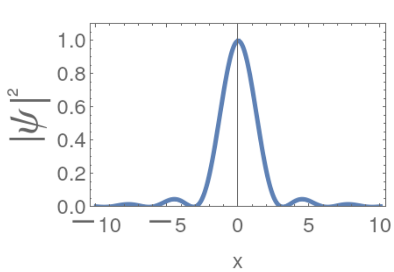
1. What is an absorption spectrum?

2. In our study of the CO molecule what is the classical energy?

3. Why do we use energy eigenstates?

4. What is Rutherford scattering?

5. Consider the spectral distribution shown below. What is the mathematical definition of the width of the distribution? Sketch on the plot what this width represents. Explain.

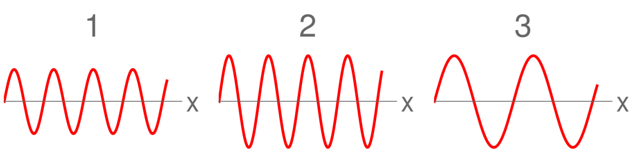


6. Cite at least two experimental measurements that violated classical physics and required quantum mechanics to explain. Discuss how they violated classical physics.

7. Why does the Sun shine? What are the important features of the process? Your answer should be descriptive and qualitative - not quantitative.

8. Recall that for a bound, one-dimensional system at $t = 0$ that $|\psi\rangle = \sum_{n=0}^{\infty} b_n |\phi_n\rangle$. What is b_n and how is it related to a measurement of the momentum?

9. The figure below shows the de Broglie waves of three equal-mass particles as a function of position x. Rank them according to their speed. Explain your reasoning. The range of x and y are the same in each plot.



10. The components of the angular momentum vector $\vec{L}$ are orthogonal to each other. Can a measurement of the x component of the angular momentum, L_x, of a state with $m_z = \ell$ produce a nonzero result? Explain.

Do not write below this line.

Problems. Clearly show all work for full credit on a separate piece of paper.

1. (10 pts.) A mass $m_0 = 0.910\ kg$ is oscillating freely on a vertical spring. The period for m_0 is $T_o = 1.10\ s$. An unknown mass m_1 replaces m_0 on the same spring and has a period of $T_1 = 1.32\ s$. What is the spring constant k and the unknown mass m_1?

2. (10 pts.) Show that the frequencies of photons due to energy decays between successive levels of a rotator with momentum of inertia I are given by the following.

$$\hbar\omega = \frac{\hbar^2}{I}(\ell+1) \quad or \quad \frac{\hbar^2}{I}\ell$$

3. (12 pts.) Find $\psi(x)$ and $P(E_n)$ at $t = 0$ relevant to a one-dimensional box with walls at $(0, a)$ for the following initial state.

$$\psi(x,0) = A_3\left(e^{i\pi(x-a)/a} - 1\right)$$

Make sure you get an expression for $\psi(x)$ valid for all eigenstates. The eigenfunctions and eigenvalues of the one-dimensional particle in a box of width a are

$$|\phi(x)\rangle = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \qquad E_n = n^2 E_1 = n^2\frac{\hbar^2\pi^2}{2ma^2} \quad .$$

4. (13 pts.) A molecule behaves like a one-dimensional harmonic oscillator with a spring constant $k = 2.42\ eV/\text{Å}^2$. It emits a photon of energy $E_\gamma = h\nu = \hbar\omega_0 = 0.1\ eV$ going from the third excited state to the second excited state. The oscillating part is a proton ($m_p = 938\ MeV/c^2$).

   a) What is the value of the classical turning point x_t if the molecule is in the $n = 2$ state?

   b) What is the numerical value of $\beta = \sqrt{m\omega_0/\hbar}$?

   c) What is the probability that a proton in this state is at a distance from the origin forbidden to it by classical mechanics? Get your answer in terms of x_t and β where β is defined below with the wave function for the $n = 2$ state.

$$|\phi_2\rangle = \frac{1}{\sqrt{8\sqrt{\pi}}}\left(4\xi^2 - 2\right)e^{-\xi^2/2} \quad \xi = \beta x \quad \beta = \sqrt{\frac{m\omega_0}{\hbar}}$$

Do not write below this line.

5. (15 pts.) Suppose a rigid rotator is in the eigenstate of $\hat{L}^2$ with $\ell = 1$ and $m_z = -1$ ($Y_1^{-1}(\theta,\phi)$). We want to find the probability of obtaining the values of $m_x = 0, \pm 1$ from a measurement of $\hat{L}_x$. We always measure eigenvalues so to get the results of a measurement of L_x we need to construct the appropriate operator $\hat{L}_x$ which satisfies $\hat{L}_x X = \alpha\hbar X$ where X is an eigenfunction of the $\hat{L}_x$ operator and α is the eigenvalue. To do that we can assume

$$X = aY_1^1 + bY_1^0 + cY_1^{-1}$$

since the spherical harmonics form a complete set and we know what they are. We restrict our attention to only $\ell = 1$ states as a consequence of angular momentum conservation. Generate the set of simultaneous equations that the coefficients (a, b, c) and α must satisfy.

Physics 309 Equations

$$R_T(\nu) = \frac{Energy}{time \times area} \quad E = h\nu = \hbar\omega \quad v_{wave} = \lambda\nu \quad I \propto |\vec{E}|^2 \quad \lambda = \frac{h}{p} \quad p = \hbar k \quad K = \frac{p^2}{2m} \quad K_{max} = h\nu - \Phi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_x = -i\hbar\frac{\partial}{\partial x} \quad \hat{A}\,|\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\,\rangle = \int_{-\infty}^{\infty}\psi^*\hat{A}\,\psi dx$$

$$\langle\phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty}\phi_{n'}^*\phi_n dx = \delta_{n',n} \quad \langle\phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty}\phi_{k'}^*\phi_k\, dx = \delta(k-k') \quad e^{i\phi} = \cos\phi + i\sin\phi$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \rightarrow b_n = \langle\phi_n|\psi\rangle \qquad |\phi\rangle = e^{\pm ikx} \qquad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \rightarrow b(k) = \langle\phi(k)|\psi\rangle$$

$$|\psi(x,t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega_n t} \quad |\psi(x,t)\rangle = \int b(k)|\phi(k)\rangle e^{-i\omega(k)t}dk \quad \Delta p\Delta x \geq \frac{\hbar}{2} \qquad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$$

The wave function, $\Psi(\vec{r},t)$, contains all we know of a system and its square is the probability of finding the system in the region $\vec{r}$ to $\vec{r}+d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued* ($\psi_1(a) = \psi_2(a)$ and $\psi_1'(a) = \psi_2'(a)$) .

$$V_{HO} = \frac{\kappa x^2}{2} \quad \omega = 2\pi\nu = \sqrt{\frac{\kappa}{m}} \quad E_n = (n+\frac{1}{2})\hbar\omega_0 = \hbar\omega \quad |\phi_n\rangle = e^{-\xi^2/2}H_n(\xi) \quad \xi = \beta x \quad \beta^2 = \frac{m\omega_0}{\hbar}$$

$$\psi_1 = \mathbf{t}\psi_3 = \mathbf{d_{12}p_2d_{21}p_1^{-1}}\psi_3 \qquad T = \frac{1}{|t_{11}|^2} \qquad R + T = 1$$

$$\mathbf{d_{ij}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_j}{k_i} & 1-\frac{k_j}{k_i} \\ 1-\frac{k_j}{k_i} & 1+\frac{k_j}{k_i}\end{pmatrix} \quad \mathbf{p_i} = \begin{pmatrix} e^{-ik_i2a} & 0 \\ 0 & e^{ik_i2a}\end{pmatrix} \quad \mathbf{p_i^{-1}} = \begin{pmatrix} e^{ik_i2a} & 0 \\ 0 & e^{-ik_i2a}\end{pmatrix}$$

$$E = \frac{\hbar^2k^2}{2m} \quad k = \sqrt{\frac{2m(E-V)}{\hbar^2}} \quad T = \frac{\text{transmitted flux}}{\text{incident flux}} \quad R = \frac{\text{reflected flux}}{\text{incident flux}} \quad \text{flux} = |\psi|^2 v$$

$$V(r) = \frac{Z_1Z_2e^2}{r} \quad E = \frac{1}{2}\mu v^2 + V(r) \quad \vec{R}_{cm} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \quad \mu = \frac{m_1m_2}{m_1+m_2}$$

$$\psi(x)=\sum_{n=1}^{\infty}a_n x^n \quad \langle K\rangle=\frac{3}{2}kT \qquad n(v)=4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2}v^2 e^{-mv^2/2k_BT} \quad \vec{L}=\vec{r}\times\vec{p}=\mathcal{I}\vec{\omega}$$

$$\mathcal{I}=\sum_i m_i r_1^2=\int r^2 dm \quad KE_{rot}=\frac{L^2}{2\mathcal{I}} \quad E_\ell=\frac{\ell(\ell+1)\hbar^2}{2\mathcal{I}} \quad V_{coul}=\frac{Z_1Z_2e^2}{r} \quad ME=\frac{p_r^2}{2\mu}+\frac{L^2}{2\mu r^2}+V(r)$$

$$L_z|nlm\rangle=m\hbar|nlm\rangle \quad L^2|nlm\rangle=\ell(\ell+1)\hbar^2|nlm\rangle$$

$$\sin A\sin B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right] \quad \cos A\cos B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$

$$\sin A\cos B=\frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right] \quad \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Constants

Speed of light (c)	$2.9979\times10^{8}\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381\times10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62\times10^{-5}\ eV/k$	electron-volt (eV)	$1.6\times10^{-19}\ J$
Planck constant (h)	$6.621\times10^{-34}\ J-s$	MeV	$10^{6}\ eV$
	$4.1357\times10^{-15}\ eV-s$	GeV	$10^{9}\ eV$
Planck constant ($\hbar$)	$1.0546\times10^{-34}\ J-s$	Electron charge (e)	$1.6\times10^{-19}\ C$
	$6.5821\times10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11\times10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67\times10^{-27}kg$	atomic mass unit (u)	$1.66\times10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68\times10^{-27}\ kg$		
	$939\ MeV/c^2$		

Integrals and Derivatives

$$\frac{df}{du}=\frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n)=nx^{n-1} \quad \frac{d}{dx}(\sin x)=\cos x \quad \frac{d}{dx}(\cos x)=-\sin x \quad \frac{d}{dx}(e^{ax})=ae^{ax}$$

$$\frac{d}{dx}(\ln ax)=\frac{1}{x} \quad \int x^n dx=\frac{x^{n+1}}{n+1} \quad \int e^{ax}dx=\frac{e^{ax}}{a} \quad \int\frac{1}{x}=\ln x \quad \int\frac{1}{\sqrt{x^2+a^2}}dx=\ln\left[x+\sqrt{x^2+a^2}\right]$$

$$\int\frac{x}{\sqrt{x^2+a^2}}dx=\sqrt{x^2+a^2} \quad \int\frac{x^2}{\sqrt{x^2+a^2}}dx=\frac{1}{2}x\sqrt{x^2+a^2}-\frac{1}{2}a^2\ln\left[x+\sqrt{x^2+a^2}\right]$$

$$\int\frac{x^3}{\sqrt{x^2+a^2}}dx=\frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2} \quad \int x^2\sin(ax)dx=\frac{2x\sin(ax)}{a^2}-\frac{\left(a^2x^2-2\right)\cos(ax)}{a^3}$$

hydrogen 1 **H** 1.0079																		helium 2 **He** 4.0026
lithium 3 **Li** 6.941	beryllium 4 **Be** 9.0122												boron 5 **B** 10.811	carbon 6 **C** 12.011	nitrogen 7 **N** 14.007	oxygen 8 **O** 15.999	fluorine 9 **F** 18.998	neon 10 **Ne** 20.180
sodium 11 **Na** 22.990	magnesium 12 **Mg** 24.305												aluminium 13 **Al** 26.982	silicon 14 **Si** 28.086	phosphorus 15 **P** 30.974	sulfur 16 **S** 32.065	chlorine 17 **Cl** 35.453	argon 18 **Ar** 39.948
potassium 19 **K** 39.098	calcium 20 **Ca** 40.078		scandium 21 **Sc** 44.956	titanium 22 **Ti** 47.867	vanadium 23 **V** 50.942	chromium 24 **Cr** 51.996	manganese 25 **Mn** 54.938	iron 26 **Fe** 55.845	cobalt 27 **Co** 58.933	nickel 28 **Ni** 58.693	copper 29 **Cu** 63.546	zinc 30 **Zn** 65.39	gallium 31 **Ga** 69.723	germanium 32 **Ge** 72.61	arsenic 33 **As** 74.922	selenium 34 **Se** 78.96	bromine 35 **Br** 79.904	krypton 36 **Kr** 83.80
rubidium 37 **Rb** 85.468	strontium 38 **Sr** 87.62		yttrium 39 **Y** 88.906	zirconium 40 **Zr** 91.224	niobium 41 **Nb** 92.906	molybdenum 42 **Mo** 95.94	technetium 43 **Tc** [98]	ruthenium 44 **Ru** 101.07	rhodium 45 **Rh** 102.91	palladium 46 **Pd** 106.42	silver 47 **Ag** 107.87	cadmium 48 **Cd** 112.41	indium 49 **In** 114.82	tin 50 **Sn** 118.71	antimony 51 **Sb** 121.76	tellurium 52 **Te** 127.60	iodine 53 **I** 126.90	xenon 54 **Xe** 131.29
caesium 55 **Cs** 132.91	barium 56 **Ba** 137.33	57-70 *	lutetium 71 **Lu** 174.97	hafnium 72 **Hf** 178.49	tantalum 73 **Ta** 180.95	tungsten 74 **W** 183.84	rhenium 75 **Re** 186.21	osmium 76 **Os** 190.23	iridium 77 **Ir** 192.22	platinum 78 **Pt** 195.08	gold 79 **Au** 196.97	mercury 80 **Hg** 200.59	thallium 81 **Tl** 204.38	lead 82 **Pb** 207.2	bismuth 83 **Bi** 208.98	polonium 84 **Po** [209]	astatine 85 **At** [210]	radon 86 **Rn** [222]
francium 87 **Fr** [223]	radium 88 **Ra** [226]	89-102 * *	lawrencium 103 **Lr** [262]	rutherfordium 104 **Rf** [261]	dubnium 105 **Db** [262]	seaborgium 106 **Sg** [266]	bohrium 107 **Bh** [264]	hassium 108 **Hs** [269]	meitnerium 109 **Mt** [268]	ununnilium 110 **Uun** [271]	unununium 111 **Uuu** [272]	ununbium 112 **Uub** [277]		ununquadium 114 **Uuq** [289]				

*Lanthanide series	lanthanum 57 **La** 138.91	cerium 58 **Ce** 140.12	praseodymium 59 **Pr** 140.91	neodymium 60 **Nd** 144.24	promethium 61 **Pm** [145]	samarium 62 **Sm** 150.36	europium 63 **Eu** 151.96	gadolinium 64 **Gd** 157.25	terbium 65 **Tb** 158.93	dysprosium 66 **Dy** 162.50	holmium 67 **Ho** 164.93	erbium 68 **Er** 167.26	thulium 69 **Tm** 168.93	ytterbium 70 **Yb** 173.04
* *Actinide series	actinium 89 **Ac** [227]	thorium 90 **Th** 232.04	protactinium 91 **Pa** 231.04	uranium 92 **U** 238.03	neptunium 93 **Np** [237]	plutonium 94 **Pu** [244]	americium 95 **Am** [243]	curium 96 **Cm** [247]	berkelium 97 **Bk** [247]	californium 98 **Cf** [251]	einsteinium 99 **Es** [252]	fermium 100 **Fm** [257]	mendelevium 101 **Md** [258]	nobelium 102 **No** [259]

Properties of the Spherical Harmonics

$$\hat{L}^2|\ell, m\rangle = \ell(\ell+1)\hbar^2\,|\ell, m\rangle$$

$$\hat{L}_z|\ell m\rangle = m\hbar\,|\ell m\rangle$$

$$\hat{L}_x|\ell, m\rangle = \frac{\hbar}{2}\sqrt{(\ell-m)(\ell+m+1)}\;|\ell, m+1\rangle + \frac{\hbar}{2}\sqrt{(\ell+m)(\ell-m+1)}\;|\ell, m-1\rangle$$

$$\hat{L}_y|\ell, m\rangle = -\frac{\hbar}{2}\sqrt{(\ell-m)(\ell+m+1)}\;|\ell, m+1\rangle + \frac{\hbar}{2}\sqrt{(\ell+m)(\ell-m+1)}\;|\ell, m-1\rangle$$

$$\hat{L}_\pm|\ell, m\rangle = \hbar\sqrt{\ell(\ell+1) - m(m\pm 1)}\;|\ell, m\pm 1\rangle$$

$$\langle \ell' m'|\ell m\rangle = \int_0^\pi \int_0^{2\pi} Y_{\ell'}^{m'*} Y_\ell^m d\Omega = \delta_{\ell\ell'}\delta_{mm'}$$