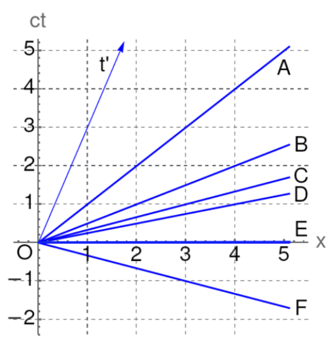
Physics 205 Test 2

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name ______________________ Signature ______________________

Questions (5 for 8 pts. apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. The Other frame is moving in the $+x$ direction with $x-$velocity $\beta = 1/3$ with respect to the Home frame. The two-observer diagram in the figure shows the t and x axes of the Home frame and the diagram t' axis of the Other frame. Which choices in the figure best correspond to the x' axis. Explain.



2. The absolute value of the x component of a particle's four-momentum vector is always either equal to or greater than its t component. T or F? Explain.

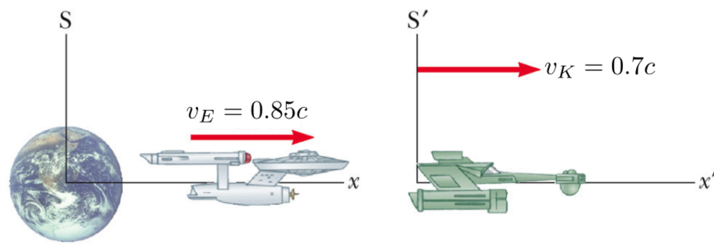
3. Length is a something we measure with a ruler. It's the distance between the markings on the ruler that are taken simultaneously. T or F. Explain.

4. Particle A has half the mass and twice the speed of Particle B. Is the 3-momentum p_A less than, greater than, or equal to the 3-momentum p_B? Explain.

5. A spherical space ship (when it is a rest) passes you at $v = 0.9c$. Describe what you see. Explain.

Problems (3). Clearly show all reasoning for full credit. Use a separate sheet for your work.

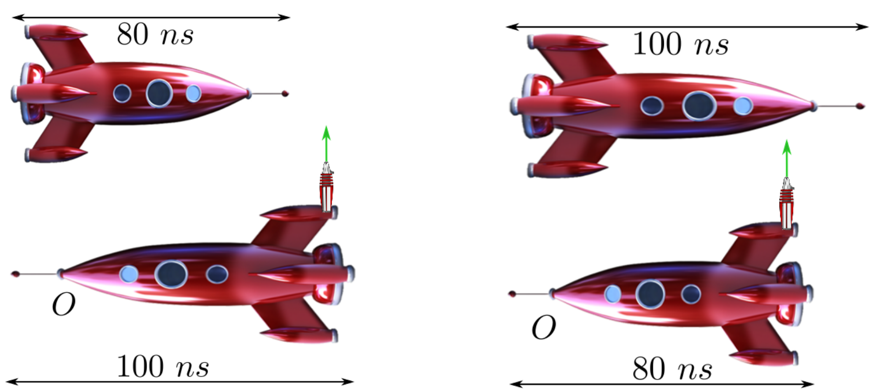
1. 15 pts. A Klingon spacecraft moves away from the Earth at a speed $v_K = 0.7c$ (see figure). The starship Enterprise pursues at a speed of $v_E = 0.85c$ relative to the Earth. Observers on the Earth measure the Enterprise overtaking the Klingon craft at a relative speed $v_{rel} = 0.15c$. With what speed is the Enterprise overtaking the Klingon craft as measured by the crew of the Enterprise?

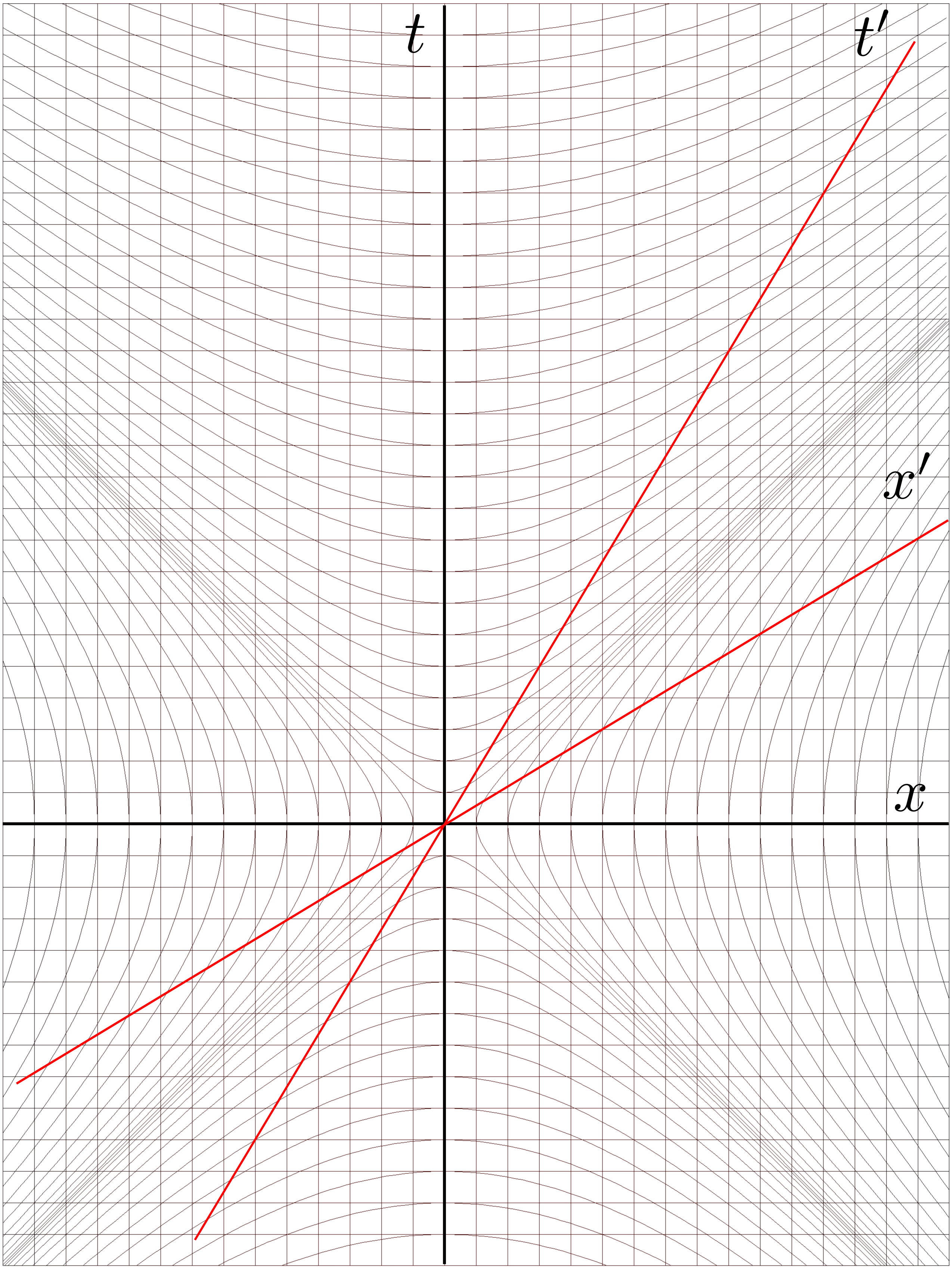


2. 20 pts. Physicists in one of the experimental halls at JLab have found evidence there may be a problem with the calibration of their measurement of the beam energy. The accelerator operators say it is $E_e = 10.81 \pm 0.04$ GeV. You have a set of events from the same hall for elastic scattering of electrons off protons. Consider one of your events with $E_{e'} = 8.773$ GeV at $\theta_e = 11.5°$. Starting from the appropriate equation on the equation sheet obtain an expression for the energy of the beam E_e in terms of the information above. Get a numerical value for 'your' beam energy and compare it with the one from the accelerator operators. Do you agree or disagree? Explain.

3. 25 pts. Two spacecraft of equal rest length $L_R = 100\ ns = 30\ m$ pass very close to each other as they travel in opposite directions at a relative speed $|\beta| = \frac{3}{5}$ $(0.6c)$. See left-hand side of figure below. The captain of ship O intends to fire a warning shot across the bow of ship O' with her laser cannon at the instant her bow is lined up with the tail of ship O'. Since O' is Lorentz-contracted to a length of $80\ ns = 24\ m$ in the frame of ship O, she expects the laser burst to miss the other ship by a distance $20\ ns = 6\ m$. However, to the captain on ship O', it is ship O that has contracted to $80\ ns$ so the laser burst will strike ship O' about $20\ ns$ behind the bow. See right-hand side of figure.

What really happens if O carries out her plan? Construct a two-observer spacetime diagram. See page 4 for hyperbolic graph paper with the x' and t' axes already drawn. Define event A to be the coincidence of the bow of ship O and the tail of O' and event B to be the firing of the laser. Let event A define the origin event in both frames. What is the worldline of the nose of the O' ship? When and where does event B occur as measured in the O' frame? What does the captain of O' see? The travel time of the laser from one ship to another is negligible. Verify your results with the Lorentz transformations.




t
t'
x'
x

Physics 205 Equations

$$\vec{F}_{net} = \sum \vec{F}_i = m\vec{a} = \frac{d\vec{p}}{dt} \quad v = \frac{dx}{dt} \quad v = \frac{\Delta x}{\Delta t} \quad x = \frac{1}{2}at^2 + v_0 t + x_0 \quad v = at + v_0 \quad a_g = -g \quad a_c = \frac{v^2}{r}$$

$$\vec{F}_{Earth} = -mg\hat{\jmath} \quad KE = \frac{1}{2}mv^2 \quad KE_0 + PE_0 = KE_1 + PE_1 \quad PE_{Earth} = mgh \quad PE_V = qV$$

$$\vec{p}_i = \vec{p}_f \quad \vec{p} = m\vec{v} \quad \vec{F}_C = k_e \frac{q_1 q_2}{r^2}\hat{r} \quad \vec{E} \equiv \frac{\vec{F}}{q_0} \quad d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{qd\vec{v} \times \hat{r}}{r^2} \qquad \vec{F}_B = q\vec{v} \times \vec{B} \qquad |\vec{F}_B| = |qvB\sin\theta|$$

Galilean Transformation SI units	Lorentz Transformation SI units	Lorentz Transformation SR units
$x' = x - vt$ $y' = y$ $z' = z$ $t' = t$ $v'_x = v_x - v$	$x' = \gamma(x - vt)$ $y' = y$ $z' = z$ $t' = \gamma(t - vx/c^2)$ $v'_x = \frac{v_x - v}{1 - v_x v/c^2}$	$x' = \gamma(x - \beta t)$ $y' = y$ $z' = z$ $t' = \gamma(t - \beta x)$ $v'_x = \frac{v_x - \beta}{1 - v_x\beta}$

Coordinate Time	Proper Time	Spacetime Interval
Time between two events in an inertial frame measured with synchronized clocks $c\Delta t$,Δt Frame dependent	Time between two events measured by the same clock at both events. $\Delta\tau_{SI}$, $\Delta\tau_{SR}$ Frame independent	Time between two events measured by the same, inertial clock at both events. Δs_{SI}, Δs_{SR} Frame independent

$$\Delta s_{SI}^2 = c^2\Delta t^2 - \Delta d^2 = \Delta {s'_{SI}}^2 \quad \text{or} \quad \Delta s_{SR}^2 = \Delta t^2 - \Delta d^2 = \Delta {s'_{SR}}^2$$

$$\Delta\tau_{SI} = \int_{t_A}^{t_B} \sqrt{1 - \frac{v^2}{c^2}}\, dt \quad \text{or} \quad \Delta\tau_{SR} = \int_{t_A}^{t_B} \sqrt{1 - \beta^2}\, dt$$

$$\Delta\tau_{SI} = \sqrt{1 - v^2/c^2}\ \Delta t \quad \text{or} \quad \Delta\tau_{SR} = \sqrt{1 - \beta^2}\ \Delta t$$

$$L_{SI} = L_R\sqrt{1 - v^2/c^2} \quad \text{or} \quad L_{SR} = L_R\sqrt{1 - \beta^2}$$

$$v_x = \frac{v'_x + v}{1 + v'_x v/c^2} \quad KE = E - mc^2 \quad \text{SI units} \qquad v_x = \frac{v'_x + \beta}{1 + v'_x \beta} \quad KE = E - m \quad \text{SR units}$$

$$\underset{\sim}{p} = m\, d\underset{\sim}{s}/d\tau = \left[m\frac{dt}{d\tau}, m\frac{dx}{d\tau}, m\frac{dy}{d\tau}, m\frac{dz}{d\tau}\right] = \frac{m}{\sqrt{1-|\vec{v}|^2}}[1,\vec{v}] \quad \underset{\sim}{p}\cdot\underset{\sim}{p} = E_r^2 - |\vec{p}|^2 = m^2 \qquad \text{SR units}$$

$$\underset{\sim}{p} = m\, d\underset{\sim}{s}/d\tau = \left[mc\frac{dt}{d\tau}, m\frac{dx}{d\tau}, m\frac{dy}{d\tau}, m\frac{dz}{d\tau}\right] = \frac{m}{\sqrt{1-v^2/c^2}}[c,\vec{v}] \quad \underset{\sim}{pc}\cdot\underset{\sim}{pc} = E_r^2 - |\vec{p}c|^2 = (mc^2)^2 \qquad \text{SI units}$$

$$\underset{\sim}{p_i} = \underset{\sim}{p_f} \quad \underset{\sim}{p_1}\cdot\underset{\sim}{p_2} = \underset{\sim}{p_3}\cdot\underset{\sim}{p_4} \quad (\underset{\sim}{p_1}+\underset{\sim}{p_2})^2 = (\underset{\sim}{p_3}+\underset{\sim}{p_4})^2 \quad E_{e'} = \frac{E_e}{1 + \frac{2E_e}{m_n}\sin^2\left(\frac{\theta_e}{2}\right)}$$

$$\frac{d}{dx}(f(u)) = \frac{df}{du}\frac{du}{dx} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int \frac{1}{x}dx = \ln x \quad \vec{A}\cdot\vec{B} = AB\cos\theta \quad |\vec{A}\times\vec{B}| = |AB\sin\theta|$$

$$\frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{de^x}{dx} = e^x \quad \frac{d}{dx}(\ln x) = \frac{1}{x} \quad \frac{d}{dx}(\cos ax) = -a\sin ax \quad \frac{d}{dx}(\sin ax) = a\cos ax$$

$$\langle x\rangle = \frac{1}{N}\sum_i x_i \quad \sigma = \sqrt{\frac{\sum_i (x_i - \langle x\rangle)^2}{N-1}} \quad C = 2\pi r \quad A = 4\pi r^2 \quad V = Ah \quad V = \frac{4}{3}\pi r^3$$

$$\frac{df(x)}{dx} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \int_a^b f(x)dx = \lim_{\Delta x\to 0}\sum_{n=1}^{N} f(x)\Delta x$$

$$\int \frac{1}{x}dx = \ln x \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax}dx = \frac{e^{ax}}{a} \quad \int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2} \quad \int \frac{x^2}{\sqrt{x^2+a^2}}dx = \frac{1}{2}x\sqrt{x^2+a^2} - \frac{1}{2}a^2\ln\left[x+\sqrt{x^2+a^2}\right]$$

$$\int \sqrt{1-ax^2}dx = \frac{x}{2}\sqrt{1-ax^2} + \frac{\arcsin(\sqrt{a}x)}{2\sqrt{a}} \quad \int \frac{x^3}{\sqrt{x^2+a^2}}dx = \frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2}$$

Physics 205 Constants and Conversions

Avogadro's number (N_A)	6.022×10^{23}	Speed of light (c)	$3 \times 10^8\ m/s$
proton/neutron mass	0.938 GeV/c^2	proton/neutron mass	$1.67 \times 10^{-27}\ kg$
Electron mass	0.511 MeV/c^2	Electron mass	$9.11 \times 10^{-31}\ kg$
1 MeV	10^6 eV	1 GeV	10^3 MeV $= 10^9$ eV
Coulomb constant (k_e)	$8.99 \times 10^9 \frac{N-m^2}{C^2}$	g	$9.8\ m/s^2$
Gravitation constant	$6.67 \times 10^{-11}\ N - m^2/kg^2$	Earth's radius	$6.37 \times 10^6\ m$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	k_B	$1.38 \times 10^{-23}\ J/K$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} \frac{kg^2}{N-m^2}$	1.0 eV	$1.6 \times 10^{-19}\ J$
1 MeV	$10^6\ eV$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
Planck's constant (h)	$6.63 \times 10^{-34}\ Js$	Planck's constant (h)	$4.14 \times 10^{-15}\ eVs$
Permeability constant (μ_0)	$1.26 \times 10^{-6}\ Tm/A$	Rydberg constant (R_H)	$1.097 \times 10^7\ m^{-1}$
Becquerel (Bq)	$1\ decay/s$	Curie (Ci)	$3.7 \times 10^{10}\ Bq$