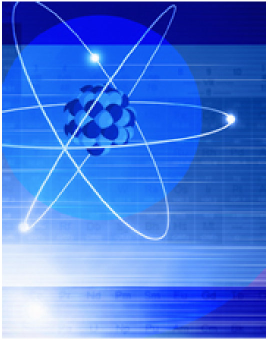
What is the Energy of the Electron? 1

The Coulomb force binds an electron and a proton into a hydrogen atom with a force that is mathematically identical to the gravitational force that binds the planets in our Solar System, the Moon to the Earth, *etc.* What is the energy of an electron?



What We Already Know.

- The Organizing Principle.

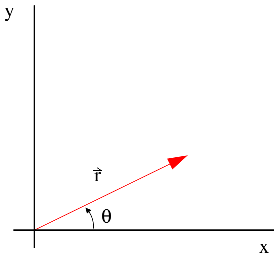
$$\begin{aligned} ME_0 &= ME_1 \\ KE_0 + PE_0 &= ME_1 + PE_1 \\ \frac{1}{2}mv_0^2 + PE_0 &= \frac{1}{2}mv_1^2 + PE_1 \end{aligned}$$

- The Forces

$$\vec{F}_{grav} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{F}_{coul} = \frac{k_e q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

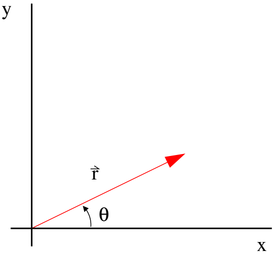
The simulation is here.

The Kinetic Energy in Polar Coordinates - 1

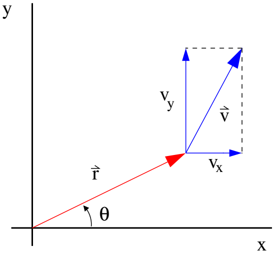


$\vec{r} = x\hat{i} + y\hat{j}$

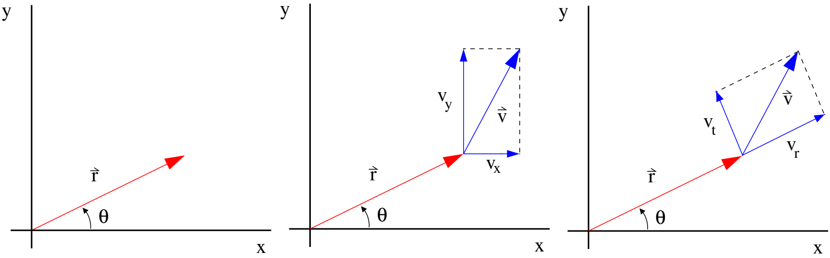
The Kinetic Energy in Polar Coordinates - 1



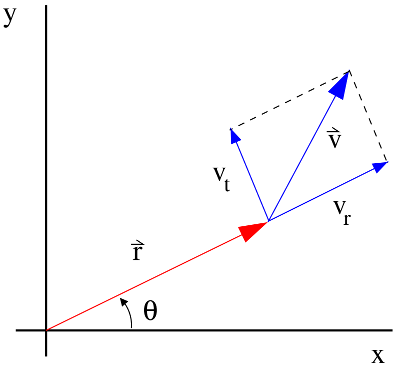
$\vec{r} = x\hat{i} + y\hat{j}$

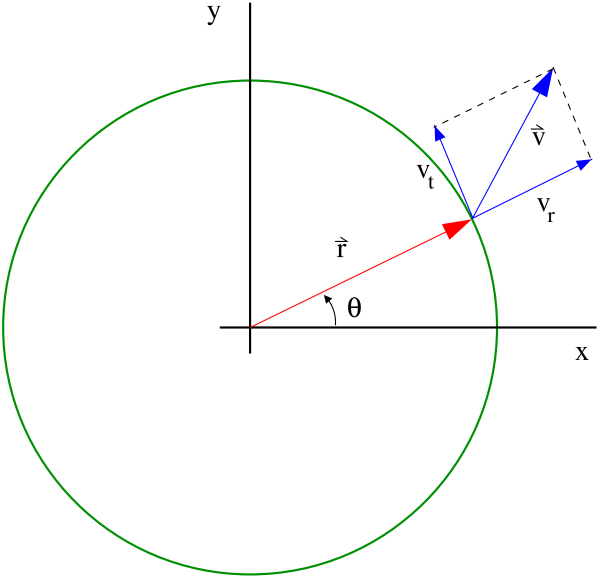


$\vec{v} = v_x\hat{i} + v_y\hat{j}$

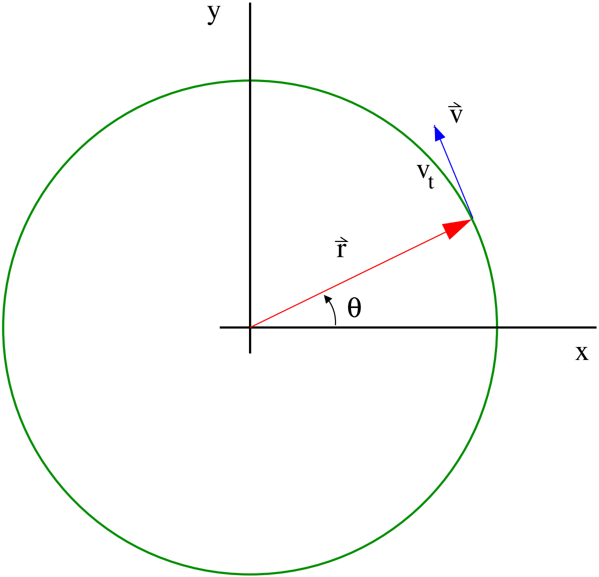


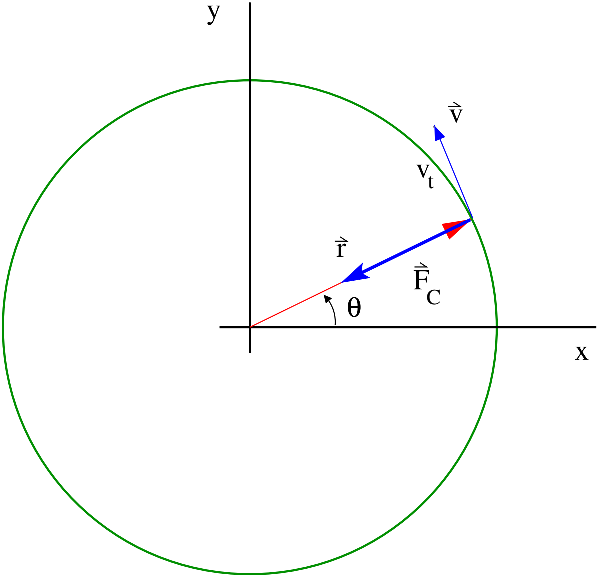
$\vec{r} = x\hat{i} + y\hat{j}$ $\vec{v} = v_x\hat{i} + v_y\hat{j}$ $\vec{v} = v_r\hat{r} + v_t\hat{\theta}$


y
$\vec{v}$
v_t
v_r
$\vec{r}$
θ
x

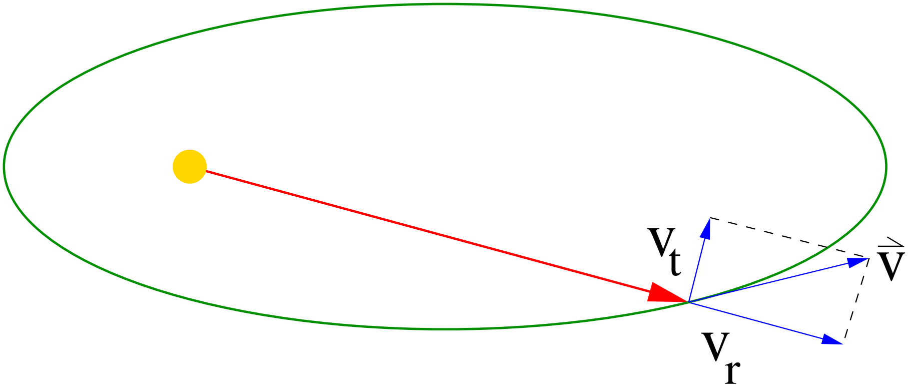


y
$\vec{v}$
v_t
v_r
$\vec{r}$
θ
x

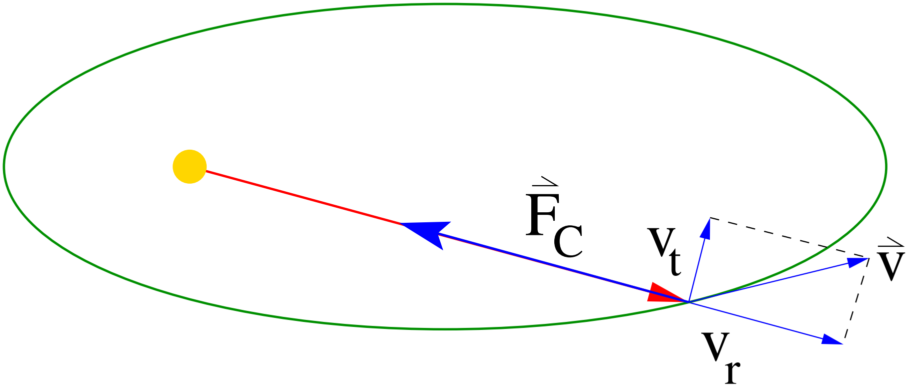

y
$\vec{v}$
v_t
$\vec{r}$
θ
x

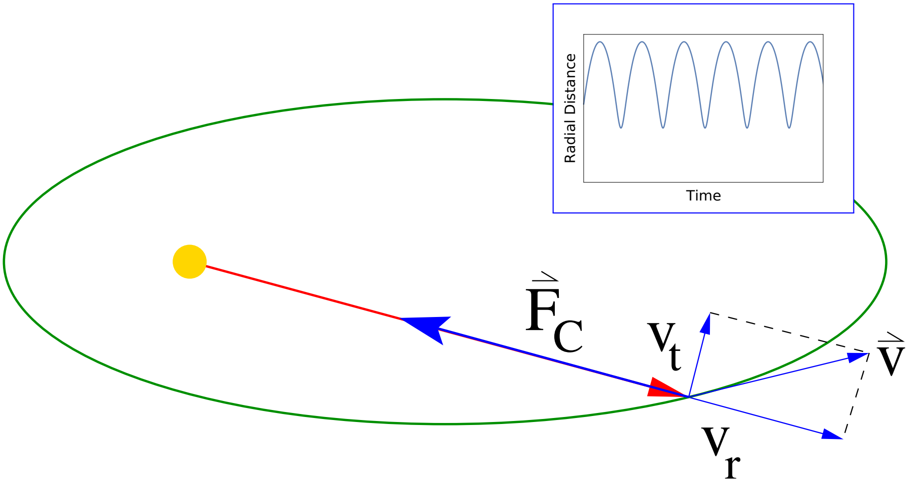


y
$\vec{v}$
v_t
$\vec{r}$
$\vec{F}_C$
θ
x


v_t
$\vec{v}$
v_r

The Kinetic Energy in Polar Coordinates - 3

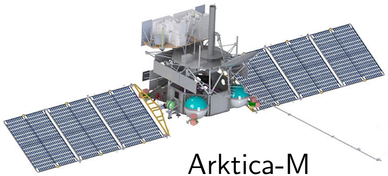



Radial Distance
Time
$\vec{F}_C$
v_t
$\vec{v}$
v_r

A Russian Artica satellite that monitors polar weather follows an elliptical orbit around the Earth at an altitude of $h = 300\ km$ above the surface (radius $r_s = 6.67 \times 10^6\ m$) at a velocity

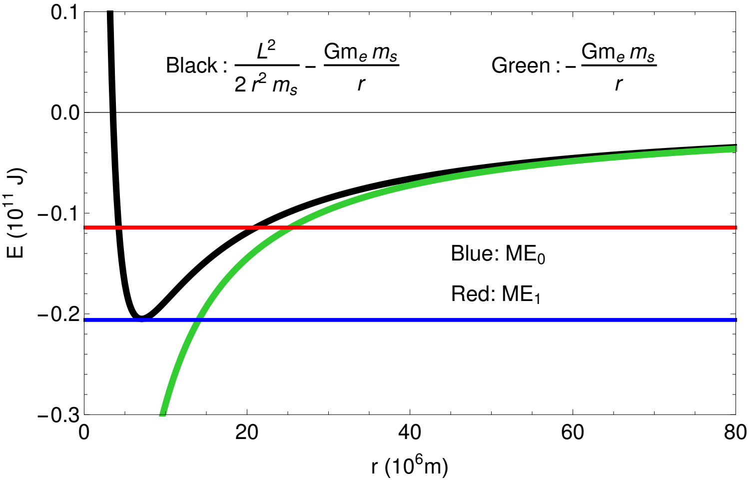
$$\vec{v} = 4.1 \times 10^3\ m/s\ \hat{r} + 7.5 \times 10^3\ m/s\ \hat{\theta} \qquad .$$

What is the angular momentum? What is the total energy? What is the distance of closest approach to the Earth? The satellite mass is $m_s = 600\ kg$.



Arktica-M

$R_{earth} = 6.37 \times 10^6\ m$
$m_{earth} = 5.97 \times 10^{24}\ kg$
$G = 6.673 \times 10^{-11}\ Nm^2/kg^2$



0.1
0.0
−0.1
−0.2
−0.3
0
20
40
60
80
E (10^11 J)
r (10^6m)
Black : $\frac{L^2}{2 r^2 m_s} - \frac{G m_e m_s}{r}$
Green : $-\frac{G m_e m_s}{r}$
Blue: ME_0
Red: ME_1

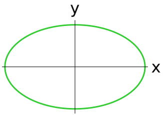
Classical Physics

1. Start with Newton's Laws.
2. Insert the force/potential.
3. Solve the differential equation with initial conditions

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}$$

where $\vec{r}$ is the position.

4. Get the position $\vec{r}(t)$ as a function of time.



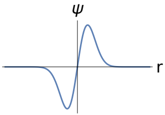
Quantum Physics

1. Start with Schroedinger's equation.
2. Insert the force/potential.
3. Solve the differential equation with initial conditions

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dr^2}+\frac{L^2}{2mr^2}\psi+V\psi = E\psi(\vec{r})$$

where ψ is a wave function.

4. Get the probability $|\psi(\vec{r})|^2$ as a function of time.



1. The quantum state of a particle is characterized by a wave function $\Psi(\vec{r}, t)$, which contains all the information about the system an observer can possibly obtain. The square of the magnitude of the wave function $|\Psi(\vec{r}, t)|^2$ is interpreted as a probability or probability density for the particle's presence.
2. The things we measure (*e.g.* energy, momentum) are called observables. Each observable has a corresponding mathematical object called an operator that does 'something' to the wave function $\Psi(\vec{r}, t)$ and we obtain the value of the observable. The radial dependence of the wave function $\Psi(\vec{r}, t)$ is governed by the energy operator which generates a famous expression called the Schrödinger equation.

$$-\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2}\right)\Psi(r) + \frac{L^2}{2mr^2}\Psi(r) + V\Psi(r) = E\Psi(r)$$

What did your ground-state wave function look like?

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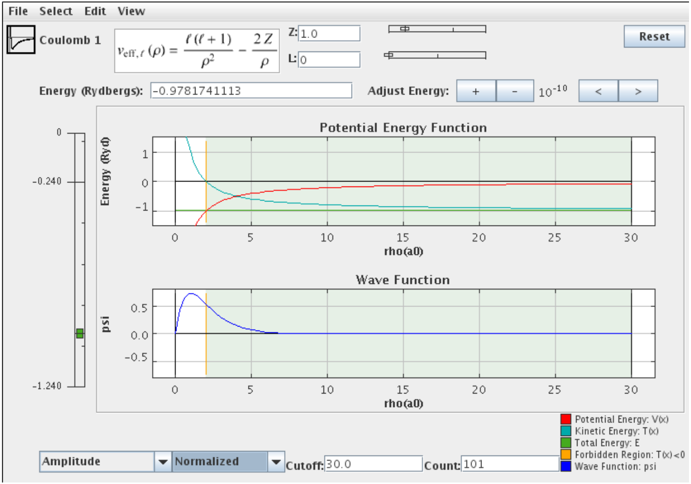
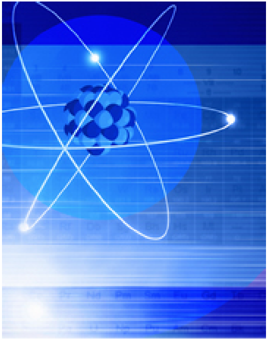
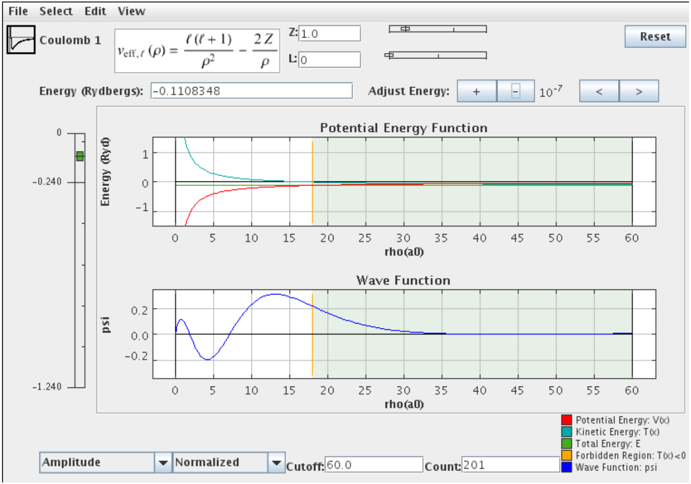


The Coulomb force binds an electron and a proton into a hydrogen atom with a force that is mathematically identical to the gravitational force that binds the planets in our Solar System, the Moon to the Earth, *etc.* For an electron with energy E_e where can it be found as a function of r where r is the distance from the proton?

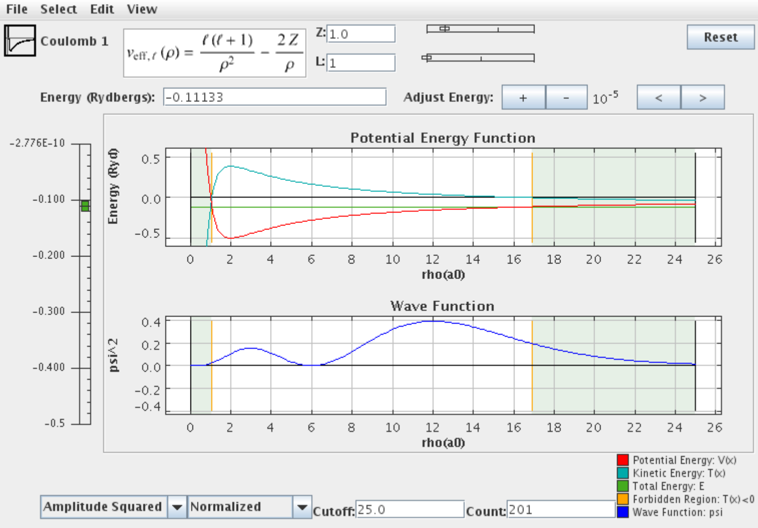


What did your n=3 wave function look like?

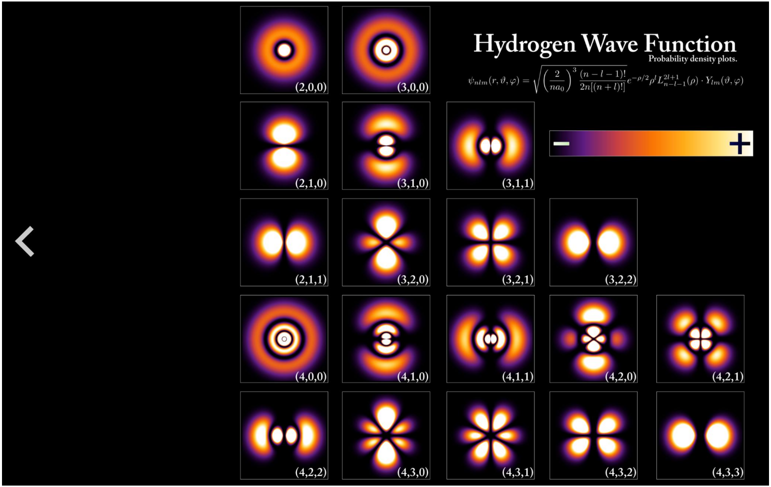
What did your n=3 wave function look like?



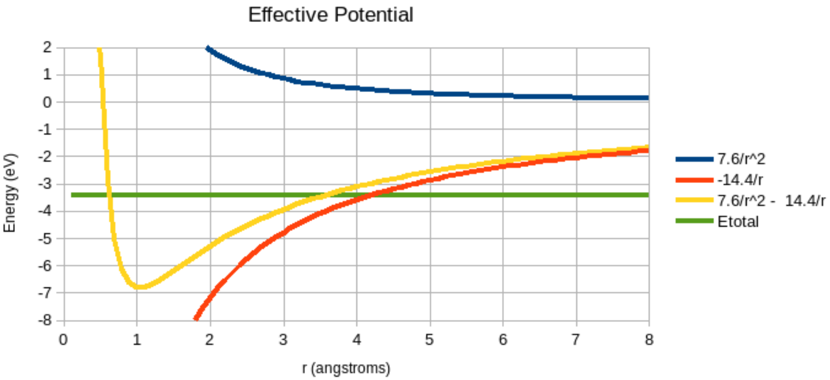
For $n = 3$, $L = 1$.

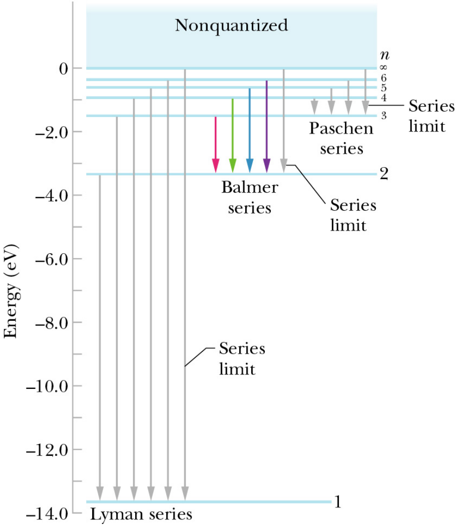


File Select Edit View
Coulomb 1
$v_{\mathrm{eff},\ell}(\rho) = \frac{\ell(\ell+1)}{\rho^2} - \frac{2Z}{\rho}$
Z: 1.0
L: 1
Reset
Energy (Rydbergs): -0.11133
Adjust Energy: + - 10^{-5} < >
-2.776E-10
-0.100
-0.200
-0.300
-0.400
-0.5
Potential Energy Function
Energy (Ryd)
0.5
0.0
-0.5
0 2 4 6 8 10 12 14 16 18 20 22 24 26
rho(a0)
Wave Function
psi^2
0.4
0.2
0.0
-0.2
-0.4
0 2 4 6 8 10 12 14 16 18 20 22 24 26
rho(a0)
Potential Energy: V(x)
Kinetic Energy: T(x)
Total Energy: E
Forbidden Region: T(x)<0
Wave Function: psi
Amplitude Squared
Normalized
Cutoff: 25.0
Count: 201

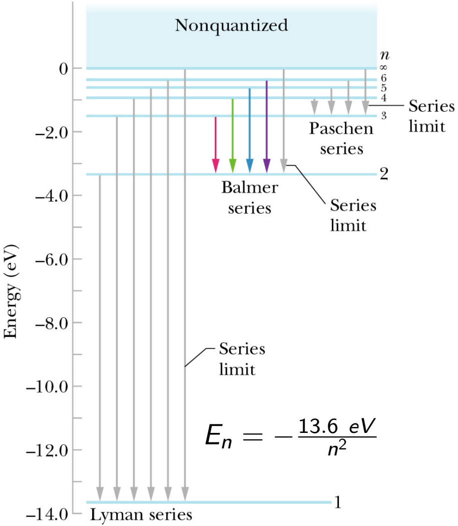


The electron probability density for the first few hydrogen atom electron orbitals shown as cross-sections. These orbitals form an orthonormal basis for the wave function of the electron. Different orbitals are depicted with different scale.


Effective Potential
Energy (eV)
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
0
1
2
3
4
5
6
7
8
r (angstroms)
7.6/r^2
-14.4/r
7.6/r^2 - 14.4/r
Etotal

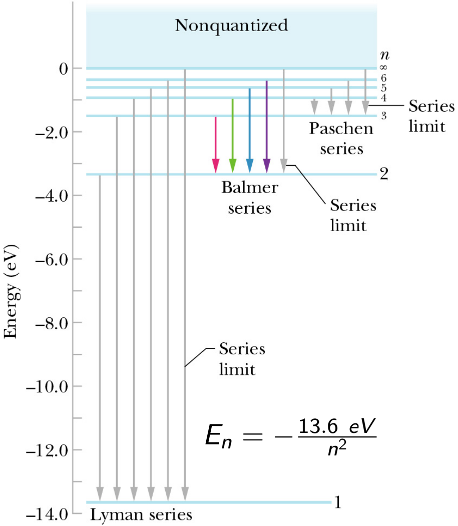


Nonquantized
n
∞
6
5
4
3
2
1
0
–2.0
–4.0
–6.0
–8.0
–10.0
–12.0
–14.0
Energy (eV)
Series limit
Paschen series
Balmer series
Series limit
Series limit
Lyman series

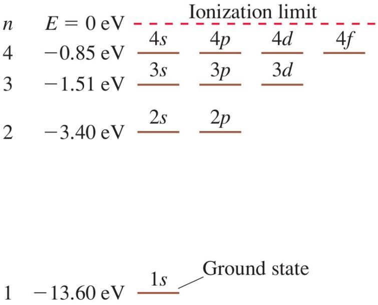


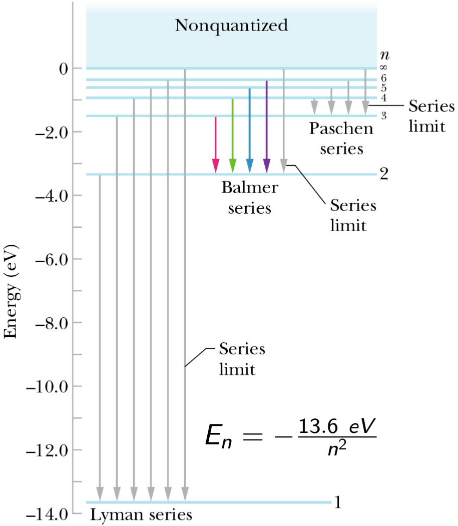
$$E_n = -\frac{13.6\ eV}{n^2}$$

Hydrogen Quantum Numbers



$$E_n = -\frac{13.6\ eV}{n^2}$$





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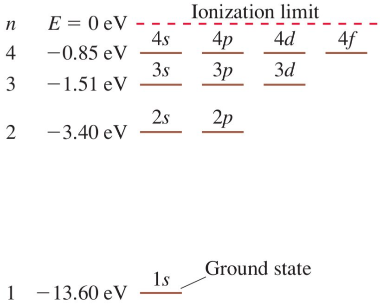

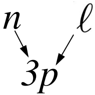

n - principal quantum number

ℓ - angular momentum

n_i	n_f	Energy (eV)
5	2	2.82 ± 0.04
4	2	2.55 ± 0.08
3	2	1.86 ± 0.02