

All of Electromagnetism!

Gauss' law for electricity:
$$\oint_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \quad (1)$$

Gauss' law for magnetism:
$$\oint_{\text{closed surface}} \vec{B} \cdot d\vec{A} = 0 \quad (2)$$

Faraday's law:
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt} \quad (3)$$

Ampere-Maxwell law:
$$\oint \vec{B} \cdot d\vec{s} = \mu_0\epsilon_0 \frac{d\phi_E}{dt} + \mu_0 i_{enc} \quad (4)$$

Note: \oint is used to specify a line integral meaning the integration must form a closed loop. These equations hold in the absence of magnetic materials.

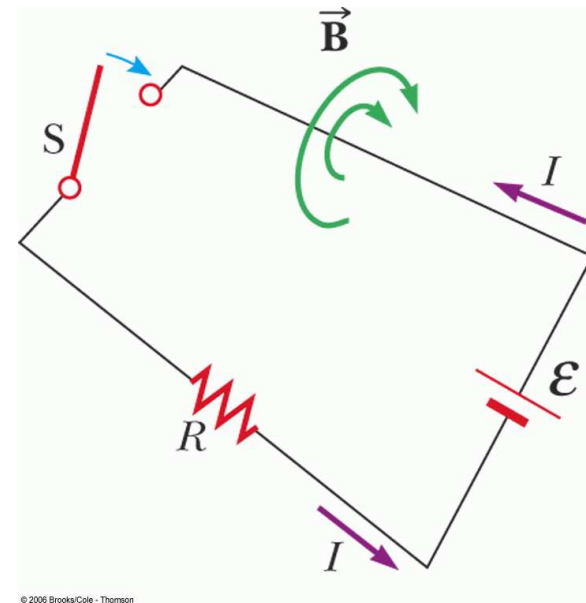
Faraday's Law for a Wire Coil

$$\oint \vec{E} \cdot d\vec{S} = -\frac{d\phi_B}{dt}$$



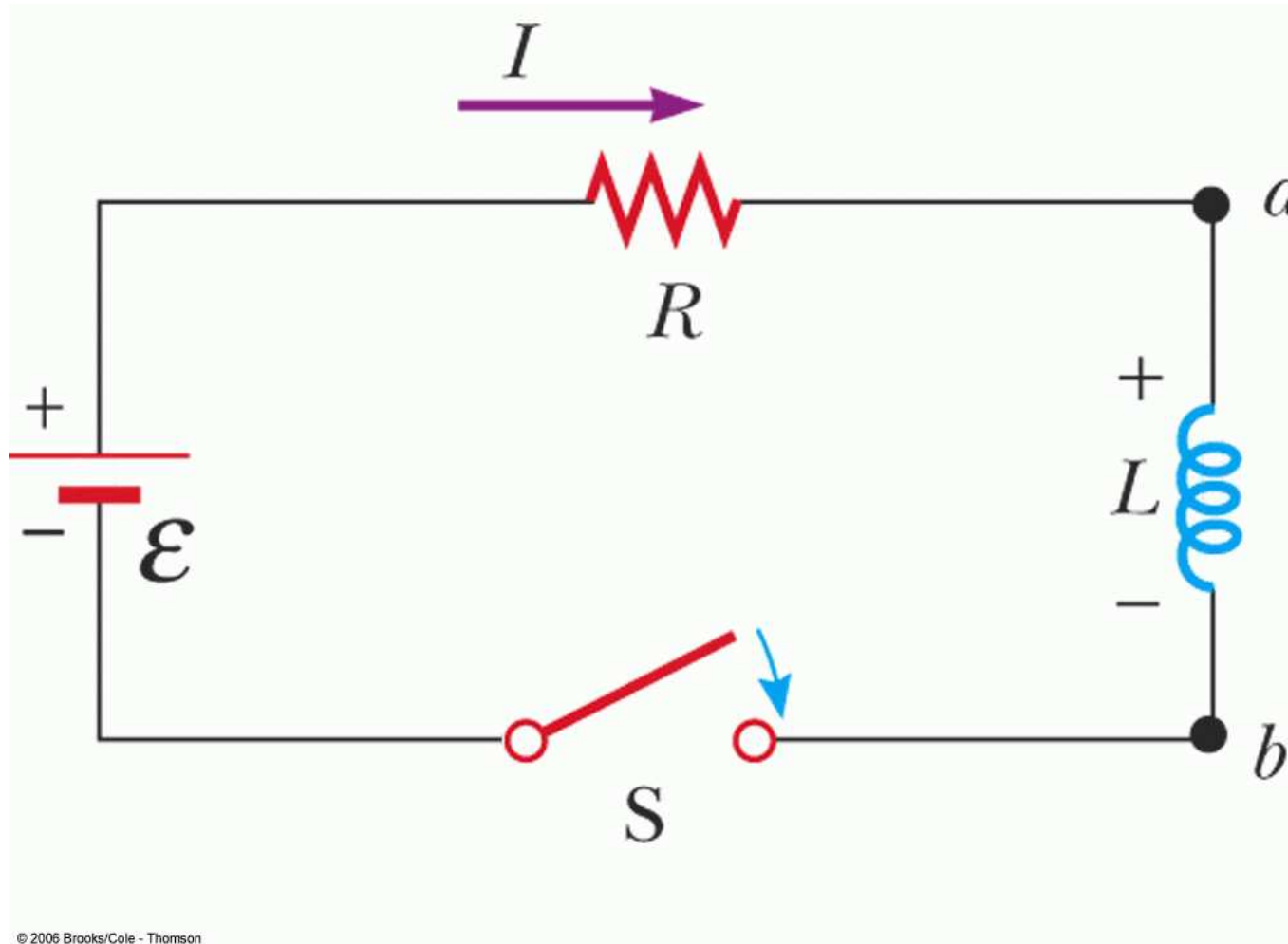
$$V_L = -L \frac{di}{dt}$$

When the switch is closed a current begins to flow creating a magnetic field. This magnetic field is changing as the current rises and induces a *back emf* or *back voltage* that resists the change in the original magnetic field.

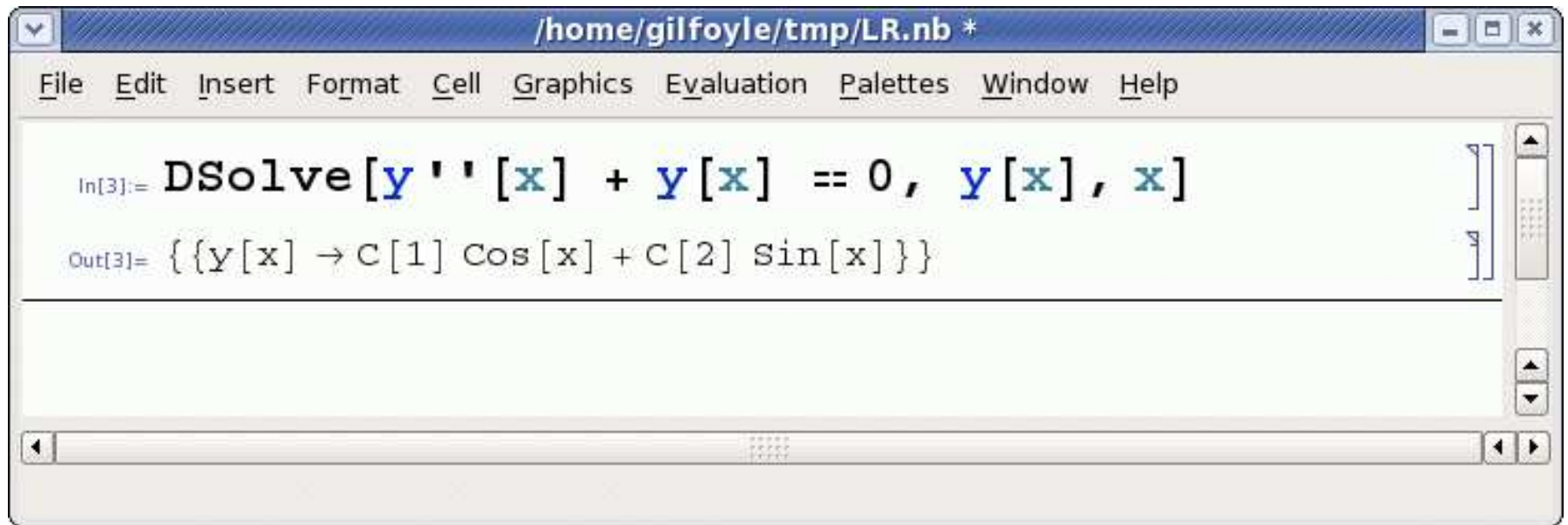


What happens to the current?

Analyze the circuit below for a constant input voltage $\epsilon = V_1$ to obtain the differential equation describing the current.



Using DSolve in Mathematica



The screenshot shows a Mathematica notebook window titled `/home/gilfoyle/tmp/LR.nb *`. The menu bar includes `File`, `Edit`, `Insert`, `Format`, `Cell`, `Graphics`, `Evaluation`, `Palettes`, `Window`, and `Help`. The input cell contains the command `DSolve[y''[x] + y[x] == 0, y[x], x]`. The output cell displays the result `{{y[x] -> C[1] Cos[x] + C[2] Sin[x]}}`. The notebook interface includes a scroll bar on the right and a horizontal scroll bar at the bottom.

```
In[3]:= DSolve[y''[x] + y[x] == 0, y[x], x]
```

```
Out[3]= {{y[x] -> C[1] Cos[x] + C[2] Sin[x]}}
```

An Example of Applying Faraday's Law

Consider the circuit shown below where $\epsilon = V = 6.0\text{ V}$, $L = 8.0 \times 10^{-3}\text{ H}$, and $R = 4.0\ \Omega$. (a) What is the inductive time constant of the circuit? (b) What is the current at a time $t_1 = 250\ \mu\text{s}$ after the switch is closed? (c) What is the final, steady-state current? (d) When does the current reach 80% of its maximum value?

