

# Measuring the Gravitational Constant with a Torsion Balance

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November 12, 2004

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# 1 Introduction

The inquisition for the Higgs particle is the current manifestation of mankind’s yearning to understand gravitation. Over four hundred years ago Isaac Newton performed research with macroscopic and astronomical objects to build the basis of gravitational theory. In 1665, at age 23, Newton “showed that the force that holds the Moon in its orbit is the same force that makes an apple fall.”[2] In his proposition is a force law known now as Newton’s Law of Gravitation:

$$F_G = G \frac{m_1 m_2}{r^2} \tag{1}$$

The law states that every body attracts every other body with a force whose magnitude is given by equation 1. This force includes a constant  $G$  called the universal constant of gravitation.

Early tests of Newton’s law involved measuring the “Attraction of Mountains” since very large masses were required to produce an appreciable deflection of measurement equipment available at the time. This is a consequence of the extreme weakness of the gravitational force at the human scale. In comparison to one’s attraction to Earth, forces due to nearby masses are inconsequential. With this in mind, a very delicate piece of equipment was designed and built by the Revd. John Michell that could measure very weak forces. Unfortunately Michell died before he could use his own design in an experiment.

Despite its name, Cavendish’s experiment was made possible through the work of astronomers, geologists, mathematicians and other scientists that came before him. To his credit, Henry Cavendish was the diligent chap that obtained the final result of the experiment; Cavendish completed the measurement in 1798, five years after Michell’s death.[1] His diligence in experimental techniques and error determinations has lead some to call this experiment the “first modern physics experiment.”[4] As a testament to this claim, Cavendish’s results from this experiment were not improved for over a century.

## 2 Experimental Apparatus

The apparatus used in our experiment is much like that which was used by Cavendish; we use a torsion balance with an optical method of measurement. In our case the optics are improved (incorporating a He-Ne laser) and the read out is supplemented by a PC-based DAQ system. We also improved the isolation of our experiment with an active vibration damping system.

### 2.1 Torsion Balance and Readout

Central to the experiment is the torsion balance. The balance used in our experiment is pictured in figure 1. This design incorporates clever use of symmetry to prevent the Earth’s gravitational force from adversely affecting its measurement. It is extremely sensitive to torsional forces; since it incorporates a tungsten thread as its torsion element, it offers only a very faint resistance to movement. The torsion element is also sensitive to environmental conditions including temperature and air flow. To protect the balance from external contact and air flow it is encased in an aluminum structure with removable glass windows.

Our balance is fundamentally the same as that which was used in Cavendish’s experiment: a suspended boom with two small lead spheres in axial alignment with a boom with two larger lead spheres. The boom with the large weights is fixed along two axes to prevent any pendulous movements; this outer boom can only be rotated along the same axis as the suspended boom. The suspended boom hangs freely from  $\sim 6$  cm of tungsten thread. This thread is carefully tied to upper and lower posts, each allowing for some adjustment. The upper post allows one to raise and lower the inner boom, and rotate it along the vertical axis. The lower post slides and rotates within a groove in the suspended boom.

Our balance allows two different methods of readout. A small mirror is fixed to the center post of the balance for use as an “optical lever.” A symmetric differential capacitive sensor is built into the base of our balance to provide an analog readout. This output provides a signal whose voltage varies as the inner balance rotates. The sensor is especially resistant to pendulous modes (see figure 2 for a graphical explanation) of motion and electrical noise.

An external piece of DAQ hardware (also pictured in figure 1) provides power to the readout electronics and allows for zero-adjustment of the signal from the balance. This hardware feeds the signal to a DAQ card in a PC that records the voltage in two second intervals. Measurements are output in a text file that is analyzed with statistical software including ROOT and Gnuplot.

## 2.2 Vibration Isolation Table

We employ a vibration isolation table in this experiment to minimize errors in measurement caused by forces external to our apparatus. We also wanted to minimize movements in the suspended boom, as our theoretical model of this experiment does not account for such motion. The table also provides us (once properly adjusted) with a very uniform and level surface. This also improves our experiment’s correlation to our theoretical model, which assumes the apparatus is perfectly aligned with the horizon.

The table is a large aluminum slab that rests on four pneumatic pistons. Attached to each piston is a small lever with an adjustable foot that rests against the bottom of the slab. As the slab moves up and down, the lever tracks its movement and adjusts the valve that regulates the corresponding piston’s internal pressure. An air compressor with reservoir sits adjacent to the table, sourcing the levelling pistons. The compressor has an automatic regulator that runs the pump as needed to maintain sufficient pressure in the reservoir.

## 2.3 Laser and Optics

A 0.6 mW Helium-Neon laser made by Uniphase (Model 1508) was incorporated to produce an “optical lever” arrangement. Figure 3 illustrates our arrangement; note the error of  $\pm 0.7$  cm in the measurements. Our primary concern during our design phase was accuracy of measurement. We aimed to minimize the laser-mirror distance to minimize effects of beam spread, and maximize the mirror-yardstick distance to maximize our angular resolution. Another factor minimizing beam spread was a small convex lens held in position by a lab stand. A jack stand was used to center the laser beam within the small mirror on the center post (seen in figure 2).

## 2.4 Other Measurements

We weighed the lead spheres with a triple-beam balance and measured their diameters with a vernier caliper. The caliper was also used to measure dimensions of the balance. The yardstick in figure 3 is fixed to the wall with double-stick tape, and a thin strip of black masking tape is affixed to the portion of the ruler on which the laser beam is incident. The use of black tape reduced the glare from the incident beam, allowing for (slightly) more precise measurement.

## 3 Procedure

### 3.1 Setup

Our first task was to ensure our apparatus had a level surface on which to rest. We opened the air valves to all of the pistons under the isolation table and allowed them to fill until the adjustment lever stopped the flow. We then used a small circular level to check the pitch of the table at various locations. After adjusting the feet on the levers, we achieved a uniformly level table. We set up our equipment as in figure 3 and left it alone overnight so all motion within our balance ceased. The next day we levelled our torsion balance via adjustable feet on the balance’s base. The circular level indicated we had a level balance, now assumed to be on the same plane as our table.

Upon further inspection, we found the suspended boom was not properly aligned with the center post. Using a small pin, John pushed the boom from side to side through small holes in either side of our balance’s enclosure. After some work we left the system alone for another night. Later, we verified the suspended boom appeared to be parallel with the table, but the boom’s equilibrium point was at some large angle to the glass walls of the balance. The upper support was carefully adjusted several times before we achieved a centered boom (note our “centering” here is not exact; this was a difficult task and we allowed for about 5 degrees of error in our determination of “center”). We noted the mirror on the center post was not parallel to the boom, so we corrected for this misalignment by rotating the entire balance. After more settling time we verified we had achieved our intended beam path.

After affixing the yardstick to the far wall and masking it with black tape, we attempted to minimize the beam spot size (to aid measurement) by incorporating a convex lens in the stationary portion of the beam. We avoided putting the lens in the moving portion of the beam to minimize errors caused by optical aberration. While adjusting the position of the lens in the beam, we noticed the spot size at the yardstick shrank as the lens approached the balance. We could only move the lens a certain amount, however, since at some point the reflected beam strikes the edge of our lens. Our final configuration was measured and the results appear as dimensions in our schematic (figure 3).

### 3.2 Measurement

Our DAQ software was simple to operate, although its interface was far from intuitive. We simply turned the software “on” and let it run, and it would record the signal level from our balance once every two seconds. We placed the large masses in position I (see figure 4 for a plan view of the positions) and recorded data as the torsion balance settled into equilibrium

over a 24 hour period. As shown in figure 6, the equilibrium position of the balance changed over time. With this in mind we decided to record only about an hour of data for each adjustment of the large masses.

In our final data run we performed two measurements in position II and one measurement in position I. Each measurement began with a “swap” in position of the large masses (from I to II or II to I) and ended about an hour later. Before swapping the large masses we noted the position of the laser beam by making a pencil mark where the center of the beam was incident on the yardstick. We later used a vernier caliper to measure the distance between our marks, as well as the apparent size of the laser beam.

At the conclusion of our balance measurements we recorded the dimensions of the balance and masses for use in the determination of our final result. Since the masses were not uniformly spherical we took several diameter measurements for each sphere and used an average result in our calculations. Some dimensions could not be directly measured with the caliper (i.e. with the jaws of the caliper touching the apparatus), so we held the caliper up to the dimension in question and aligned the jaws with the apparatus by “eyeball.” This introduced some uncertainty due to parallax; we accounted for our gross measurement technique in our determination of measurement uncertainty.

## 4 Results

An enormous amount of raw data (over 6,300 data points in our final run) was collected during the course of our experiment. To save patience and paper, we present only portions of raw data that are relevant to our final result. Furthermore our raw data will only appear in graphical form. Numerical values that appear in the following tables are results of fitting algorithms or analytical manipulations thereof.

Figure 6 is our first sample of data from the experiment. Shortly after this data was gathered we made our final data run; see figure 7 for our final results with the balance. For reference, time “0” in our data set corresponds to November 3, 2004 6:30:09 PM PST. Our measurements of the spheres’ diameters appear in table 1 and the rest of our balance’s dimensions are shown in table 2.

## 5 Analysis and Interpretation

### 5.1 Theory

The theoretical model of our experiment include many assumptions, made to simplify the equations used in determining  $G$ :

- Only two equal and opposite forces are acting to torque our balance’s suspended boom. There is no drag or friction experienced within the balance.
- Our balance is perfectly symmetric:
  - The large and small masses are perfectly spherical.
  - The large spheres have identical masses; the small spheres have identical masses.

- The separation between the large and small spheres is identical for both equilibrium positions (I and II) and for both sides of our balance.
- The large spheres are positioned such that the lever arm of its attractive force is at a right angle to the boom and in a plane perpendicular to the thread.
- The suspended boom moves only in a plane perpendicular to the thread.
- The suspended boom is parallel to the glass windows when the sphere separation distance is determined.
- We can neglect the gravitational and inertial effects of the suspended boom and center post.
- We can neglect the gravitational effect of the outer boom.
- We can neglect any other external gravitational effects, such as the attractive force of the nearby experimentalists.

Correction factors can be derived to account for all of our assumptions, however, these factors can be negligible compared to the accuracy of our measurements. We will use only one correction factor in our determination of  $G$ .

As described above, we have two forces acting within our balance: the attraction of the nearby large mass, and the restoring force of the torqued thread. Individually, these forces are:

$$\tau_G = 2F_G d \quad (2)$$

$$\tau_\kappa = -\kappa\theta \quad (3)$$

In opposition, these forces equate as  $\tau_G = -\tau_\kappa$ . Including equation 1 this gives us the steady-state equation

$$\kappa\theta = 2dG \frac{m_l m_s}{r^2} \quad (4)$$

which we can rearrange to determine  $G$ . We also define  $b$  as the sphere separation distance (see figure 4).

$$G = \frac{\kappa\theta b^2}{2dm_l m_s} \quad (5)$$

The two unknowns in this equation are  $\kappa$  and  $\theta$ . We use geometrical analysis to reduce  $\theta$  to a linear variable (see figure 5): making the small-angle approximation  $\theta \approx \tan\theta$  we achieve a relation to the beam displacement  $\Delta S$ . Keep in mind the  $\theta$  we observe is actually twice the rotation angle of the boom, due to the law of reflection.

$$2\theta = \frac{\Delta S}{2}L \quad \text{or} \quad \theta = \frac{\Delta S}{4L} \quad (6)$$

The torsional spring constant  $\kappa$  is determined from the solution to classical mechanics' infamous simple harmonic oscillator problem:

$$\omega_0 = \frac{2\pi}{T_0} = \sqrt{\frac{\kappa}{I}} \quad (7)$$

As stated in our assumptions, we will model the suspended boom as massless and only consider the small spheres in determining  $I$ .

$$I = 2m_s(d^2 + \frac{2}{5}r^2) \quad (8)$$

Combining equations 7 and 8 and solving for  $\kappa$ , we achieve our spring constant:

$$\kappa = 2m_s(d^2 + \frac{2}{5}r^2)\omega_0^2 \quad (9)$$

It should be noted that our balance is a *damped* simple harmonic oscillator; its behavior is modeled with the following solution:

$$V(t) = Ae^{-\beta(t-t_0)}\cos(\omega_1(t-t_0) - \delta) + C \quad (10)$$

We define an offset time  $t_0$  at which oscillation begins, and a phase factor  $\delta$ . Since our damping is small (<5%, see table 3 for an example) we make the approximation: [6]

$$\omega_1 = \sqrt{\omega_0^2 - \beta^2} \approx \omega_0 \quad (11)$$

With this approximation we can use our experimental results immediately in determining  $G$ , otherwise, we would have to incorporate another correction factor. A quick calculation shows this factor offers a correction of less than one part in 2000 (using data from table 3).

We have now reduced our determination of  $G$  to a set of experimentally obtainable parameters:  $\omega_0$  and  $\Delta S$ . The rest of the variables in our experiment are obtained simply with a ruler or caliper. Collecting equations 5, 6 and 9 and solving for  $G$  we produce our first approximation:

$$G = \Delta S\omega_0^2 b^2 \frac{d^2 + \frac{2}{5}r^2}{4Lm_1d} \quad (12)$$

As promised we introduce a correction factor to account for the opposite large sphere's attraction to each small sphere. In effect, this correction reduces the attraction between a small sphere and the adjacent large sphere by a factor  $\gamma$ . Using equation 1 and a little more geometry we determine the reduced attraction:

$$F_{net} = F_g - \gamma F_g = F_g(1 - \gamma) \quad \text{where} \quad \gamma = \frac{b^3}{(4d^2 + b^2)^{\frac{3}{2}}} \quad (13)$$

We use this factor to adjust our first approximation thusly:

$$G_1 = \frac{G}{1 - \gamma} \quad (14)$$

Errors are propagated through our analysis using a standard error combination technique:

$$\sigma_G^2 = \sum_{i=1}^n \left(\frac{\partial G}{\partial x_i}\right)^2 \sigma_i^2 \quad (15)$$

In cases where data is averaged, the errors are added in quadrature:

$$\sigma_{\bar{x}} = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \dots \quad (16)$$



## 5.2 Experiment

Some trial and error was needed to fit our damped S.H.O. model (equation 10) to our experimental data as the data contained additional, unmodeled influences. We saw nonuniform acceleration and deceleration of the boom, and small ( $\sim 5\%$ ) impulses that affected the amplitude of oscillation during its expected decay. We had to cut our data down to a few oscillations, beginning at least one-half period after the beginning of the boom's movement. See figures 8, 9 and 10 for the data that survived our cuts. Gnuplot was used to fit equation 10 to this data, and good results were achieved. Looking at the fit results in tables 3, 4 and 5, we saw small fractional errors in all parameters, comparable to the fractional errors in our dimensional measurements (table 2). By this criteria (small fractional errors) and visual inspection of our fits (to ensure errors were not systematic), we concluded our fits were acceptable for use in determining  $G$ . I could not figure out how to produce  $\chi^2$  using Gnuplot and didn't take the time to do it "by hand."

We averaged our repeated measurements ( $r_l$ ,  $r_s$ ,  $m_l$ ,  $m_s$ ,  $\omega_0$ ) and collected them in table 6 with their respective uncertainties. We noted our tick marks accurately represented (as best as our eyeballs could determine) the equilibrium positions of the balance, measured their separation with a vernier caliper, and called it  $\Delta S$ . We now have the data in hand to "plug and chug" through to our result, using equation 12. Careful error analysis reveals the differential error on each parameter (each term in equation 15); see table 7 to verify our largest errors came from uncertainties in measuring the beam spot displacement  $\Delta S$  and the separation of the large and small spheres  $b$ . These were indeed difficult measurements to make!

Before corrections we achieved the result:

$$G = 10.72 \cdot 10^{-11} + / - 2.35 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} \quad (17)$$

This is a 21.9% fractional error. Including our correction (equation 14) we achieve our second result:

$$G_1 = 11.09 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} \quad (18)$$

Since the correction uses terms already included in our determination of  $\sigma_G$ , we assume our corrected result has the same error.

The accepted value of  $G$  is  $6.67300 \cdot 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$ . Our value is 60.6% higher than this value, and our lower error bound is still 25.4% higher. Our inability to even "cover" our expected value is puzzling, given our generous measurement uncertainties.

## 6 Conclusion

This experiment provides many opportunities for error, and the theory requires many accurate measurements to produce a decent result. As the setup was presented to us (in shambles), we could not achieve the accuracy required to come even close to the 1% result featured in the manual for our torsion balance.[5] Throughout the setup and measurement process, we found ways the experiment could be improved:

- Place the yardstick further away from the table (perhaps across the hall) to achieve a more accurate measurement of  $\Delta S$ .
- Add a linear amplifier to the DAQ system to provide sensor measurements with better precision.
- Mechanically isolate the vibration table from the air compressor.
- Regulate the temperature in the lab to prevent “equilibrium drift.”
- Use a different lens to achieve a smaller laser spot.
- Fix the suspended boom to the center post such that the mirror is parallel to the boom.

...and the list goes on. We could also have been more diligent in our analysis, avoiding approximations to prevent further inaccuracies in our final result. Such diligence, however, is only called for when our measurement errors compare to these approximation errors (fractions of a percent).

Sadly, we were not given enough time to implement any of these improvements and repeat our measurements. Since our access to the lab was limited and the setup required long waiting periods while equilibrium was achieved, we had few “windows of opportunity” in which to perform the actual experiment. Our poor results are a testament to our limited resources of time, equipment and experience. Should we repeat this experiment, we are confident that better accuracy can be achieved in all measurements. This experiment was more of a learning experience than an actual determination of a physical constant.

## 7 Tables and Figures

Large sphere 1 (cm)	Large sphere 2 (cm)	Small sphere 3 (cm)	Small sphere 4 (cm)
5.43	5.47	1.36	1.35
5.49	5.48	1.36	1.36
5.46	5.41	1.36	1.36
5.41	5.49	1.36	1.36
5.46	5.46		

Table 1: Diameters of the spheres used in our experiment.



Figure 1: The torsion balance used in our experiment. *Source: TEL-Atomic Incorporated*

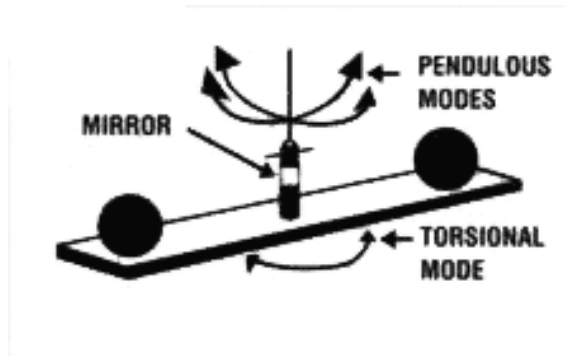


Figure 2: The modes of movement of the inner boom. *Source: TEL-Atomic Incorporated*

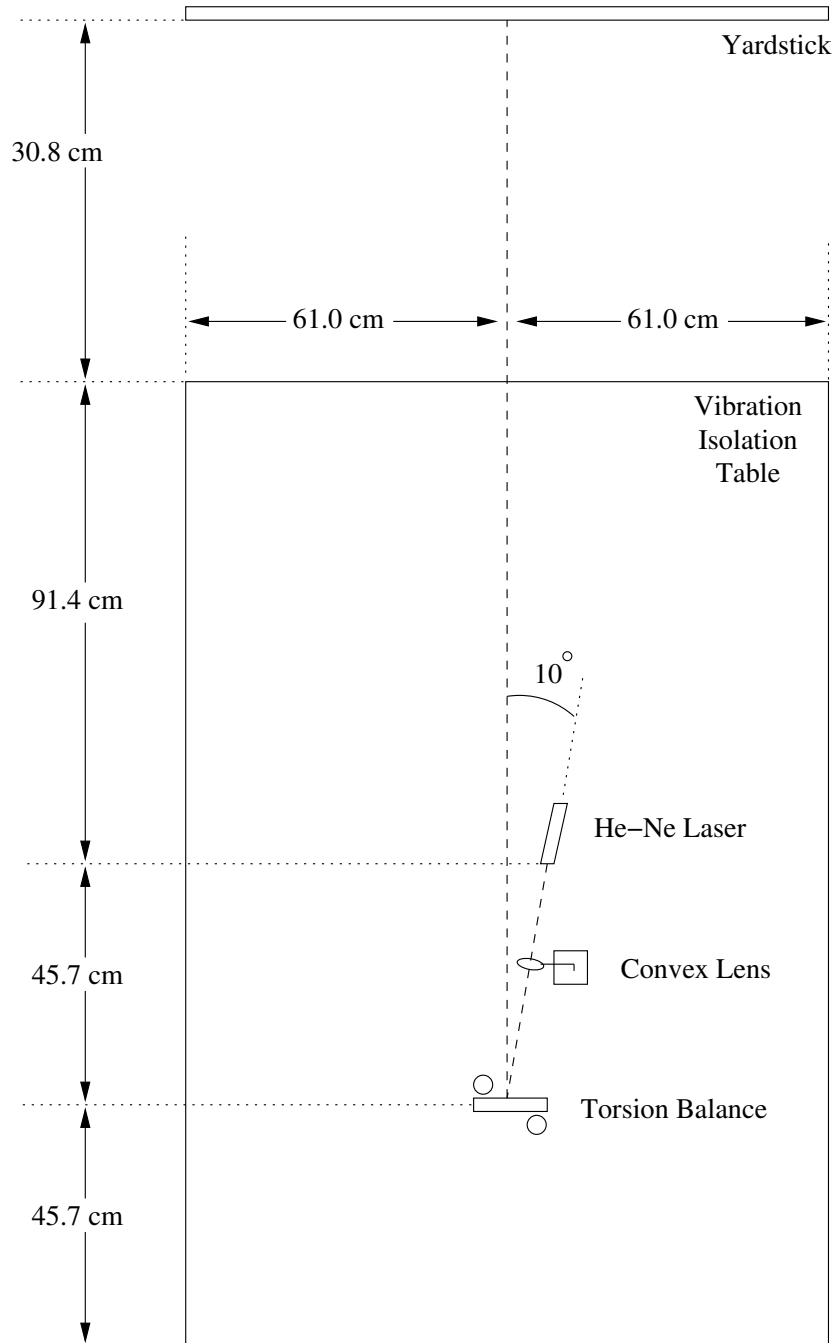


Figure 3: A schematic diagram of our optical lever arrangement. Our intended beam path is indicated with a dashed line. All measurements were made with a yardstick and are subject to an error of  $\pm 0.7$  cm (1/4 inch).

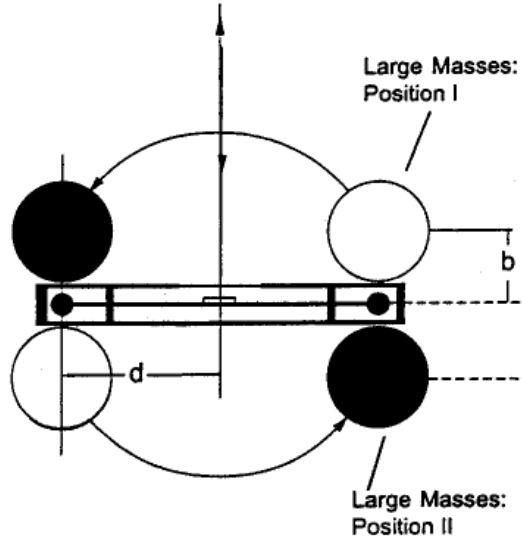


Figure 4: The positions of the large spheres, with dimensions. *Source: PASCO Scientific*

Var.	Dimension	Measurement	Uncertainty	Units
	Beam spot size	3.8	0.2	mm
$\Delta S$	Beam displacement	3.2	0.2	mm
$2 \cdot d$	Dist. betw. small spheres (OC)	13.3	0.1	cm
$b$	Dist. betw. small and large sphere (horizontal, OC)	4.52	0.2	cm
	Dist. betw. small and large sphere (radial, OC)	0	0	cm
	Dist. betw. small and large sphere (vertical, top surface to top surface)	1.31	0.2	cm
	Mass of large sphere 1	917.5	0.1	g
	Mass of large sphere 2	918.3	0.1	g
	Mass of small sphere 3	14.7	0.1	g
	Mass of small sphere 4	14.6	0.1	g

Table 2: Dimensions of the balance used in our experiment. “OC” stands for On Center.

Var.	Value	Absolute error	Fractional error	Units
A	0.0191787	0.0002067	1.078%	V
$\beta$	0.00113951	4.417e-05	3.876%	$\frac{1}{\text{sec}}$
$\omega_1$	0.0390873	3.861e-05	0.09878%	$\frac{\text{rad}}{\text{sec}}$
$t_0$	82700	0	0	sec
$\delta$	1.77528	0.009584	0.5399%	rad
C	-1.01427	6.119e-05	0.006033%	V

Table 3: Parameters of fit in figure 8.

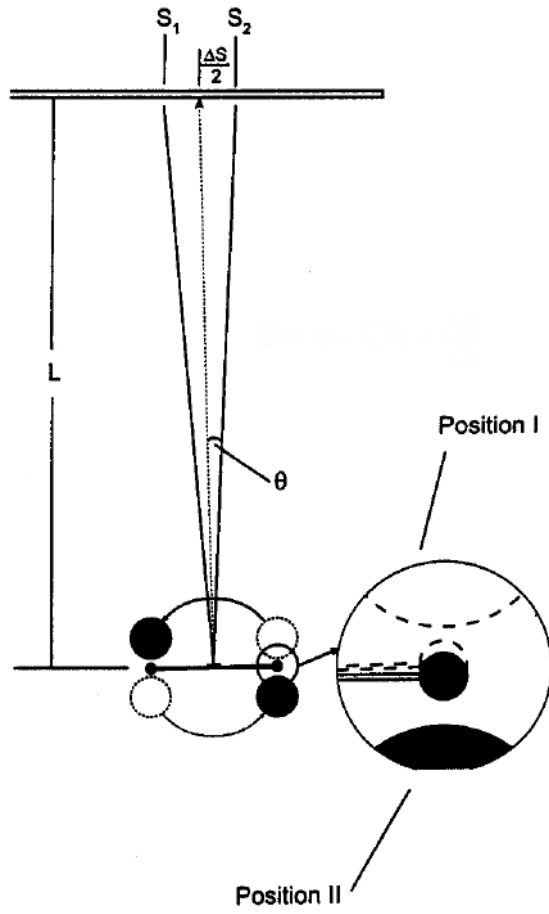


Figure 5: The dimensions of our optical lever arrangement, in two positions. *Source: PASCO Scientific*

Var.	Value	Absolute error	Fractional error	Units
A	-0.029903	0.0002861	0.9569%	V
$\beta$	0.0012693	4.817e-05	3.795%	$\frac{1}{\text{sec}}$
$\omega_1$	0.0386047	4.981e-05	0.129%	$\frac{\text{rad}}{\text{sec}}$
$t_0$	86350	0	0	sec
$\delta$	1.22208	0.0102	0.8344%	rad
C	-0.98473	9.348e-05	0.009493%	V

Table 4: Parameters of fit in figure 9.

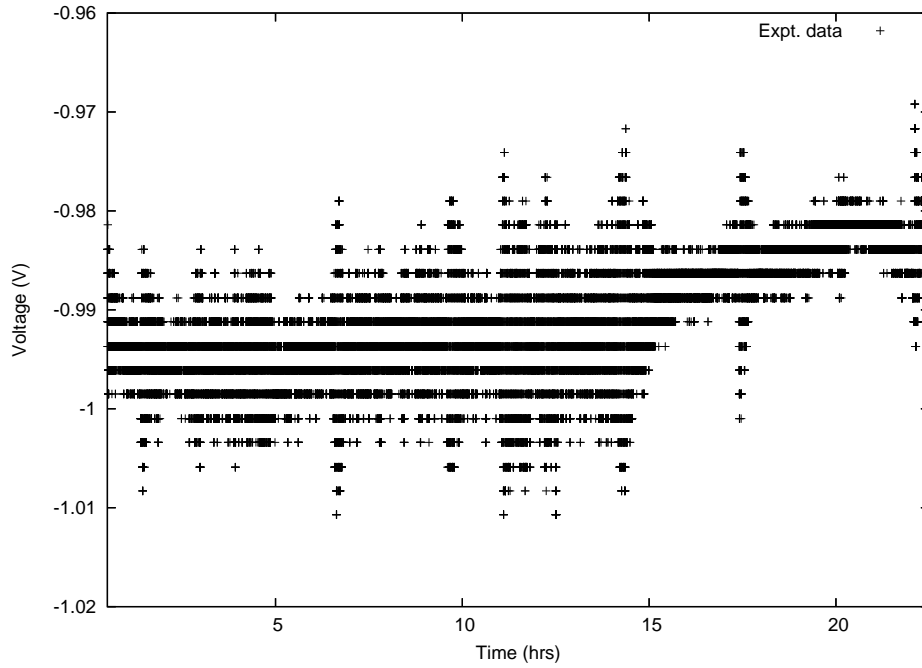


Figure 6: A long period of measurement showed the balance’s equilibrium point drifted over time.

Var.	Value	Absolute error	Fractional error	Units
A	0.0265612	0.0002192	0.8254%	V
$\beta$	0.00113724	3.372e-05	2.965%	$\frac{1}{\text{sec}}$
$\omega_1$	0.0389085	3.087e-05	0.07933%	$\frac{\text{rad}}{\text{sec}}$
$t_0$	89730	0	0	sec
$\delta$	1.84155	0.007355	0.3994%	rad
C	-1.01772	6.546e-05	0.006433%	V

Table 5: Parameters of fit in figure 10.

Var.	Dimension	Measurement	Uncertainty	Units
	Diameter of large sphere	5.46	0.03	cm
$2 \cdot r$	Diameter of small sphere	1.36	0.01	cm
$m_l$	Mass of large sphere	917.9	0.1	g
$m_s$	Mass of small sphere	14.7	0.1	g
	Angular frequency of oscillation	2.3320	0.0042	$\frac{\text{rad}}{\text{min}}$
$\omega_0$		0.03887	0.00007	$\frac{\text{rad}}{\text{sec}}$

Table 6: Averaged dimensions used to obtain final results.

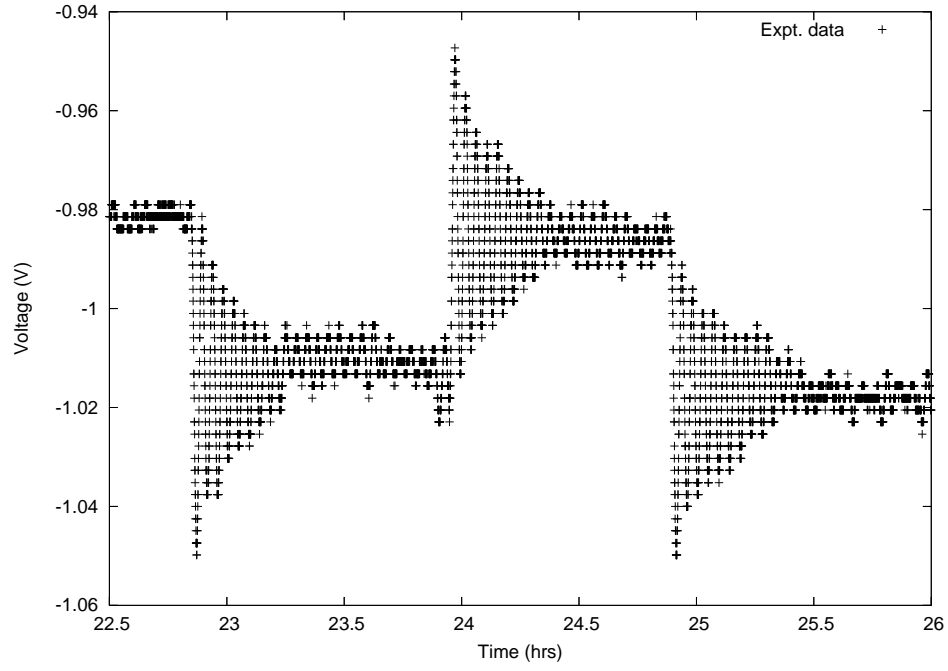


Figure 7: Measurements used to obtain our final result.

Dimension	Value
$\sigma_S$	$1.795893 \cdot 10^{-22}$
$\sigma_b$	$3.600506 \cdot 10^{-22}$
$\sigma_d$	$1.022378 \cdot 10^{-23}$
$\sigma_r$	$1.724830 \cdot 10^{-28}$
$\sigma_\omega$	$1.487970 \cdot 10^{-25}$
$\sigma_m$	$1.363877 \cdot 10^{-28}$
$\sigma_L$	$5.871128 \cdot 10^{-25}$
$\sigma_G$	$2.346487 \cdot 10^{-11}$

Table 7: Differential and final error calculations, for comparison.



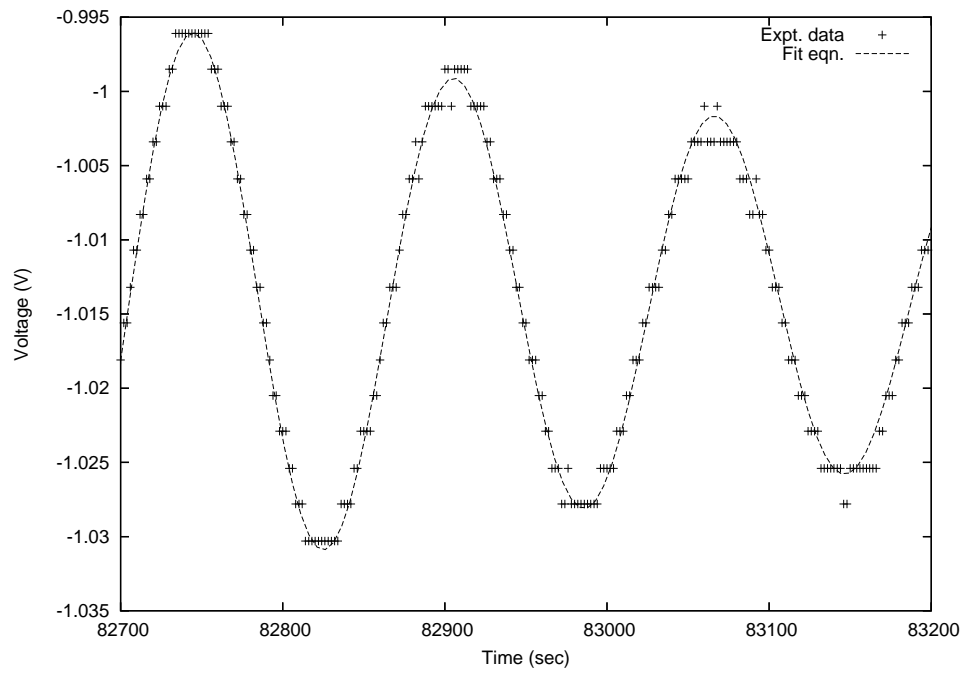


Figure 8: Measurements used to obtain our final result, with fitted equation (see text).

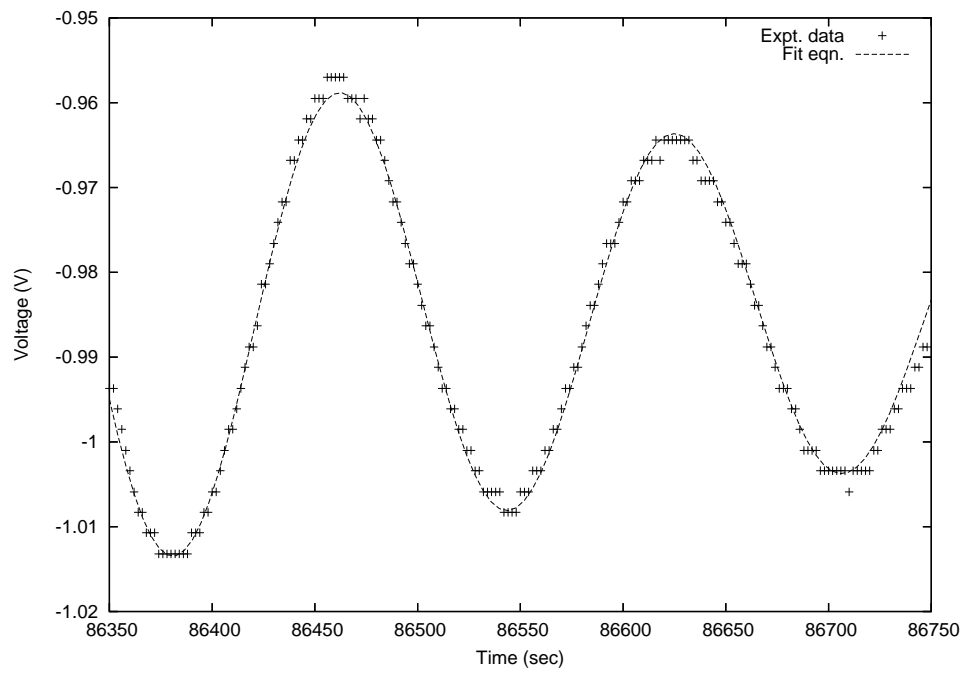


Figure 9: Measurements used to obtain our final result, with fitted equation.

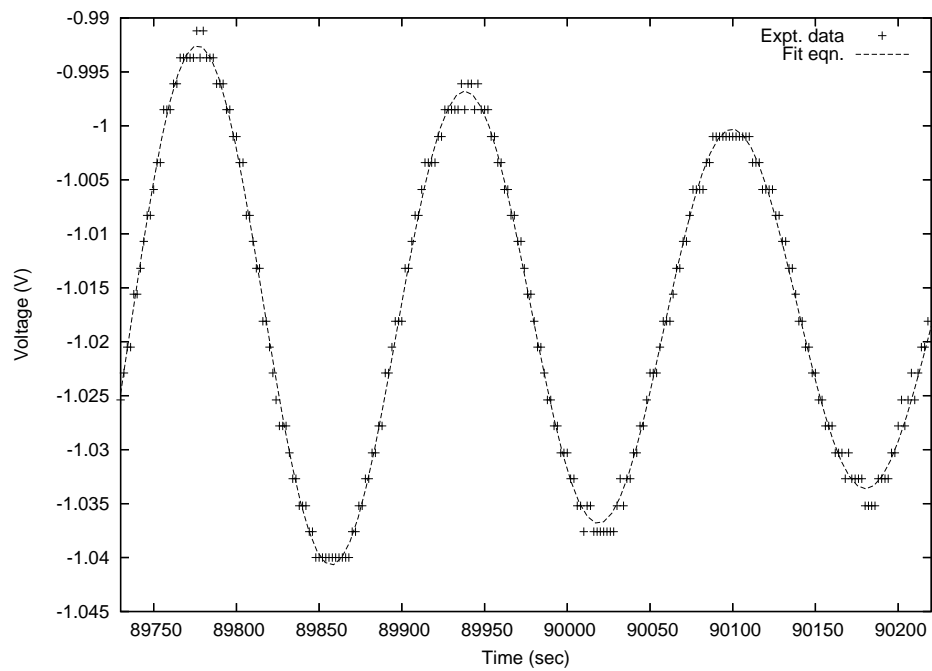


Figure 10: Measurements used to obtain our final result, with fitted equation.

## Acknowledgments

This experiment can be likened (in more ways than one) to a journey into outer space. It was by the work of John Wray and myself, alone, that our mission was accomplished. As such, I would like to acknowledge ourselves for a dedicated and inspired effort. I would also like to acknowledge our patience that was quickly burnt up in the process of rethreading the torsion balance. I take my hat off to those who tie fishing flies.

## References

- [1] Clarendon Laboratory Archive, *History of Boys' experiment*, Retrieved Nov. 09, 2004 from <http://www1.physics.ox.ac.uk/History/BigGHistory.html>
- [2] Halliday, D., Resnick, R., and Walker, J., *Fundamentals of Physics*, 6th ed. (2001) New York: John Wiley and Sons, Inc.
- [3] PASCO Scientific, "012-06802B Gravitational Torsion Balance" manual. (1998) Roseville, CA.
- [4] Saeta, P., *The Cavendish Experiment*, Retrieved Nov. 09, 2004 from <http://kossi.physics.hmc.edu/Courses/p23a/Experiments/Cavendish.html>
- [5] TEL-Atomic, Incorporated, "TEL-RP2111 Computerized Cavendish Balance" manual. (1994) Jackson, MI.
- [6] Thornton, S. and Marrison, J., *Classical Dynamics of Particles and Systems*, 5th ed. (2004) Belmont, CA: Brooks / Cole.