Homework 3 Einstein Solid

1. Consider the following 'gas'. It consists of four atoms in a cubical box. At any instant, there is a 50% chance of each atom being in the left half of the box (L) or the right half (R). Make a table showing all the microstates of this system. (Hint: There are 16.) How many macrostates are there? How many microstates are in each macrostate?

2. Show that for N gas atoms in a box, the number of possible microstates is 2^N when microstates are defined by whether a given molecule is in the left half of the box or the right half of the box. The volumes of each half are equal.

3. Imagine that we have an ideal gas consisting of 15 molecules. We can flip the signs of each of the three velocity components of a given molecule w without changing its overall energy (and thus without changing the gas's macrostate). How many possible patterns of sign choices are there?

4. What is the multiplicity of an Einstein solid with $N = 1$ and $E_{int} = 4\hbar\omega$? List all the microstates.

5. Calculate the multiplicity of an Einstein solid with $N = 1$ and $E_{int} = 5\hbar\omega$ by directly listing and counting the microstates. Check your work by using equation 3 from the lab and shown below.

$$\Omega(N_A, q_A) = \frac{(q_A + 3N_A - 1)!}{q_A!(3N_A - 1)!} \tag{1}$$

6. Calculate the multiplicity of an Einstein solid with $N = 1$ and $E_{int} = 6\hbar\omega$ by directly listing and counting the microstates. Check your work by using equation 3 from lab and shown in Problem 5 above.

7. Use equation 3 from lab and shown in Problem 5 above to calculate the multiplicity of an Einstein solid with $N = 4$ and $E_{int} = 10\hbar\omega$.

8. Use equation 3 and shown in Problem 5 above to calculate the multiplicity of an Einstein solid with $N = 3$ and $E_{int} = 15\hbar\omega$.

9. How many times more likely is that the combined system of solids described in the table below will be found in macropartition 3:3 than in macropartition 0:6, if the fundamental assumption is true?

10. How many times more likely is it that the combined system of solids describe in the table below will not be found in macropartition 3:3 than it is to be found in macropartition 0:6, if the fundamental assumption is true?

Macropartition	E_A	E_B	Ω_A	Ω_B	Ω_{AB}
0:6	0	6	1	28	28
1:5	1	5	3	21	63
2:4	2	4	6	15	90
3:3	3	3	10	10	100
4:2	4	2	15	6	90
5:1	5	1	21	3	63
6:0	6	0	28	1	28
				Total=	462

Table 1: Possible macropartitions for $N_A = 1$, $N_B = 1$, $E_{int} = 6\hbar\omega$.

11. Consider the system consisting of a pair of Einstein solids in thermal contact. A certain macropartition has a multiplicity of 3.7×10^{1024}, while the total number of microstates available to the system in all macropartitions is 5.9×10^{1042}. If we look at the system at a given instant of time, what is the probability that we will find it to be in our certain macropartition?

12. Consider the system consisting of a pair of Einstein solids in thermal contact. A certain macropartition has a multiplicity of 1.2×10^{346}, while the total number of microstates available to the system in all macropartitions is 5.9×10^{362}. If we look at the system at a given instant of time, what is the probability that we will find it to be in our certain macropartition?

13. Consider the system consisting of a pair of Einstein solids in thermal contact. Imagine that it is initially in a macropartition that has a multiplicity of 8.8×10^{123}. The adjacent macrostate closer to the equilibrium macrostate has a multiplicity of 4.2×10^{1234}. If we look at the system a short later, how many times more likely is it to have moved to the second macropartition than to have stayed with the first?

14. Consider the system consisting of a pair of Einstein solids in thermal contact. Imagine that it is initially in a macropartition that has a multiplicity of 7.6×10^{3235}. The adjacent macropartition closer to the equilibrium macropartition has a multiplicity of 4.1×10^{3278}. If we look at the system a short time later, how many times more likely is it to have moved to the second macropartition than to have stayed with the first?

15. Suppose you put 100 pennies in a cup, shake it up, and toss them all into the air. (a) After landing, how many different head-tail arrangements (microstates) are possible for the hundred pennies? (b) What is the probability of finding exactly 50 heads? (c) 49 heads? (d) 1 head?

16. You ask your roommate to clean up a mess he or she made in your room. Your roommate refuses, because cleaning up the mess would violate the second law of thermodynamics, and campus security's record of your roommate's legal violation is already excessive. Gently but firmly explain why complying will not put your roommate at risk of such an infraction.

17. The classic statement of Murphy's law reads, 'If something can go wrong, it will.' Explain how this is really a consequence of the second law of thermodynamics. (Hint: What is the entropy of 'wrong' in a given context compared to the entropy of 'right'?)

18. Run the *StatMech* program to answer the questions below.

    (a) For two Einstein solids in contact with $N_A = N_B = 100$ and $E_{int} = 200\hbar\omega$ answer the following questions. (1) How many times more likely is the system to be found in the center macropartition than in the extreme macropartition where $E_A = 0$ and $E_B = 200\hbar\omega$ (2) What is the range of values that E_A is likely to have more than 99.98% of the time? (3) if E_A were initially to have the extreme value 0, how many times more likely is it to move to the next macropartition nearer the center than to remain in the extreme one?

    (b) Answer the same question as in (a) for a run where you scale everything up by a factor of 10, so that $N_A = N_B = 1000$ and $E_{int} = 2000\hbar\omega$.

    (c) Answer the same question as in (a) for a run where $N_A = N_B = 1000$ and $E_{int} = 200\hbar\omega$. Comment on the effect that increasing just the size of the system by a factor of 10 has on these answers.

    (d) Answer the same question as in (a) for a run where $N_A = N_B = 100$ and $E_{int} = 2000\hbar\omega$. Comment on the effect that increasing just the energy available to the system by a factor of 10 has on these answers.

19. Consider two Einstein solids in thermal contact. The solids have different values of N but are identical in all other respects. It is plausible, since every atom in the combined system is identical, that in equilibrium the energy will be distributed among the solids in such a way that the average energy per atom is the same. Use *StatMech* to test this hypothesis in the situation where $E_{int} = 1000\hbar\omega$ and N_A and N_B have various different values such that $N_A + N_B = 1000$. (Set `Max Rows` to 1000 so that you can see every macropartition).

(a) Is it true in most cases that in the most probable macropartition the solids have energies such that the average energy per atom in each is the same? Is it strictly true in every case? Answer these questions by discussing the values N_A and N_B you tested, and whether the actual most probable macropartition is the same as that predicted by the hypothesis.

(b) In any case where the hypothesis does not work, does increasing both N_A and N_B by a factor of 10 or 100 (but leaving U alone) yield a result more or less consistent with the hypothesis?

(c) Speculate as to the value of this hypothesis in the large-N limit.

20. For the following questions, you will find that using *StatMech* is by far the fastest way to calculate the multiplicity.

    (a) What is the entropy of an Einstein solid with 5 atoms and an energy of $15\hbar\omega$? Express your answer as a multiple of k_b.

    (b) What is the entropy of an Einstein solid with 50 atoms and an energy of $100\hbar\omega$? Express your answer as a multiple of k_b.

21. A certain macropartition of two Einstein solids has an entropy of $305.2k_b$. The next macropartition closer to the most probable one has an entropy of $335.5k_b$. If the system is initially in the first macropartition and we check it again later, how many times more likely is it to have moved to the other than to have stayed in the first?

22. My calculator cannot display e^x for $x > 230$. One can calculate e^x for larger values of x as follows. Define y such that $x = y \ln 10$. This means that $e^x = e^{y \ln 10} = (e^{\ln 10})^y = 10^y = 10^{x \ln 10}$. Note that we can calculate 10 raised to a non-integer power (for example, 103.46) as follows: $10^{3.46} = 10^{3+0.46} = 10^3(10^{0.46}) = 2.9 \times 10^3$. Use these techniques to solve the following problem. The entropy of the most probably macropartition for a certain system of Einstein solids is $6025.3k_b$, while the entropy of an extreme macropartition is only $5755.4k_b$. What is the probability of finding the system at a given time in the extreme macropartition compared to that of finding it in the most probable macropartition?

23. In principle, the entropy of a isolated system decreases a little bit whenever random processes cause its macropartition to fluctuate away from the most probable macropartition. We can certainly see this with small systems. But is this really a possibility for a typical macroscopic system? Imagine that we can measure the entropy of a system of two solids to within 2 parts in 1 billion. This means that we could just barely distinguish a system that has an entropy of 4.99999999 J/K (eight 9s!) from one that has 5.00000000 J/K. (This is a reasonable entropy for a macroscopic system).

    (a) Imagine that the entropy of the equilibrium macropartition is 5.00000000 J/K. Show that the approximate probability that at any given time later we will find the system in a macropartition with entropy 4.99999999 J/K (*i.e.*, with an entropy that is only barely measurably smaller) is about 10315,000,000,000,000 times smaller that the probability we will still find it to have entropy 5.00000000 J/K. (Hint: See problem 17.)

    (b) Defend the statement that the entropy of an isolated system in thermal equilibrium never decreases.