Homework 9   Quanta

1. What is the wavelength, in nm, of a photon with energy (a) 0.30 eV, (b) 3.0 eV, and (c) 30 eV? For each, is this wave- length visible, ultraviolet, or infrared light?

2. For what wavelength of light does a 100 mW laser deliver 2.50×10^{17} photons per second?

3. Through what potential difference must an electron be accelerated from rest to have a de Broglie wavelength of 500 nm?

4. What is the de Broglie wavelength of a neutron that has fallen 1.0 m in a vacuum chamber, starting from rest?

5. The figure is an energy-level diagram for a simple atom. What wavelengths appear in the atom's (a) emission spectrum and (b) absorption spectrum?

6. The allowed energies of a simple atom are 0.00 eV, 4.00 eV, and 6.00 eV.

    (a) Draw the atom's energy-level diagram. Label each level with the energy and the quantum number.

    (b) What wavelengths appear in the atom's emission spectrum?

    (c) What wavelengths appear in the atom's absorption spectrum?

$n = 3$ ——————— $E_3 = 4.00$ eV

$n = 2$ ——————— $E_2 = 1.50$ eV

$n = 1$ ——————— $E_1 = 0.00$ eV

7. How much energy does it take to ionize a hydrogen atom that is in its first excited state?

8. What is the third-longest wavelength in the absorption spectrum of hydrogen?

9. Two hydrogen atoms collide head-on. The collision brings both atoms to a halt. Immediately after the collision, both atoms emit a 121.6 nm photon. What was the speed of each atom just before the collision?

10. A beam of electrons is incident upon a gas of hydrogen atoms.

    (a) What minimum speed must the electrons have to cause the emission of 656 nm light from the $3 \rightarrow 2$ transition of hydrogen?

    (b) Through what potential difference must the electrons be accelerated to have this speed?

11. The wave function for a particle is

$$\psi(x) = \sqrt{\frac{a}{\pi(x^2 + a^2)}}$$

    for $a > 0$ and $-\infty < x < +\infty$. Determine the probability that the particle is located somewhere between $x = -a$ and $x = +a$.

12. An electron is contained in a one-dimensional box of length 0.100 nm. (a) Draw an energy level diagram for the electron for levels up to $n = 4$. (b) Find the wavelengths of all photons that can be emitted by the electron in making downward transitions that could eventually carry it from the $n = 4$ state to the $n = 1$ state.

13. The nuclear potential energy that binds protons and neutrons in a nucleus is often approximated by a square well. Imagine a proton confined in an infinitely high square well of length 10.0 fm, a typical nuclear diameter. Calculate the wavelength and energy associated with the photon emitted when the proton moves from the $n = 2$ state to the ground state. In what region of the electromagnetic spectrum does this wavelength belong?

14. A photon with wavelength λ is absorbed by an electron confined to a box. As a result, the electron moves from state $n = 1$ to $n = 4$. (a) Find the length of the box. (b) What is the wavelength of the photon emitted in the transition of that electron from the state $n = 4$ to the state $n = 2$?

15. The wave function of a particle is given by

$$\psi(x) = A\cos(kx) + B\sin(kx)$$

where A, B, and k are constants. Show that ψ is a solution of the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

assuming the particle is free ($U = 0$), and find the corresponding energy E of the particle.

16. The wave function of an electron is

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$$

Calculate the probability of finding the electron between $x = 0$ and $x = L/4$.

17. A particle of mass $m = 2.0 \times 10^{-28}$ kg is confined to a one-dimensional box of length $\ell = 1.0 \times 10^{10}$ m. For $n = 1$, what are (a) the particle's wavelength, (b) its momentum, and (c) its ground-state energy?

18. Imagine that a particle has a wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{a}}e^{-x/a} & \text{for} > 0 \\ 0 & \text{for} < 0 \end{cases}$$

19. An atom (not a hydrogen atom) absorbs a photon whose frequency is $f = 6.2 \times 10^{14}$ Hz. How much does the energy of the atom increase?

20. What is the energy of the hydrogen atom electron whose probability density is represented by the dot plot shown below? What minimum energy is needed to remove this electron from the atom?
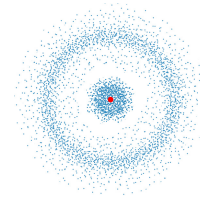
21. What are the energy and wavelength of a photon emitted when a hydrogen atom undergoes a transition from the $n = 3$ state to the $n = 1$ state?

22. What is the wavelength of the least energetic photon emitted in the Lyman series (where $n_f = 1$) of the hydrogen atom spectrum lines?

23. What is the series limit for the Lyman series (where $n_f = 1$)?

24. A Russian Arktica satellite that monitors polar weather follows an elliptical orbit around the Earth at an altitude of $h = 300$ km above the surface (radius $r_s = 6.67 \times 10^6$ m) at a velocity

$$\vec{v} = 4.1 \times 10^3 \ m/s \ \hat{r} + 7.5 \times 10^3 \ m/s \ \hat{\theta} \qquad .$$

What is the angular momentum? What is the total energy? What is the distance of closest approach to the Earth? The satellite mass is $m_s = 600$ kg.

Arktica-M

25. The WorldView-4 satellite is in a circular orbit around the Earth at an altitude of $h = 681$ km above the surface at a speed $v_0 = 7.52 \times 10^3$ m/s. It quickly burns it rockets pointing in a direction away from the Earth and parallel to the direction of the gravitational force. It reaches a new mechanical energy $ME_1 = -1.14 \times 10^{10}$ J. What will be its new angular momentum and new distance of closest approach to the Earth? Is this rocket burn a good idea? The satellite mass is $m_s = 726$ kg.
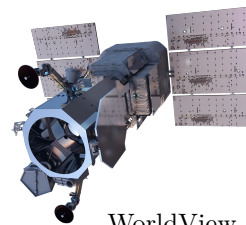
WorldView-4

26. A satellite of mass $m_s = 726$ kg is orbiting the Earth and has a perigee (closest point to the center of the Earth) $r_p = 1.31 \times 10^7$ m with a velocity $v_p = 7.5 \times 10^3$ m/s at that point. How far away is apogee r_a (farthest point from the center of the Earth)?

27. A satellite of mass $m_s = 800$ kg is orbiting the Earth with a velocity $\vec{v}_i = 5.3 \times 10^3$ m/s $\hat{r} + 6.8 \times 10^3$ m/s $\hat{\theta}$ at a distance $r_s = 10^7$ m from the Earth. It has to burn its rocket so the apogee r_a (farthest point from the center of the Earth) is $r_a = 1.5 \times 10^8$ m. How much does its speed have to increase?