Homework 4 Entropy

1. (E) An object's entropy is measured to increase by 0.1 J/K when we add 35 J of energy. What is its approximate temperature? (Assume that the object's temperature does not change much when we add the 35 J.)

2. (E) A certain Einstein solid's entropy changes from $305.2k_b$ to $338.1k_b$ when we add 1 unit $\hbar\omega$ of energy. What is the value (and units) of $k_bT/\hbar\omega$ for this solid? If $\hbar\omega = 1.0\ eV$, what is its temperature T?

3. (E) Does it make sense to talk about the temperature of a vacuum? If so, how could you measure or calculate it? If not, why not?

4. (E) An Einstein solid in a certain macrostate has a multiplicity of 3.8×10^{280}. What is its entropy (expressed as a multiple of k_B)?

5. (E) A pair of Einstein solids in a certain macropartition has multiplicities of 4.2×10^{320} and 8.6×10^{132}. What are the entropies of each solid? What is the total entropy of the system in this macropartition? (Express entropies as multiples of k_b.)

6. (E) Is it really true that the entropy of an isolated system consisting of two Einstein solids never decreases? (Consider a pair of very small solids.) Why is this statement more accurate for large systems than for small systems? Explain in your own words.

7. (E) A certain macropartition (call it microstate 1) of two Einstein solids has an entropy of $180k_b$. The next macropartition closer to the most probable one has an entropy of $205k_b$ (call it microstate 2). If the system is initially in microstate 1 and we check it again later, Will the system be more likely to be in microstate 1 or 2? By what factor? In other words, what is the ratio of the probabilities for being in microstates 1 and 2?

8. (P) In lab we argued on fairly fundamental grounds that $dS/dE = f(T)$. In principle, we could define $f(T)$ to be anything that we like: this would amount to defining temperature and its scale. Still, some definitions would violate deeply embedded preconceptions about the nature of temperature. For example, the simplest definition of temperature would be $dS/dE = T_{new}$. Show that this definition
   (a) Would imply that T_{new} has units of K^{-1} and
   (b) Would imply that heat would flow spontaneously from objects with low T_{new} to objects with high T_{new}. This would imply that object with low values of T_{new} are hot, while objects with high values T_{new} are cold (we might want to call T_{new} so defined *coolness* instead of *temperature*). While we could define temperature in this way, it would really fly in the face of convention (if not intuition).
   (c) If we did define coolness T_{new} in this way, what ordinary temperature T would an object with absolutely zero coolness ($T_{new} = 0$) have? What about something that is infinitely cool ($T_{new} = \infty$)?

9. (P) Imagine that the entropy of a certain substance as a function of N and E is given by the formula $S = Nk_b \ln E$. Using the definition of temperature, show that the thermal energy of this substance is related to its temperature by the expression $E = Nk_bT$.

10. (P) Imagine that the multiplicity of a certain substance is given by $\Omega(E, N) = Ne^{\sqrt{NE/\hbar\omega}}$, where $\hbar\omega$ is some unit of energy. How would the energy of an object made out of this substance depend on its temperature? Would this be a 'normal' substance in our usual sense of temperature.

11. (P) Consider an Einstein solid having $N = 20$ atoms.
    (a) What is the solid's temperature when it has an energy of $10\hbar\omega$, assuming that $\epsilon = h\omega = 0.02eV$? Calculate this directly from the definition of temperature by finding S at $10\hbar\omega$ and $11\hbar\omega$, computing $dS/dE \approx [S(11\hbar\omega) - S(10\hbar\omega)]/\hbar\omega$, and then applying the definition of temperature. (You will find that your work will go faster if you use *StatMech* to tabulate the multiplicities.)

(b) How does this compare with the result from the formula $E = 3Nk_bT$ (which is only accurate if N is large and $E/3N\hbar\omega > 1$)?

(c) If you have access to *StatMech*, repeat for $N = 200$ and $E = 100\hbar\omega$. (Hint: If your calculator cannot handle numbers in excess of 10^{100}, use the fact that in $(a \times 10^b) = \ln a + b \ln 10$).

12. (P) A newly-created material has a multiplicity

$$\Omega = \alpha N E$$

where N is the number of atoms in the solid, E is the total, internal energy in the solid, and α is a constant of proportionality.

(a) How does the temperature of the new material depend on the internal energy?

(b) What is the molar heat capacity for this solid?

(c) Could this material really exist? Why or why not?

13. (P) A newly-created material has a multiplicity

$$\Omega = \beta M E^2$$

where N is the number of atoms in the solid, E is the total, internal energy in the solid, and α is a constant of proportionality.

(a) How does the temperature of the new material depend on the internal energy?

(b) What is the molar heat capacity for this solid?

(c) Could this material really exist? Why or why not?

14. A newly-created material has a multiplicity

$$\Omega = \alpha N E^{3/2}$$

where N is the number of atoms in the solid, E is the total internal energy in the solid, and α is a constant. How is the energy E of the material related to the temperature T? What is the molar specific heat? Does this result make sense? Explain.

15. Imagine that the entropy of a certain substance as a function of N and E is given by the formula $S = \alpha N k_b E^3$. Using the definition of temperature, find an expression for the thermal energy E of this substance in terms of its temperature T, the number of particle N, and any other necessary quantities. Is this result well behaved?

16. Consider the multiplicity of an Einstein solid.

$$\Omega = \frac{(q + 3N - 1)!}{q!(3N - 1)!}$$

We need to obtain the derivative of Ω with respect to the number of quanta q. To do that approximate the derivative as the slope between adjacent values of $\Omega(q)$ where q is the number (integer) of quanta so

$$\frac{d\Omega}{dq} \approx \frac{\Omega(q+1) - \Omega(q)}{(q+1) - q}$$

and show the following assuming $q >> 1$.

$$\frac{d\Omega}{dq} = \frac{3N}{q}\Omega$$