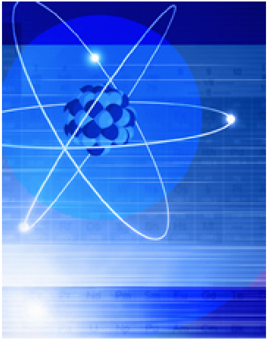
What is the Energy of an Electron? 1

The Coulomb force binds an electron and a proton into a hydrogen atom with a force that is mathematically identical to the gravitational force that binds the planets in our Solar System, the Moon to the Earth, *etc.* What is the energy E_e of an electron?



- The Organizing Principle.

$$
\begin{aligned}
ME_0 &= ME_1 \\
KE_0 + PE_0 &= ME_1 + PE_1 \\
\frac{1}{2}mv_0^2 + PE_0 &= \frac{1}{2}mv_1^2 + PE_1
\end{aligned}
$$

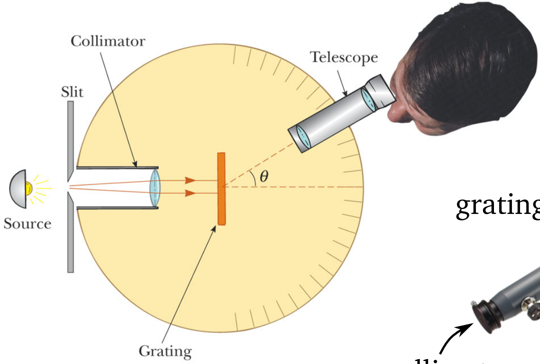
- The Forces

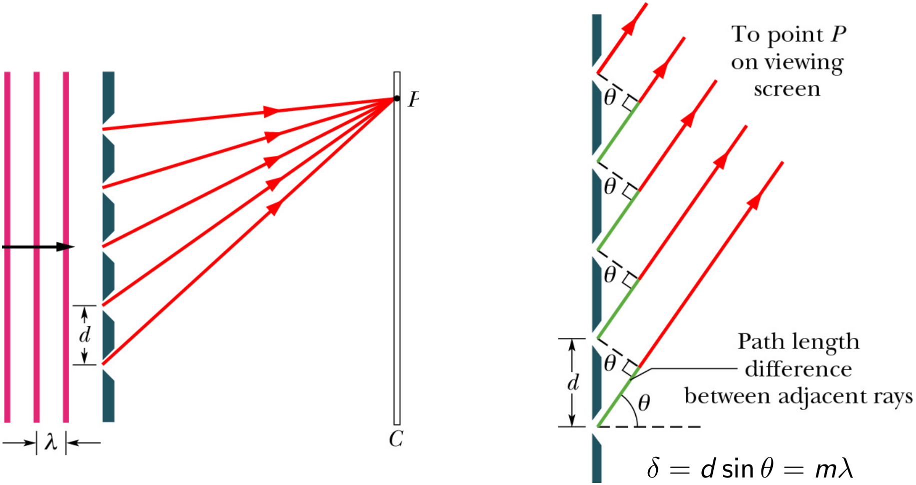
$$
\vec{F}_{grav} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{F}_{coul} = \frac{k_e q_1 q_2}{r_{12}^2}\hat{r}_{12}
$$

The simulation is here.

Atomic Spectroscopy - 1 3

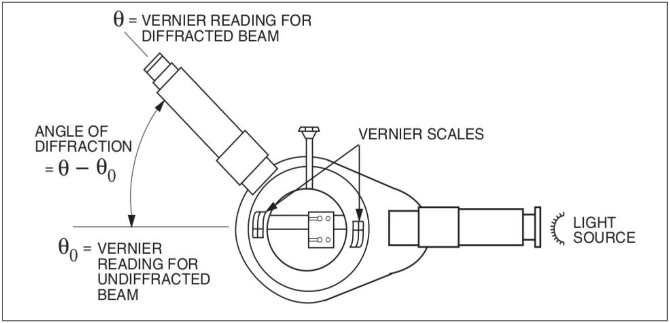
Light from a hydrogen spectrum tube is incident on a diffraction grating in a spectrometer. A narrow, red line appears at $\theta_1 = 23.2^\circ$. The grating has a line density of 600 lines/mm. What is the wavelength of the light? What is the energy of the photons?





$$\delta = d \sin\theta = m\lambda$$
$$m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

When analyzing a light source, angles of diffraction are measured using the vernier scales. However, the scales only measure the relative rotational positions of the telescope and the spectrometer table base. Therefore, before making a measurement, it's important to establish a vernier reading for the undeflected beam. All angles of diffraction are then made with respect to that initial reading (see figure). To obtain a vernier reading for the undeflected beam, first align the vertical cross-hair with the fixed edge of the slit image for the undeflected beam. Then read the vernier scale. This is the zero point reading θ_0.



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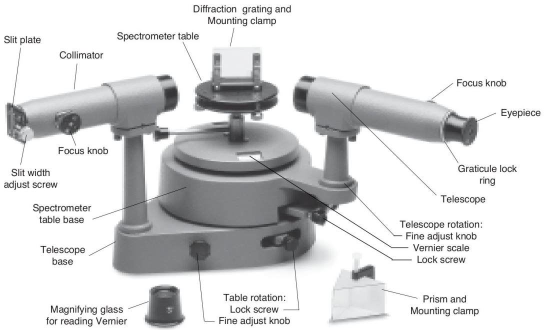
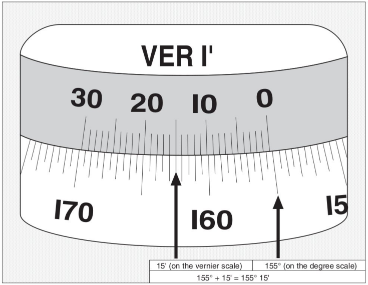
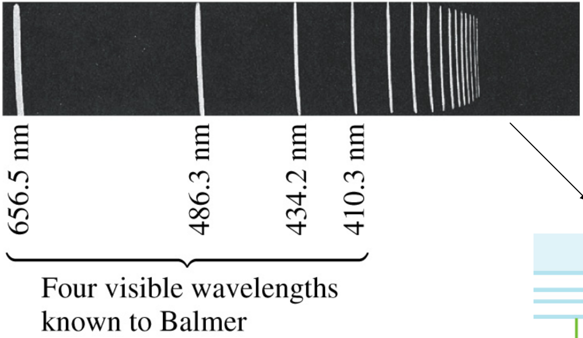
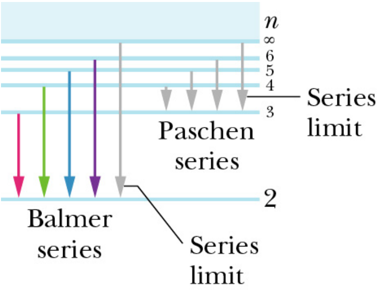


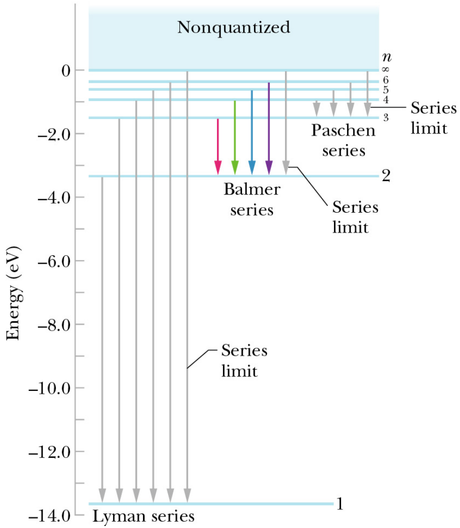
1. To obtain a vernier reading for the undeflected beam, first align the vertical cross-hair with the fixed edge of the slit image for the undeflected beam. Then read the vernier scale. This is the zero point reading θ_0.
2. To read the angle, first find where the zero point of the vernier scale aligns with the degree plate and record the value. If the zero point is between two lines, use the smaller value. In the figure below, the zero point on the vernier scale is between the 155° and $155^\circ 30'$ marks on the degree plate, so the recorded value is 155°.
3. Now use the magnifying glass to find the line on the vernier scale that aligns most closely with any line on the degree scale. In the figure, this is the line corresponding to a measurement of 15 minutes of arc. Add this value to the reading recorded above to get the correct measurement to within 1 minute of arc: that is, $155^\circ + 15' = 155^\circ\ 15'$.



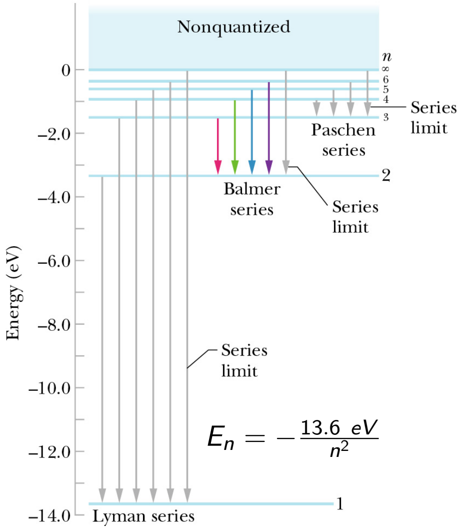


Transition	Energy (eV)
$5 \to 2$	2.856
$4 \to 2$	2.550
$3 \to 2$	1.889



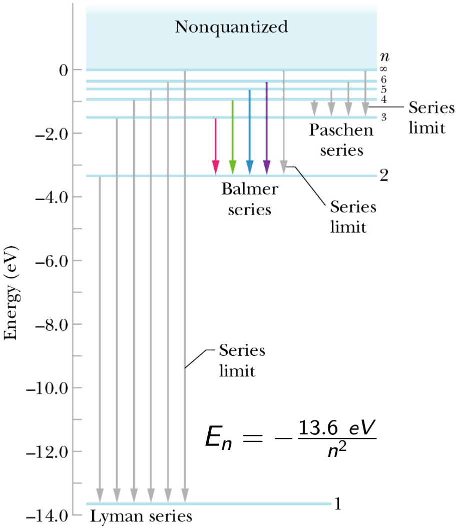

Nonquantized
n
∞
6
5
4
3
2
1
Series limit
Paschen series
Balmer series
Series limit
Series limit
Lyman series
Energy (eV)
0
–2.0
–4.0
–6.0
–8.0
–10.0
–12.0
–14.0

Hydrogen Quantum Numbers 10



$$E_n = -\frac{13.6\ eV}{n^2}$$

Hydrogen Quantum Numbers



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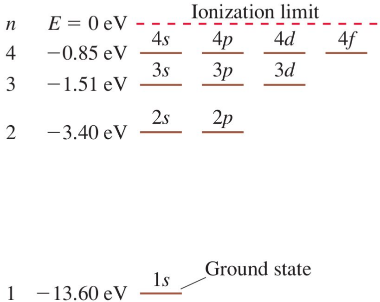
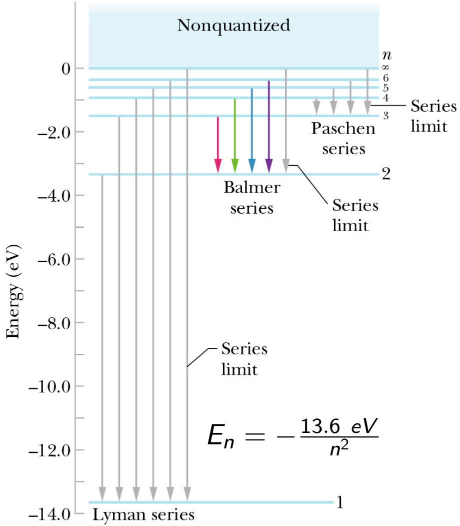


Hydrogen Quantum Numbers



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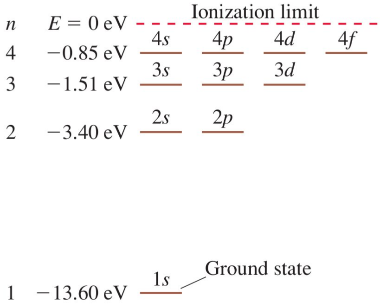
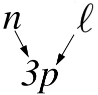




n - principal quantum number

ℓ - angular momentum

Our galaxy is filled with large gas clouds left over from its formation. The light emitted from these clouds can tell us about their composition and the nature of the processing going inside them. An astronomer has measured an emission line with a wavelength $\lambda = 1216$ Å (see figure). Does this line indicate the presence of hydrogen in the cloud?

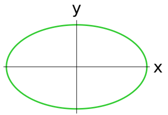
Transition	Energy (eV)
$5 \to 2$	2.856
$4 \to 2$	2.550
$3 \to 2$	1.889

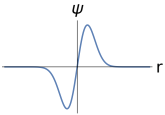
Electron Scattering on Hydrogen. 14

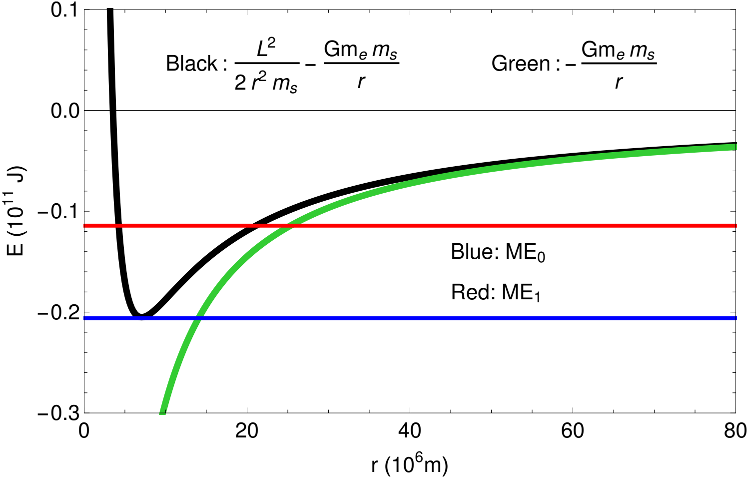
An electron beam strikes a gas of hydrogen atoms.

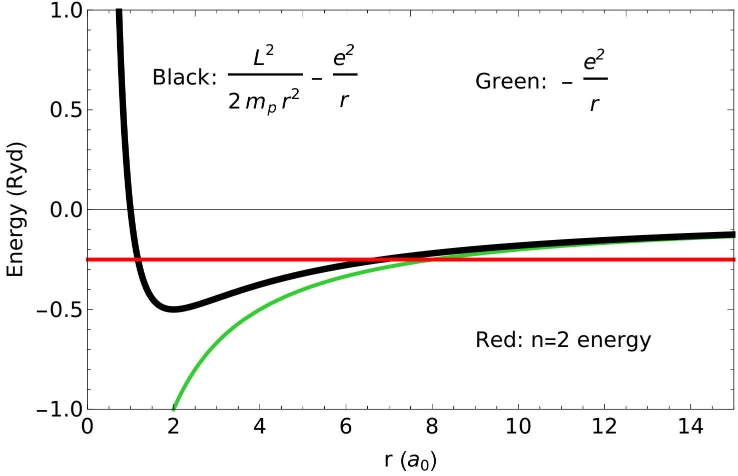
1. What is the minimum speed the electrons must have to cause the emission of $\lambda = 656\ nm$ light from the $3 \rightarrow 2$ transition of hydrogen?
2. What is the electric potential difference the electrons must fall through to be accelerated to this speed?

	Classical Physics	**Quantum Physics**
1	Start with Newton's Laws.	Start with Schroedinger's equation.
2	Insert the force/potential.	Insert the force/potential.
3	Solve the differential equation with initial conditions $$\vec{F} = m\frac{d^2\vec{r}}{dt^2}$$ where $\vec{r}$ is the position.	Solve the differential equation with initial conditions $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dr^2} + \frac{L^2}{2mr^2}\psi + V\psi = E\psi(\vec{r})$$ where ψ is a wave function.
4	Get the position $\vec{r}(t)$ as a function of time.	Get the probability $\lvert\psi(\vec{r})\rvert^2$ as a function of time.



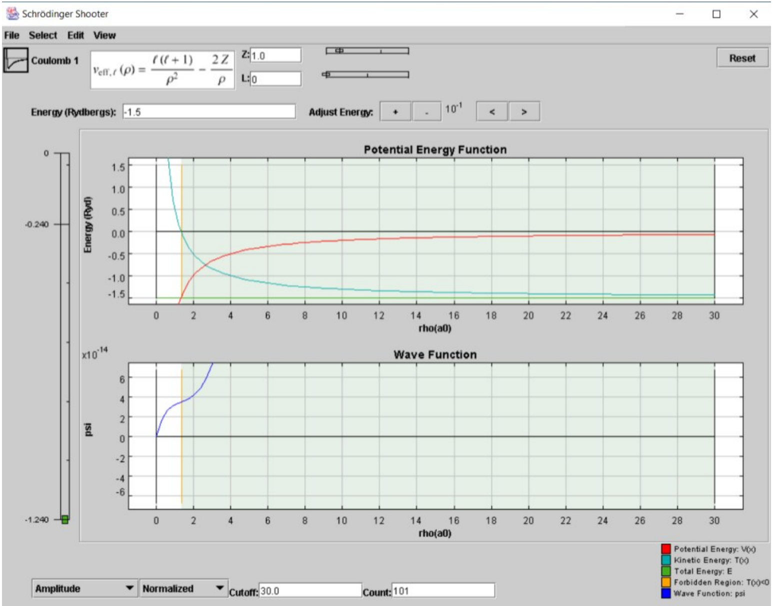



Black : $\frac{L^2}{2 r^2 m_s} - \frac{Gm_e m_s}{r}$
Green : $-\frac{Gm_e m_s}{r}$
Blue: ME_0
Red: ME_1
0.1
0.0
−0.1
−0.2
−0.3
E (10^{11} J)
0
20
40
60
80
r (10^6m)



Black: $\frac{L^2}{2 m_p r^2} - \frac{e^2}{r}$
Green: $-\frac{e^2}{r}$
Red: n=2 energy
Energy (Ryd)
r (a_0)
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10
12
14

Schroedinger Shooter 18



A Theory for the Hydrogen Atom - Results 1

What did your ground-state wave function look like?

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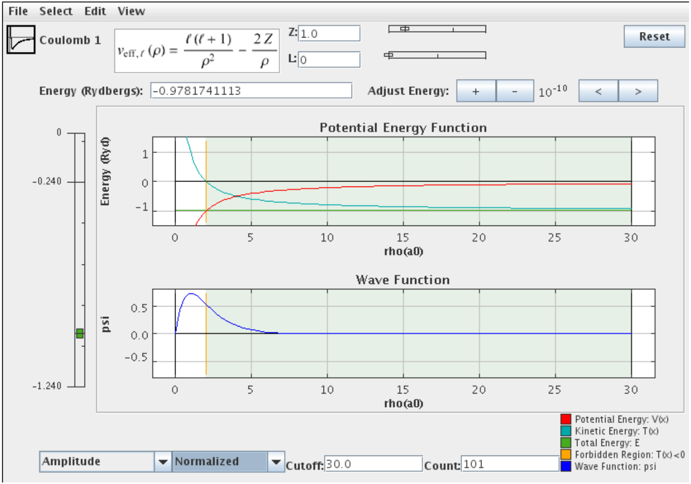

A Theory for the Hydrogen Atom - Results 3

What did your n=3 wave function look like?

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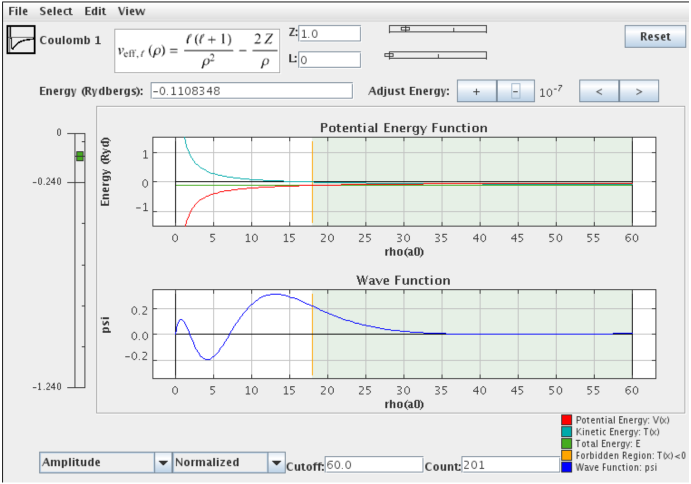
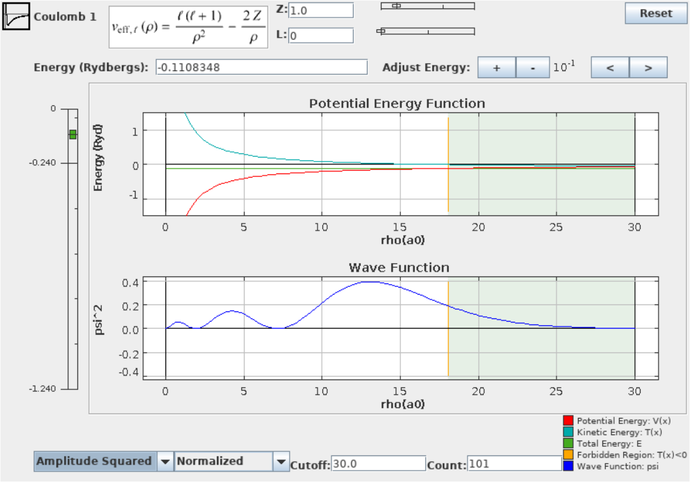


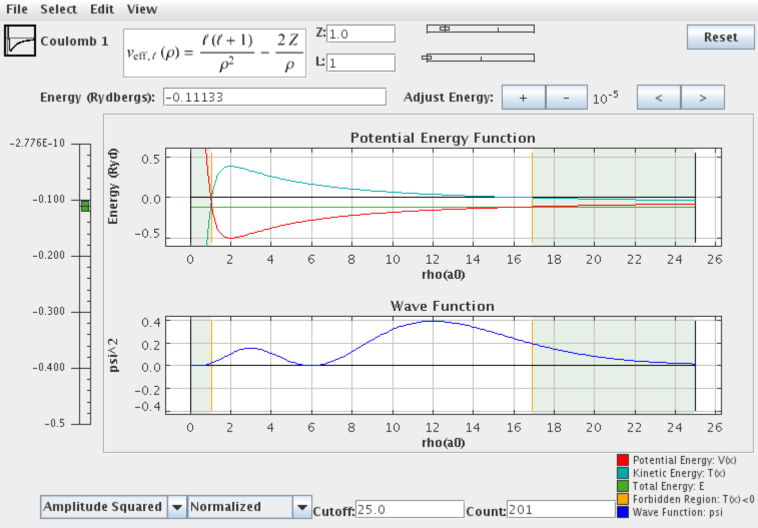
A Theory for the Hydrogen Atom - Results 3

What did your n=3 probability distribution look like?



A Theory for the Hydrogen Atom - Results 4

For $n = 3$, $L = 1$.



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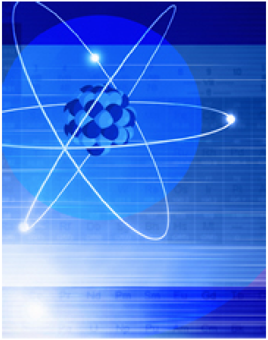


- The Organizing Principle.

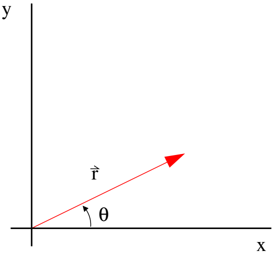
$$\begin{aligned} ME_0 &= ME_1 \\ KE_0 + PE_0 &= ME_1 + PE_1 \\ \frac{1}{2}mv_0^2 + PE_0 &= \frac{1}{2}mv_1^2 + PE_1 \end{aligned}$$

- The Forces

$$\vec{F}_{grav} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{F}_{coul} = \frac{k_e q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

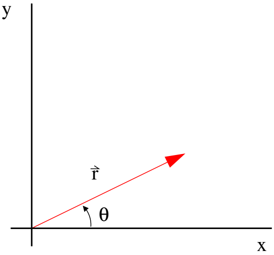
The simulation is here.

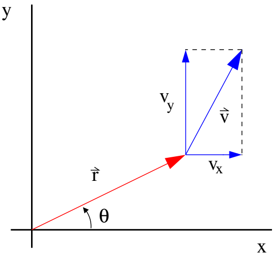
The Kinetic Energy in Polar Coordinates - 1 27



$\vec{r} = x\hat{i} + y\hat{j}$

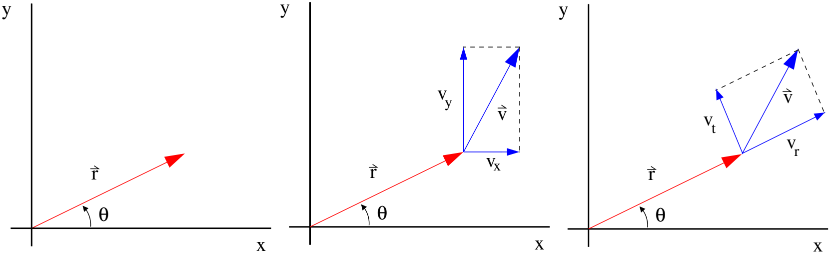
The Kinetic Energy in Polar Coordinates - 1   28





$\vec{r} = x\hat{i} + y\hat{j}$ $\vec{v} = v_x\hat{i} + v_y\hat{j}$

The Kinetic Energy in Polar Coordinates - 1



$\vec{r} = x\hat{i} + y\hat{j}$ $\vec{v} = v_x\hat{i} + v_y\hat{j}$ $\vec{v} = v_r\hat{r} + v_t\hat{\theta}$

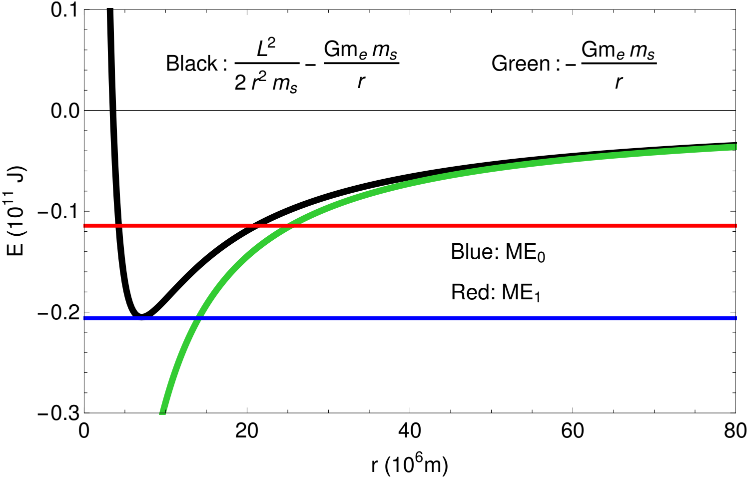
A Russian Artica satellite that monitors polar weather follows an elliptical orbit around the Earth at an altitude of $h = 300\ km$ above the surface (radius $r_s = 6.67 \times 10^6\ m$). At one point in its orbit its velocity is measured to be

$$\vec{v} = 4.1 \times 10^3\ m/s\ \hat{r} + 7.5 \times 10^3\ m/s\ \hat{\theta} \qquad .$$

What is the angular momentum? What is the total energy? What is the distance of closest approach to the Earth? The satellite mass is $m_s = 600\ kg$.

Arktica-M

$R_{earth} = 6.37 \times 10^6\ m$
$m_{earth} = 5.97 \times 10^{24}\ kg$
$G = 6.673 \times 10^{-11}\ Nm^2/kg^2$


Black : $\frac{L^2}{2 r^2 m_s} - \frac{G m_e m_s}{r}$
Green : $-\frac{G m_e m_s}{r}$
Blue: ME_0
Red: ME_1
E (10^{11} J)
r (10^6m)
0.1
0.0
−0.1
−0.2
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0
20
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60
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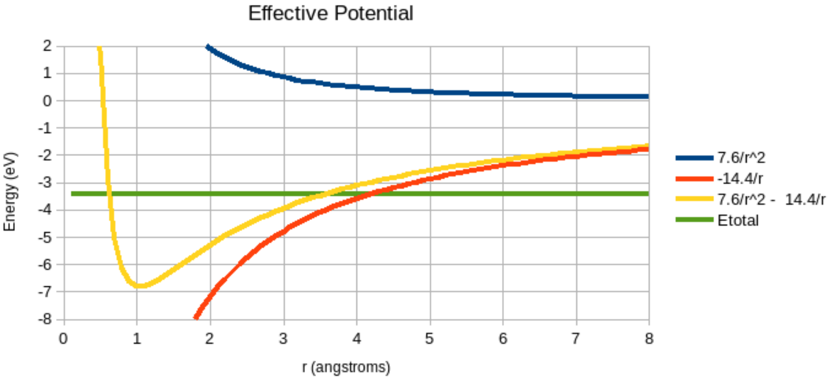
Effective Potential
Energy (eV)
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
0
1
2
3
4
5
6
7
8
r (angstroms)
7.6/r^2
-14.4/r
7.6/r^2 - 14.4/r
Etotal