

## Homework 1 Kinematics

- The position of a particle moving with uniform acceleration is  $x = Ca^m t^n$ , where  $C$  is dimensionless. Use dimensional analysis to what values of  $m$  and  $n$  are correct. What is the value of  $C$ ?
- A hydrogen atom has a radius  $r_a = 0.53 \times 10^{-10} \text{ m}$ . The hydrogen nucleus has a radius  $r_n = 1.2 \times 10^{-15} \text{ m}$ . If the diameter of the hydrogen atom was that of a football field (100 yards), what would be the diameter of the nucleus in millimeters? The atom is how many times larger in volume than its nucleus?
- How many significant figures are in the following numbers: (a)  $78.9 \pm 0.2$ , (b)  $3.788 \times 10^9$ , (c)  $2.46 \times 10^{-6}$ , (d) 0.0053?
- Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2.5; (b) the product  $0.0032 \times 356.3$ ; (c) the product  $5.620 \times \pi$ .
- The position of a pinewood derby car is measured at different times and the results compiled into the table. What is the average velocity of the car for (a) the first second, (b) during the interval  $t = 2.0 \text{ s} - 4.0 \text{ s}$ , and (c) the entire period of observation.

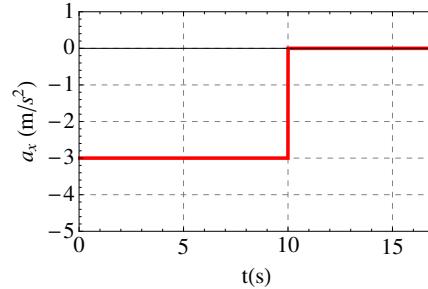
$t(\text{s})$	0	1.0	2.0	3.0	4.0	5.0
$x(\text{m})$	0	2.3	9.2	20.7	36.8	57.5

- The position of a particle moving along the  $x$  axis varies in time according to the expression

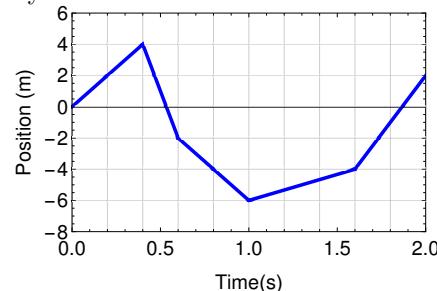
$$x = At^2 + B$$

Evaluate the limit of  $\Delta x/\Delta t$  as  $\Delta t$  approaches zero. Do NOT use any derivative formulas for specific functions you might remember from calculus.

- A particle's position is described by  $x = 10t^2$ , where  $x$  is in meters and  $t$  is in seconds. (a) What is the average velocity for the time interval from 1.5 s to 3.00 s? (b) What is the average velocity for the time interval from 1.00 s to 1.60 s.
- A particle starts from the origin and at rest and accelerates as shown in the red line in the figure. What is the particle's speed at  $t = 5 \text{ s}$  and  $t = 15 \text{ s}$ . What is the distance traveled in the first 15 s?



- A person walks at a constant speed of 5.0 m/s in a straight line from point A to point B and then back along the line from B to A at a speed of 3.0 m/s. (a) What is the average speed over the entire trip? (b) What is the average velocity over the entire trip?
- The position of a particle moving along the x axis is described by  $x = 3t^2$ , where  $x$  is in meters and  $t$  is in seconds. (a) What is its position at  $t = 2.0 \text{ s}$  and at  $t = 2.0 \text{ s} + \Delta t$ ? (b) What is the limit of  $\Delta x/\Delta t$  as  $\Delta t$  approaches zero? (c) What is the particle's velocity at  $t = 2.0 \text{ s}$ ?
- What is the instantaneous velocity of the particle shown in figure at: (a)  $t = 0.2 \text{ s}$ , (b)  $t = 0.5 \text{ s}$ , (c)  $t = 0.8 \text{ s}$ .

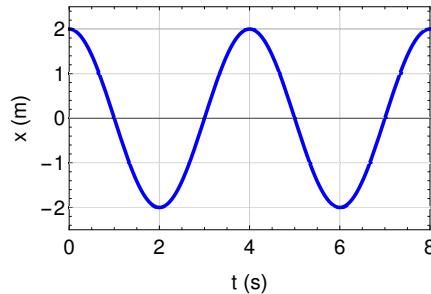


12. A particle's position is given by  $x = (t^2 - 4t + 2)m$  where  $t$  is in seconds. (a) What is the particle's velocity at  $t = 1.0s$ ? Do NOT use any derivative formulas for specific functions you might remember from calculus. (b) Where is the particle when  $v = 4.0 \text{ m/s}$ ? (c) Sketch a diagram of the particle's position versus time.

13. Use the data in Problem 5 to construct a smooth graph of position versus time. Construct tangents to the  $x(t)$  curve and get the instantaneous velocity of the car at several times. Plot the instantaneous velocity versus time. What is the average acceleration of the car? What was the initial velocity of the car?

14. A particle's velocity is described by the function  $v = t^2 - 7t + 10 \text{ m/s}$ , where  $t$  is in seconds. (a) At what times, if any, does the particle's velocity go to zero? (b) What is the acceleration at those times? Do NOT use any derivative formulas for specific functions you might remember from calculus.

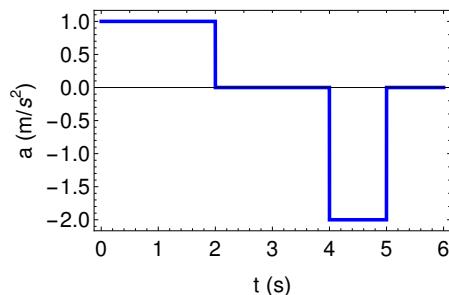
15. A block is hung from a spring, pulled down, and released. Its position as a function of time is shown in the figure. (a) When is the velocity zero? (b) When is the velocity most positive? (c) When is it most negative? Explain your reasoning.



16. A hare and a tortoise race over a course of length 1.00 km. The tortoise crawls straight and steadily at its maximum speed of 0.200 m/s toward the finish line. The hare runs at its maximum speed of 8.00 m/s toward the goal for 0.800 km and then stops to taunt the tortoise. How close to the goal can the hare let the tortoise approach before resuming the race, which the tortoise wins in a photo finish? Assume that, when moving, both animals move steadily at their respective maximum speeds.

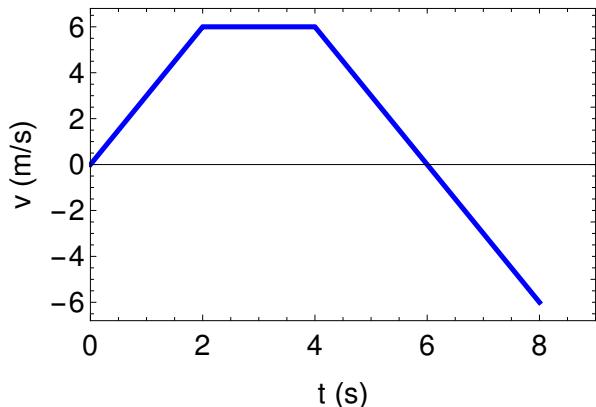
17. A 50.0-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval? (Note: 1 ms =  $10^{-3}$  s.)

18. A particle starts from rest and accelerates as shown in the figure. Determine (a) the particle's speed at  $t = 1.0 \text{ s}$  and at  $t = 3.0 \text{ s}$ , and (b) the distance traveled in the first 6.0 s.



19. An object moves in the  $x$  direction as  $x(t) = (3.0t^2 - 2.0t + 3.00)m$ , where  $t$  is in seconds. What is (a) the average speed between  $t = 2.0 \text{ s}$  and  $t = 3.0 \text{ s}$ , (b) the instantaneous speed at  $t = 2.0 \text{ s}$  and at  $t = 3.0 \text{ s}$ , (c) the average acceleration between  $t = 2.0 \text{ s}$  and  $t = 3.0 \text{ s}$ , and (d) the instantaneous acceleration at  $t = 2.0 \text{ s}$  and  $t = 3.0 \text{ s}$ .

20. A student rides a bike on a straight road described by the velocity versus time graph in the figure. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the  $v_x - t$  graph, again aligning the time coordinates. On each graph, show the numerical values of  $x$  and  $a_x$  for all points of inflection. (c) What is the acceleration at  $t = 6$  s? (d) Find the position (relative to the starting point) at  $t = 6$  s. (e) What is the students final position at  $t = 8$  s?



21. A truck covers a distance of 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) What is its original speed? (b) What is its acceleration?

22. A drowsy state trooper is measuring the speed of cars with a radar gun as they appear over the top of a hill. You come zooming over the hill at 80 mph (36 m/s), but the officer doesn't check your speed until two seconds have passed. Your car can decelerate at  $5.0 \text{ m/s}^2$ , but it takes you 0.5 s to get your foot on the brake. Will the officer catch you if the speed limit is 65 mph (29 m/s)?

23. The minimum distance required to stop a car moving at 35.0 mi/h is 40.0 ft. What is the minimum stopping distance for the same car moving at 70.0 mi/h, assuming the same rate of acceleration?

24. A particle moves in the x direction. Its position is given by  $x = 2 + 3t - 4t^2$  with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at  $t_0$ , *i.e.* when its position is equal to its initial position.

25. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of  $-5.60 \text{ m/s}^2$  for 4.20 s, making straight skid marks 62.4 m long ending at the tree. What is the speed of the car when it strikes the tree?

26. A car moving at 30.0 m/s, enters a one-lane tunnel. The driver then observes a slow-moving van 155 m ahead traveling at 5.0 m/s. The driver brakes, but can accelerate only at  $-2.00 \text{ m/s}^2$  because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between the car and the van.

27. A student throws a set of keys vertically upward to her roommate, who is in a window 4.00 m above. The keys are caught 1.50 s later by the roommates outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

28. At the instant a traffic light turns green, a 'car' starts with a constant acceleration  $a = 2.2 \text{ m/s}^2$ . At the same instant a truck is 5.0 m behind the car and traveling with a constant speed  $v_t = 9.5 \text{ m/s}$ . How far does the car travel before overtaking the truck? What do the position versus time plots look like for the car and the truck?

29. A ball is thrown directly downward, with an initial speed of 8.0 m/s, from a height of 30.0 m. After what time interval does the ball strike the ground?

30. Nerve impulses typically travel through the body at about 150 *miles/hour* or about 67 m/s. Imagine someone drops a brick from one meter above your big toe. You see this and immediately your brain starts the process of moving your big toe out of harms way. Compare the time it takes for the brick to fall with the time it takes for the nerve signal to get from your brain to your toe and start moving it out of the way. Assume you are 1.75 meters tall.

31. A rancher sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The speed of the horse is constant at 10.0 m/s, and the distance from the limb to the level of the saddle is 3.0 m. (a) What must be the horizontal distance between the saddle and limb when the ranch hand makes his move? (b) How long is he/she in the air?

32. You can shoot an arrow at a speed as high as 100 m/s. (a) If friction can be ignored, how high would an arrow launched at this speed rise if shot straight up? (b) How long would the arrow be in the air?

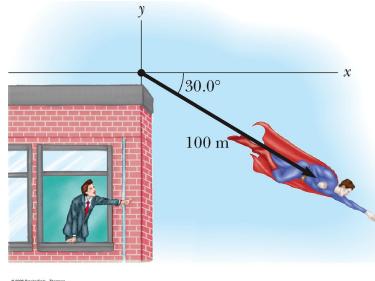
33. As soon as a traffic light turns green, a car speeds up from rest to 50.0 mi/h with constant acceleration 9.0 mi/h·s. In the adjoining bike lane, a cyclist speeds up from rest to 20.0 mi/h with constant acceleration 13.0 mi/h·s. Each vehicle maintains constant velocity after reaching its cruising speed. (a) For what time interval is the bicycle ahead of the car? (b) By what maximum distance does the bicycle lead the car?

34. Some cars now carry ‘black boxes’ like those on airplanes that collect data on speed, brake status, and other information when the air bags are activated. Suppose you are a lawyer and your client was in a car accident. Your client claims he was driving at the speed limit of  $v_0 = 15.6 \text{ m/s}$  (35 mph) when a car came out of a driveway in front of him. He immediately slammed on the brakes which decelerate the car at a rate  $|\vec{a}| = 8 \text{ m/s}^2$ . From the investigation you know your client hit the brakes at a point 22 m from the crash. The ‘black box’ recorded a speed  $v_1 = 2 \text{ m/s}$  at the moment of impact when the airbag opened. Is he lying?

35. You are an engineer determining how quickly a test driver can bring a car to a halt before a traffic light in order to properly time the lights. For the driver to stop her car there are two phases. First, there is a certain reaction time called  $t_r$  to begin braking; the car moves at constant velocity during this time. Next, after the driver applies the brakes the car slows down under constant braking deceleration. When the test driver has an initial speed of 22 m/s she stops in 57 m; the combined distance for the two phases. When the test driver has an initial speed of 13 m/s, she covers 24 m at the end of both phases. Assume the reaction time and the acceleration are the same for both trials. What is the reaction time of the driver and the magnitude of the deceleration?

36. A rock is dropped from rest into a well. (a) The sound of the splash is heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) If the travel time for the sound is ignored, what percentage error is introduced when the depth of the well is calculated?

37. Find the horizontal and vertical components of the 100-m displacement of a superhero who flies from the top of a tall building following the path shown in the figure.



38. A vector has an x component of -25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.

39. A man pushing a mop across a floor causes it to undergo two displacements. The first has a magnitude of 150 cm and makes an angle of 120° with the positive x axis. The resultant displacement has a magnitude of 140 cm and is directed at an angle of 35.0° to the positive x axis. Find the magnitude and direction of the second displacement.

40. Consider the two vectors  $\vec{A} = 2\hat{i} - 6\hat{j}$  and  $\vec{B} = -\hat{i} - 3\hat{j}$ . Calculate (a)  $\vec{A} + \vec{B}$ , (b)  $\vec{A} - \vec{B}$ , (c)  $|\vec{A} + \vec{B}|$ , (d)  $|\vec{A} - \vec{B}|$ , and (e) the directions of  $\vec{A} + \vec{B}$  and  $|\vec{A} - \vec{B}|$ .

41. A fish swimming in a horizontal plane has velocity  $v_i = (4.00\hat{i} + 1.00\hat{j}) \text{ m/s}$  at a point in the ocean where the position relative to a certain rock is  $\vec{r}_i = (10.0\hat{i} - 4.00\hat{j}) \text{ m}$ . After the fish swims with constant

acceleration for 20.0 s, its velocity is  $\vec{v}_f = (20.0\hat{i} - 5.00\hat{j}) \text{ m/s}$ . (a) What are the components of the acceleration? (b) What is the direction of the acceleration with respect to unit vector  $\hat{i}$ ? (c) If the fish maintains constant acceleration, where is it at  $t = 25.0 \text{ s}$  and in what direction is it moving?

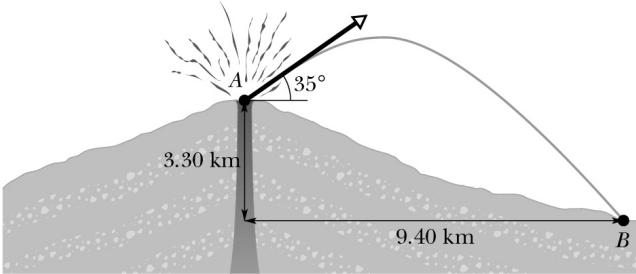
42. The current world record in the long jump is held by Galina Chistyakova of the former Soviet Union. She jumped a distance  $d = 7.52 \text{ m}$  horizontally on June 11, 1988. Assuming her take-off speed was  $v_0 = 8.6 \text{ m/s}$  and she jumped at an angle  $\theta = 45^\circ$ , how close was she to the maximum possible distance? What was her flight time?

43. A cannon with a muzzle speed of 1000 m/s is used to start an avalanche on a mountain slope. The target is 2000 m from the cannon horizontally and 800 m above the cannon. At what angle, above the horizontal, should the cannon be fired?

44. In 1987 Natalya Lisovskaya of the former Soviet Union set the current world record in the shot put with a throw of  $x_1 = 22.63 \text{ m}$ . The acceleration of gravity in Moscow is  $9.8128 \text{ m/s}^2$ . If the event were held in Mexico City (site of the 1968 Olympics), her throw might have been significantly different because the acceleration of gravity there is only  $g_M = 9.779 \text{ m/s}^2$ . What would have been the distance of her throw in Mexico City? Is the difference significant (*i.e.*, for the shot put the difference must be bigger than a millimeter or 0.01 m)? The initial velocity of the shot put is  $v_0 = 14.305 \text{ m/s}$  at a height  $y_0 = 2.13 \text{ m}$  above the ground. The initial angle of the velocity to the ground is  $45^\circ$ .

45. A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of  $20.0^\circ$  below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

46. During a volcanic eruption chunks of rock are blasted out of the volcano. These projectiles are called volcanic bombs. Consider the figure below of Mount Fuji in Japan. A volcanic bomb is launched at an angle  $\theta = 35^\circ$  from point A in the figure and it eventually lands at point B. How long would the people at point B have to get out of the way once the the rock leaves the mouth of the volcano? The mouth of the volcano is a distance  $y_0 = 3.3 \times 10^3 \text{ m}$  above the landing spot which is a distance  $x_f = 9.4 \times 10^3 \text{ m}$  downrange from the mouth of the volcano.



47. You've graduated from college and fulfilled a childhood dream to become a homicide detective in a large city. On your first day at work, you're investigating the death of a man who was found 4.5 m from the base of his apartment building and 20 m below his balcony. Do you think the death is accidental? Explain.

48. A track star rotates a 1.00-kg discus along a circular path of radius 1.06 m. The maximum speed of the discus is 20.0 m/s. What is the magnitude of the maximum radial acceleration of the discus?

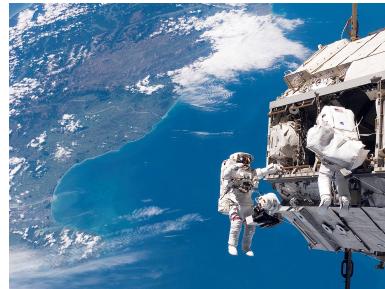
49. When a large star explodes and becomes a supernova the remaining core can become a neutron star. If a neutron star has a radius  $r = 10^4 \text{ m}$  and rotates every 1.2 s, then what is the centripetal acceleration of a particle on the star's equator?

50. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).

51. As their rocket booster separate, astronauts typically feel accelerations up to  $3g$ , where  $g = 9.80 \text{ m/s}^2$ . In their training, astronauts ride in a device where they experience such an acceleration as a centripetal acceleration. Specifically, the astronaut is fastened securely at the end of a mechanical arm, which then

turns at constant speed in a horizontal circle. Determine the rotation rate, in revolutions per second, required to give an astronaut a centripetal acceleration of  $3.0g$  while in circular motion with radius  $9.45\text{ m}$ .

52. The astronauts orbiting the Earth in the figure are working on the International Space Station (ISS). The ISS is in a circular orbit  $420\text{ km}$  above the Earth's surface, where the free-fall acceleration is  $8.56\text{ m/s}^2$ . Take the radius of the Earth as  $6400\text{ km}$ . Determine the speed of the satellite and the time interval required to complete one orbit around the Earth, which is the period of the satellite.



53. A catapult launches a rocket at an angle of  $53.0^\circ$  above the horizontal with an initial speed of  $100\text{ m/s}$ . The rocket engine immediately starts a burn, and for  $3.00\text{ s}$  the rocket moves along its initial line of motion with an acceleration of  $30.0\text{ m/s}^2$ . Then its engine fails, and the rocket proceeds to move in free-fall. Find (a) the maximum altitude reached by the rocket, (b) its total time of flight, and (c) its horizontal range.

54. An earth satellite moves in a circular orbit  $3.59 \times 10^7\text{ m}$  above the Earth's surface with a period of 1 day or  $8.64 \times 10^4\text{ s}$ . This is a special orbit called a geosynchronous one. A satellite in this orbit is always at the same point in the sky so it's easy for a ground station to find it. What is the centripetal acceleration of the satellite? How does this compare with the acceleration of gravity on Earth?

55. A Ferris wheel has a radius of  $r = 15\text{ m}$  and completes four turns every minute. The acceleration of each passenger is caused by the vector sum of the acceleration of gravity and the acceleration created by the Ferris wheel. What is the acceleration created by the Ferris wheel on a passenger at the highest point and the lowest point? Recall that centripetal acceleration points in the opposite direction as the position vector. Do your results make sense? Why?