# 1

# The Twins Paradox 2

Consider two twins. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$ where $c$ is the speed of light. The Earthbound twin goes on about her/his business accumulating the normal accouterments of advancing age (gray hair, drooping body parts, *etc.*). After twenty years have passed for the Earthbound twin, the spacefaring one returns. When they finally meet the voyager is NOT twenty years older! She/He looks only a few years older than when she/he left and shows few signs of age. How much has she/he aged during the journey?

Electrons at the speed of light.

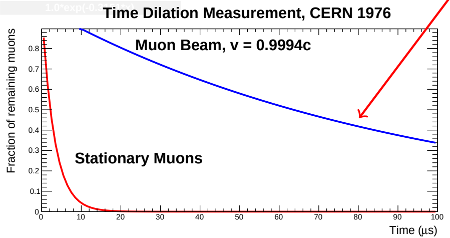
**FERMIONS** matter constituents spin = 1/2, 3/2, 5/2, ...

**Leptons** spin = 1/2

| Flavor | | Mass GeV/c$^2$ | Electric charge |
|---|---|---|---|
| $\nu_e$ | electron neutrino | <1×10$^{-8}$ | 0 |
| e | electron | 0.000511 | −1 |
| $\nu_\mu$ | muon neutrino | <0.0002 | 0 |
| $\mu$ | muon | 0.106 | −1 |
| $\nu_\tau$ | tau neutrino | <0.02 | 0 |
| $\tau$ | tau | 1.7771 | −1 |

**Quarks** spin = 1/2

| Flavor | | Approx. Mass GeV/c$^2$ | Electric charge |
|---|---|---|---|
| u | up | 0.003 | 2/3 |
| d | down | 0.006 | −1/3 |
| c | charm | 1.3 | 2/3 |
| s | strange | 0.1 | −1/3 |
| t | top | 175 | 2/3 |
| b | bottom | 4.3 | −1/3 |

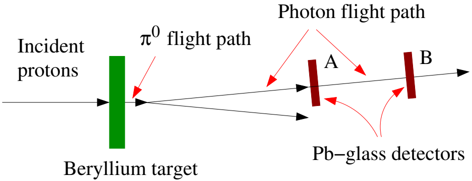



Muon half-life: $2.2 \times 10^{-6} s$

1. Physics is the same in all inertial reference frames (hopefully).
2. The speed of light is the same in all inertial reference frames.

# Testing The Second Postulate 5

1. Get on a very fast train. - At CERN in 1964 T. Alvager *et al.* created a beam of $\pi^0$'s moving close to the speed of light ($0.99975c$) by hitting a beryllium target with a high-energy proton beam.
2. The $\pi^0$'s almost immediately decayed into particles of light called photons ($t_{1/2} = 8.64 \times 10^{-17}\ s$).
3. The photons were measured at different, known locations downstream from the target.
4. $c' = (2.9977 \pm 0.0004) \times 10^8\ m/s$ versus $2.99792458 \times 10^8\ m/s$.



Alvager et al, CERN, 1964

# Testing The Second Postulate 6

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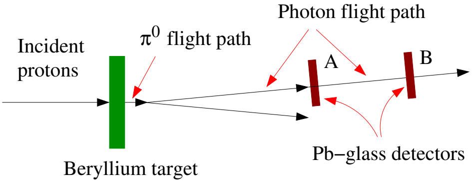



Alvager et al, CERN, 1964

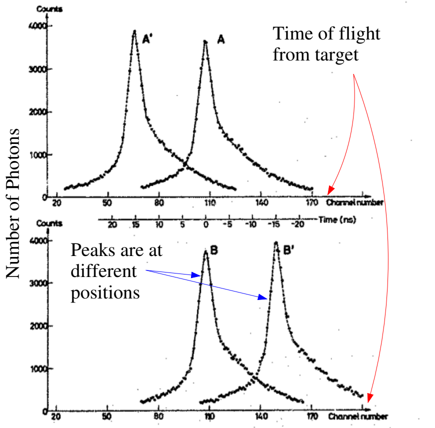



Fig. 1. The experimental arrangement and typical time spectra of the γ rays. recorded in the four detector positions A, A', B, B'. Channel width 0.35 nsec. The measuring time for 100 000 counts in the peak was about 10 min.

T.Alvager *et al.*, Phys. Lett. 12, 260 (1964)

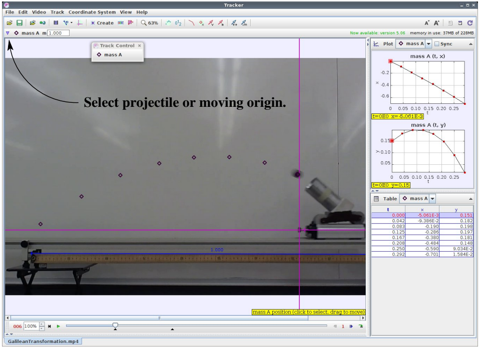
# Guidelines for Galilean Relativity Lab 7

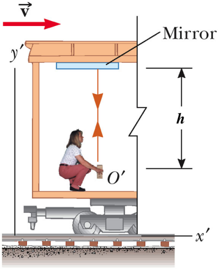
- Use the video `GalileanTransformation.mp4` available at the following site.

  https://facultystaff.richmond.edu/~ggilfoyl/genphys/131/links.html

- Measure separately the trajectory of the ball in the lab system (fixed origin) and in the launcher system (moving origin).

- To use a moving origin (1) click on the coordinates symbol in the toolbar. You should see the coordinates appear. (2) Click *Coordinate Systems* at the top of the *Tracker* GUI. (3) Make sure *Fixed Origin* is unchecked. (4) In each frame select the coordinate system in the drop-down menu in the toolbar (see figure) and set the origin. (5) Next, use the same drop-down menu to select the mass and then mark the projectile in the usual way.

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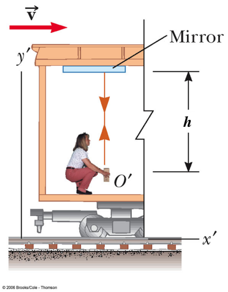
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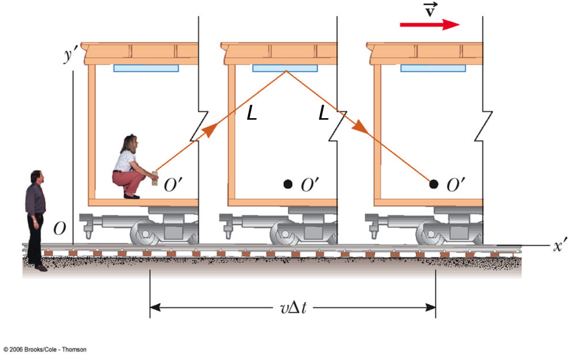
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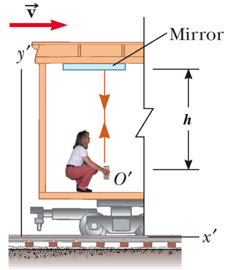
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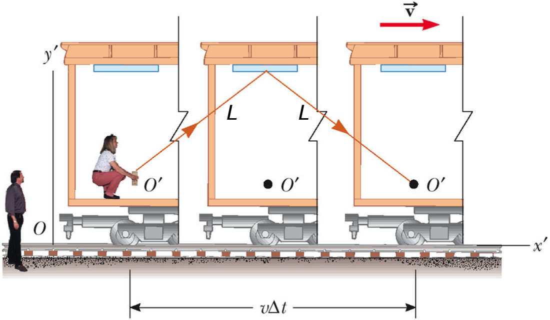
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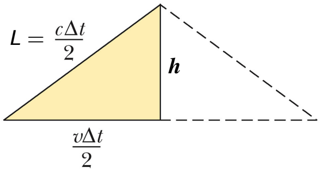
$v\Delta t$




$L = \frac{c\Delta t}{2}$
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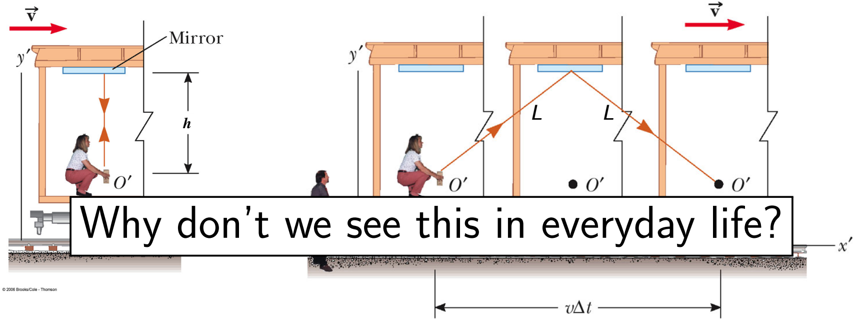
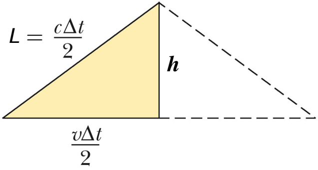
$\frac{v\Delta t}{2}$




Why don't we see this in everyday life?







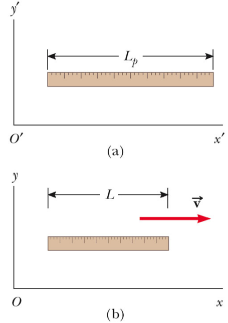
1. In 1971 Hafele and Keating at the old National Bureau of Standards (now National Institute for Standards and Technology) took four cesium-beam atomic clocks aboard commercial airliners and flew twice around the world, first eastward, then westward, and compared the clocks against those of the United States Naval Observatory.

| | nanoseconds gained | | | |
|---|---|---|---|---|
| | predicted | | | measured |
| | gravitational (general relativity) | kinematic (special relativity) | total | |
| eastward | $144 \pm 14$ | $-184 \pm 18$ | $-40 \pm 23$ | $-59 \pm 10$ |
| westward | $179 \pm 18$ | $96 \pm 10$ | $275 \pm 21$ | $273 \pm 7$ |

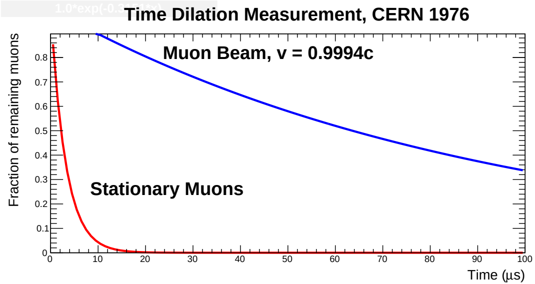
2. Mountaintop muon decay measurements.
3. Electron beam at JLab.
4. GPS and many others.

Consider two twins. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$ where $c$ is the speed of light. The Earthbound twin goes on about her/his business accumulating the normal accouterments of advancing age (gray hair, drooping body parts, *etc.*). After twenty years have passed for the Earthbound twin, the spacefaring one returns. When they finally meet the voyager is NOT twenty years older! She/He looks only a few years older than when she/he left and shows few signs of age. How much has she/he aged during the journey?

Consider the two twins again. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$. After twenty years have passed for the Earthbound twin, the spacefaring one returns. What does the Earthbound twin estimate for the distance traveled by the wandering sibling? What is the mileage on the spacefaring twin's spaceship? The spacefarer traveled outward from the Earth at 0.99c, turned around at the midpoint of her/his trip, and returned directly to Earth.





| Galilean | Lorentz |
|---|---|
| $x' = x - vt$<br>$y' = y$<br>$z' = z$<br>$t' = t$<br>$u'_x = u_x - v$<br>$u'_y = u_y$<br>$u'_z = u_z$ | |

primes refer to the frame moving with velocity $v$.

$v$ - velocity of moving frame.

$u_i$ - $i^{th}$ component of the velocity of an object in the stationary frame.

$u'_i$ - $i^{th}$ component of the velocity of an object in the moving frame.

| Galilean | Lorentz |
|---|---|
| $x' = x - vt$ | $x' = (x - vt)\sqrt{1 - v^2/c^2}$ |
| $y' = y$ | $y' = y$ |
| $z' = z$ | $z' = z$ |
| $t' = t$ | $t' = (t - vx/c^2)/\sqrt{1 - v^2/c^2}$ |
| $u'_x = u_x - v$ | $u'_x = (u_x - v)/(1 - u_x v/c^2)$ |
| $u'_y = u_y$ | $u'_y = u_y$ |
| $u'_z = u_z$ | $u'_z = u_z$ |

primes refer to the frame moving with velocity $v$.

$v$ - velocity of moving frame.

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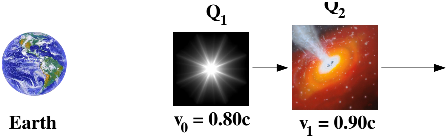
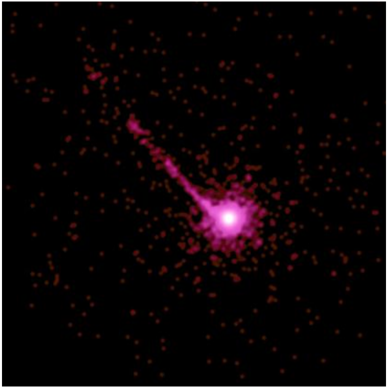
# Addition of Velocities 17

Quasars are galaxies in the early throes of birth (we think). They have been observed to be receding from us at high speeds and at great distances. Quasar $Q_1$ is found to have a recessional velocity $v_0 = 0.80c$ relative to the Milky Way ($c$ is the speed of light). Another quasar $Q_2$ is receding from the Earth at a speed of $v_1 = 0.90c$ along approximately the same line of sight as measured from Earth (see figure below). An alien who lives in galaxy $Q_1$ measures the speed of quasar $Q_2$. What speed does the alien measure?

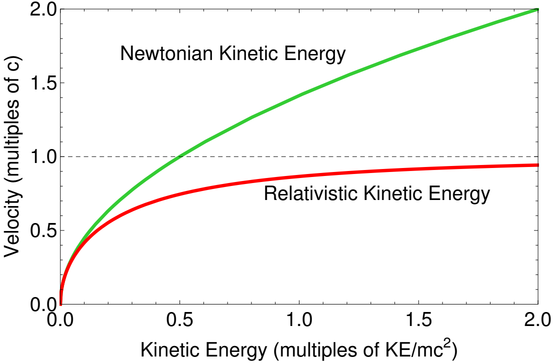


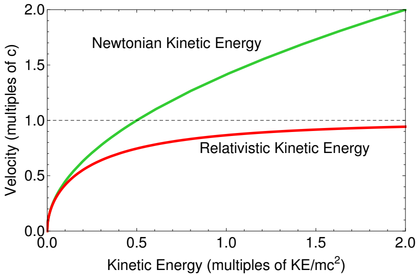
X-ray image of the quasar PKS 1127-145 10 billion light years from Earth. The jet is at least a million light years from the quasar.

A large armada of Federation spaceships moves with a speed $0.95c$ relative to the nearby Kronos system. A scout ship launched from the trailing ship in the armada moves at a speed $0.7c$ towards the front of the fleet. The scout ship's speed is measured relative to the fleet. What is the speed of the scout ship as measured on Kronos?

$$E = m_R c^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$



An electron is accelerated to an energy $E = 6\ \mathrm{GeV}$ where $1\ \mathrm{GeV} = 10^9\ \mathrm{GeV}$ at the Thomas Jefferson National Accelerator Facility in Newport News. What is the electron's speed, relativistic mass, and kinetic energy?

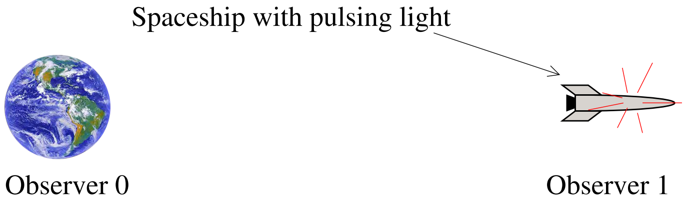



# Adding Relativistic Velocities



A fast-moving train with speed $v_0 = 2.5 \times 10^8\ m/s$ passes an observer standing on the ground. A girl on the train kicks a soccer ball at her big brother sitting in front of her with a speed $v_1 = 10^8\ m/s$ as measured by her father (much to his horror!). What speed does the stationary observer measure for the speed $v_2$ of the thrown ball?

A spaceship (Observer 1 in the figure) is moving away from an Earth-bound observer (0) at a high speed $v_0$ as measured by Observer 0. It emits a periodic light pulse the observer on the Earth (0) detects. The time between pulses measured by Observer 1 is $\Delta t_1$. The time between pulses measured by Observer 0 is $\Delta t_0$. How is $\Delta t_0$ related to $\Delta t_1$?



# The Universal Speed Limit (Part 2) 23

Two spaceships (1 and 2 in the figure) are moving away from an Earth-bound observer (0) at different speeds. The fast, lead ship (2) emits a periodic light pulse the observer on the second, slow ship (1) receives and immediately relays to Earth (0). The speeds and time intervals are defined below.

| | |
|---|---|
| $v_0$: speed of 1 from 0 | $\Delta t_0$: time interval on 0 |
| $v_1$: speed of 2 from 1 | $\Delta t_1$: time interval on 1 |
| | $\Delta t_2$: time interval on 2 |
| $v_2$: speed of 2 from 0 | |

1. How is $\Delta t_0$ related to $\Delta t_1$?
2. How is $\Delta t_1$ related to $\Delta t_2$?
3. How is $\Delta t_0$ related to $\Delta t_2$?
4. What is $v_2$ in terms of $v_0$ and $v_1$?