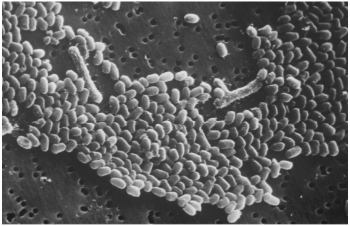
What are These?



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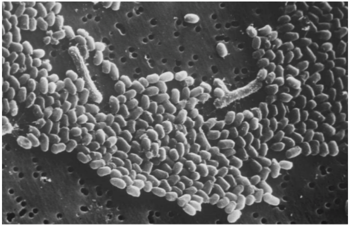

Anthrax spores

1. Until the 20th century, anthrax killed hundreds of thousands of people and animals each year.

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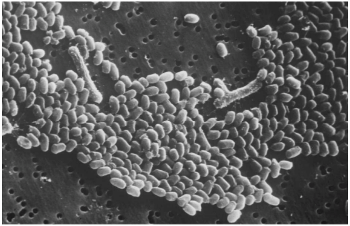

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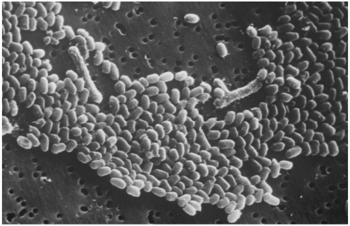

Anthrax spores

1. Until the 20th century, anthrax killed hundreds of thousands of people and animals each year.
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3. A long-standing fear is a biological attack using an agent like anthrax or smallpox.
4. The natural spread of the disease and its indiscriminate nature can amplify the impact.

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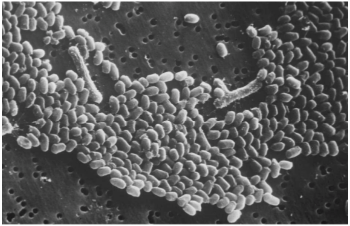

Anthrax spores

6

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4. The natural spread of the disease and its indiscriminate nature can amplify the impact.
5. Some weaponized forms could cause mass casualties.
6. Defense against such an attack is focused on rapid identification and mitigation.

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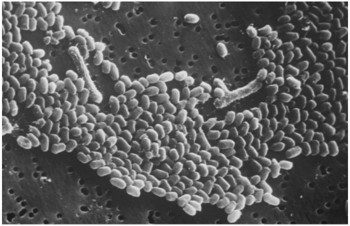
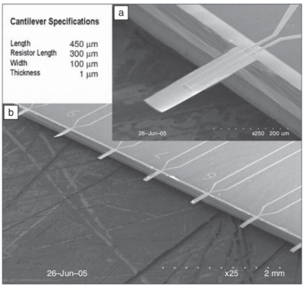
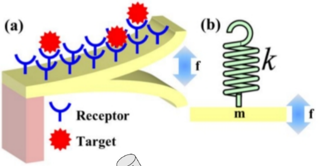


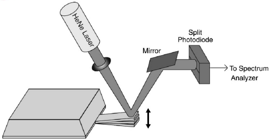
Anthrax spores

1. The attack will not be obvious; it may take hours or days to know.
2. Current biological diagnostics are very effective, but they're slow.
3. Fast response time is essential to avoid overwhelming the health-care system $\Rightarrow$ **rapid response is vital**.

Have We Been Attacked?







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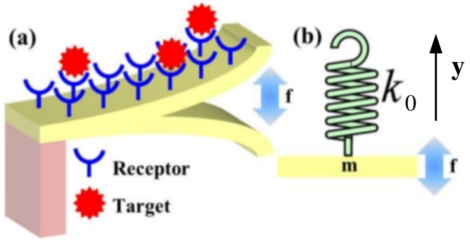
---

4. Nanosized oscillators - cantilevers can be coated with antibodies to bind to spores of specific diseases.
5. As the spores bind, the oscillations of the device change.
6. The change is measured by deflection of a laser beam shining on the cantilever.

You're a program manager for DARPA and you're evaluating a proposal to use a nano-sized cantilever to detect the presence of anthrax spores. To test the validity of the proposal consider the following problem. The cantilever can be treated as a simple harmonic oscillator of mass m_c (see below). Suppose $n_a = 300$ anthrax spores each with mass $m_a = 10^{-15}$ kg accumulate on the cantilever beam. What is the change $\Delta\omega$ in the angular frequency of the cantilever? We can detect angular frequency changes of $\approx 10^6$ rad/s. Is this change detectable? WILL IT WORK?



$$L_c = 100\mu m$$
$$m_c = 1.49 \times 10^{-12} \ kg$$
$$k_0 = 370 \ kg/s^2$$

1. The Force: $F_s = -kx$ where x is the displacement from equilibrium.

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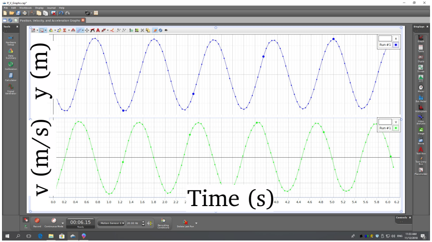

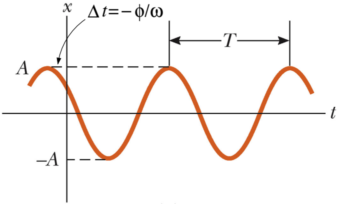

(a)

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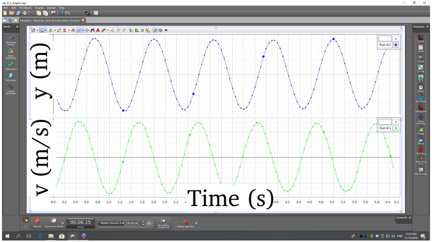

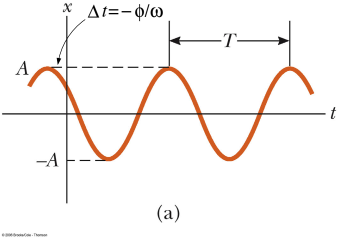

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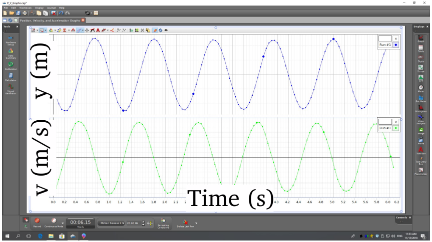

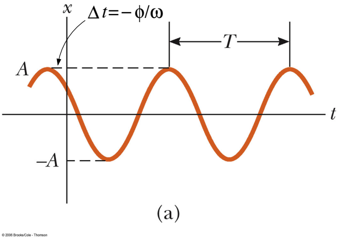

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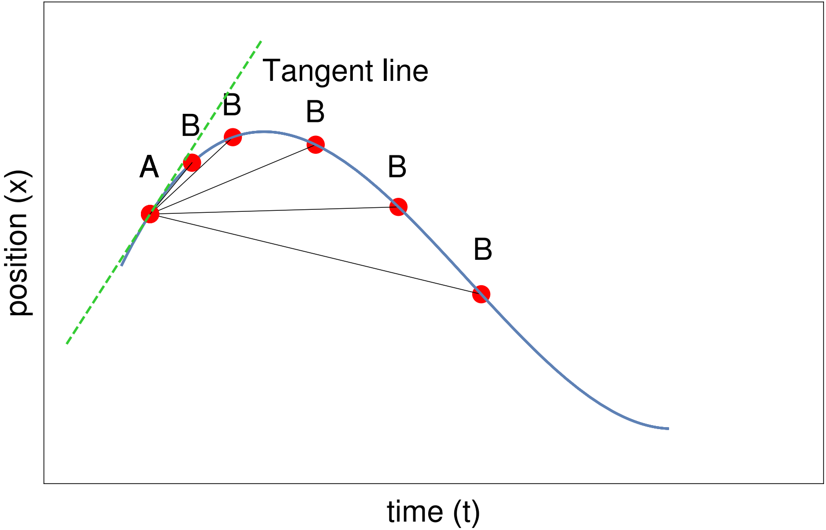
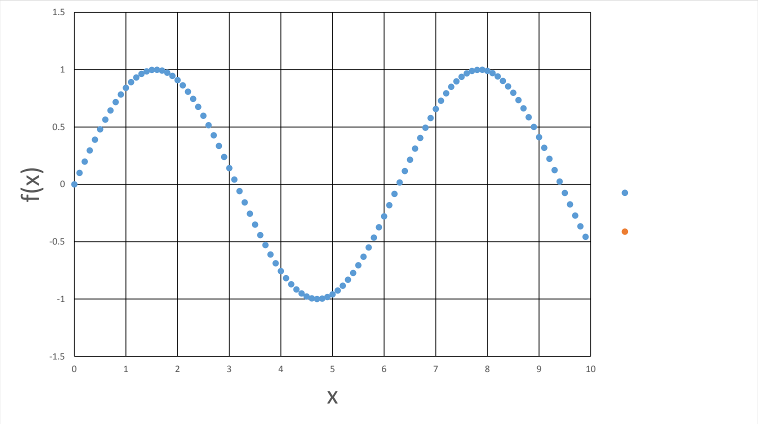
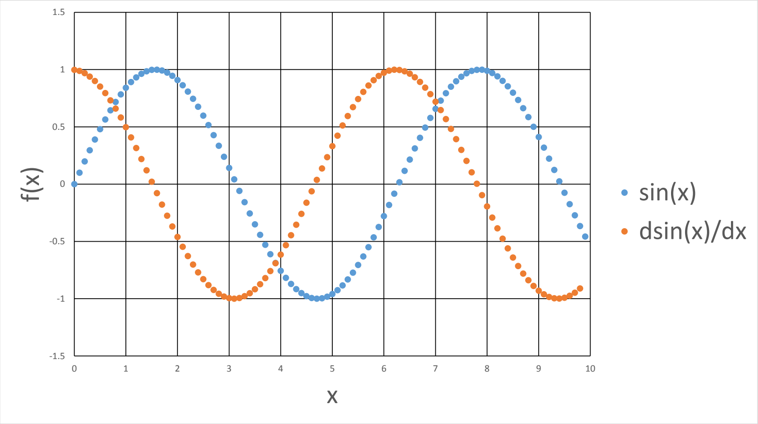
4. Newton's Second Law yields

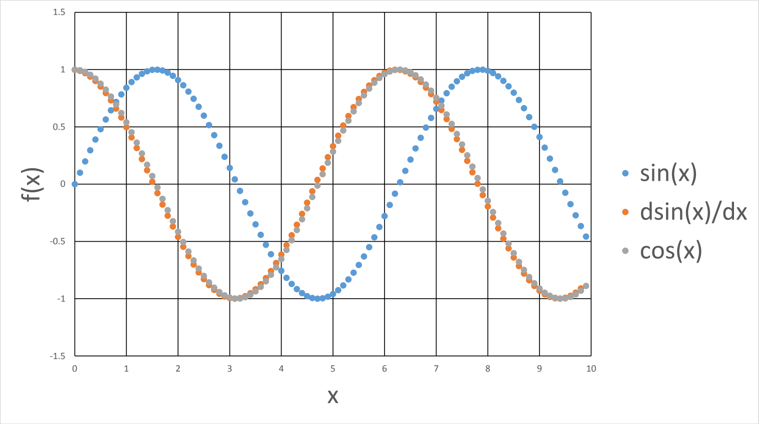
$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

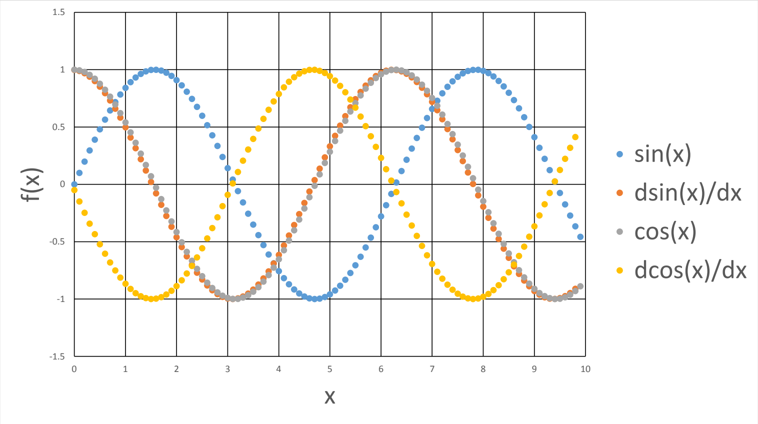

Tangent line
B
B
B
A
B
B
position (x)
time (t)

The Derivative of the Sine 15




1.5
1
0.5
0
-0.5
-1
-1.5
f(x)
0 1 2 3 4 5 6 7 8 9 10
x
sin(x)
dsin(x)/dx


1.5
1
0.5
0
-0.5
-1
-1.5
f(x)
0
1
2
3
4
5
6
7
8
9
10
x
sin(x)
dsin(x)/dx
cos(x)


1.5
1
0.5
0
-0.5
-1
-1.5
f(x)
0
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cos(x)
dcos(x)/dx

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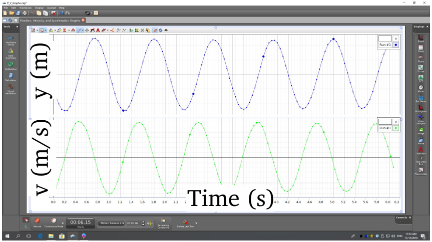

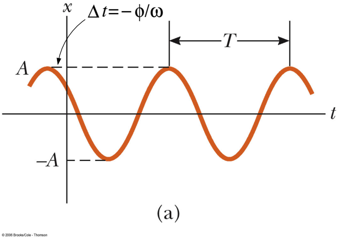

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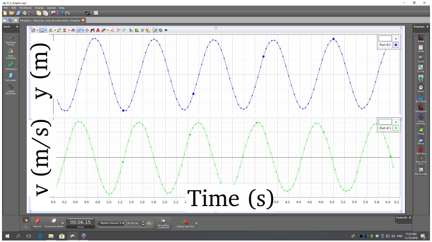

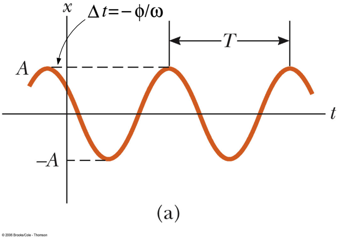

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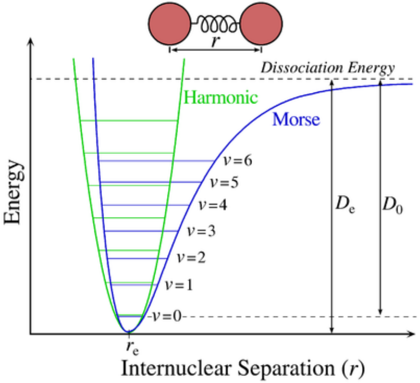
5. Parameters:

$$\omega = \sqrt{\frac{k}{m}} \quad T = \frac{2\pi}{\omega} \quad f = \frac{1}{T}$$

A and ϕ are initial conditions.

The end of the prong of a tuning fork that executes simple harmonic motion with a frequency of 1024 Hz has an amplitude $A = 0.4\ mm$. What is the maximum velocity v_{max} and maximum acceleration a_{max} of the end of a prong? What is the angular frequency? How long does it take for the prong to go from the equilibrium point to $x_1 = 0.1\ mm$?

Carbon and oxygen are bound together by a force that can be modeled as a harmonic oscillator (see below). If the angular frequency is $\omega = 3.8 \times 10^{14}\ rad/s$ and the mass is $m = 1.14 \times 10^{-26}\ kg$, then what is the spring constant k? If the energy of the ground state is $E = 2 \times 10^{-20}\ J$, then what is the amplitude of the oscillation?



To weigh astronauts on the International Space Station NASA uses a chair of mass m_c mounted on a spring of spring constant $k_c = 605.6\ N/m$ that is anchored to the spacecraft. The period of the oscillation of the empty chair is $T_c = 0.90149\ s$. When an astronaut is sitting in the chair the new period is $T_a = 2.12151\ s$. What is the mass of the astronaut?

A transducer used in medical ultrasound imaging is a very thin disk ($m = 0.10\ g$) oscillating back and forth at a frequency $f = 10^6\ Hz$ driven by an electromagnetic coil. The maximum restoring force that can be applied to the disk without breaking it is $F_{max} = 40,000\ N$. (a) What is the maximum oscillation amplitude that won't rupture the disk? (b) What is the disk's maximum speed at this amplitude?

Human cadavers have been used to measure the moments of inertia of different body parts for orthopedics and biomechanics. Consider the center of mass of a lower leg $m = 5.2\ kg$ was found to be $\ell = 0.19\ m$ from the knee. When the leg was allowed to pivot at the knee and swing freely as a pendulum, the oscillation frequency was $f = 1.6\ Hz$. What was the moment of inertia of the lower leg about the knee joint?

The Anatomy Lesson of Dr. Nicolaes Tulp by Rembrandt

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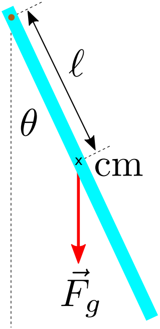

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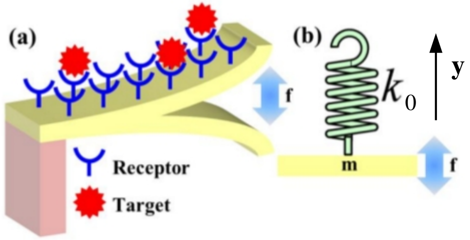

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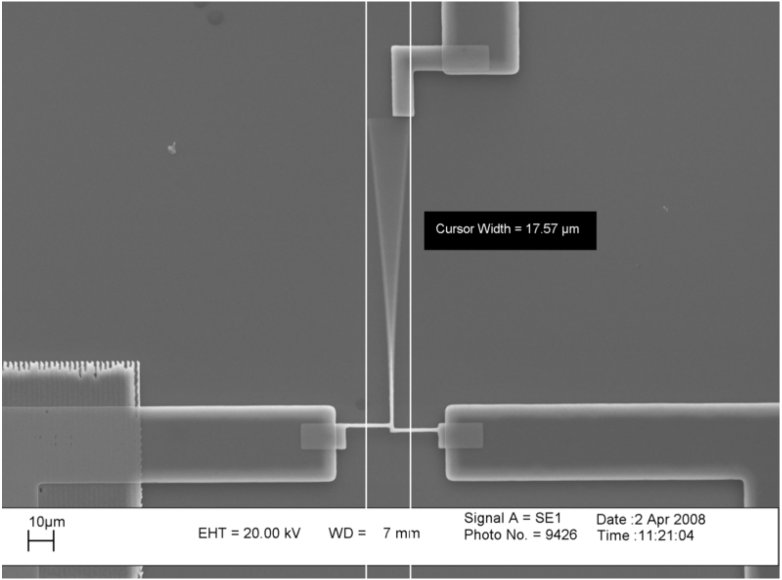
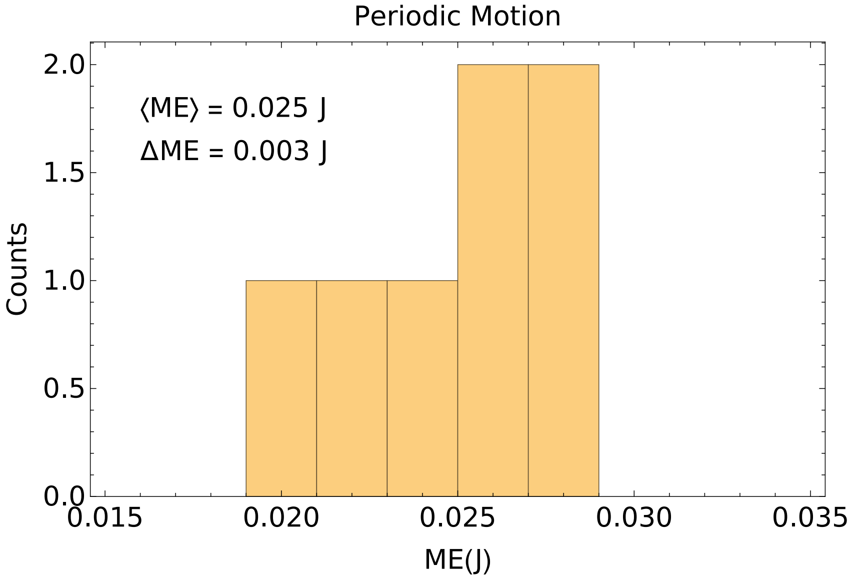
An Oscillating Cantilever




Periodic Motion
⟨ME⟩ = 0.025 J
ΔME = 0.003 J
Counts
2.0
1.5
1.0
0.5
0.0
0.015
0.020
0.025
0.030
0.035
ME(J)


Tracker
File Edit Video Tracks Coordinate System Views Help
Create 50%
mass B m 1.000
Now available: version 4.84 memory in use: 36MB of 247MB
Track Control
mass A mass B
Plot mass B
mass B (t, x)
x 10^2
t=2.87 x=-5.727E2
Table mass B
t x y
2.27 856.894 -274.485
2.37 659.042 -174.165
2.404 472.337 -51.553
2.437 313.498 96.139
2.471 179.739 266.125
2.503 121.219 405.457
2.57 84.993 533.643
2.603 82.206 533.643
2.636 90.566 522.496
2.67 59.913 477.91
2.703 18.113 391.524
2.737 -40.406 271.698
2.77 -140.726 149.086
2.803 -266.125 45.98
2.837 -411.03 -68.273
2.87 -572.656 -157.446
2.904 -723.135 -218.752
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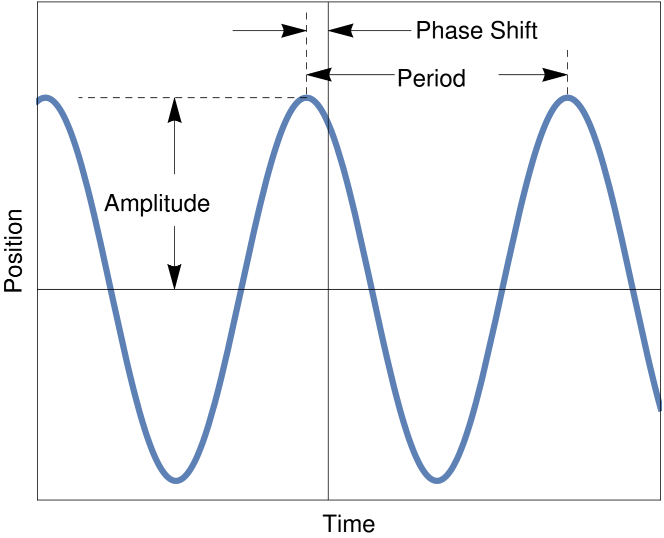
mass B selected (set mass on toolbar, shift-click to re-mark highlighted position)
085 100%
Molecule2.mp4


Phase Shift
Period
Amplitude
Position
Time

The Harmonic Oscillator 32

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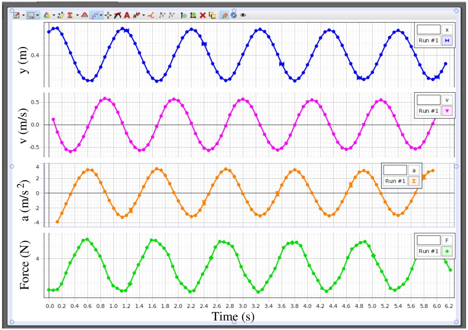
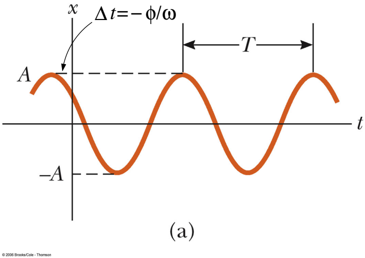
2. The Potential Energy: $V_s(x) = \frac{1}{2}kx^2$
3. Measurements:





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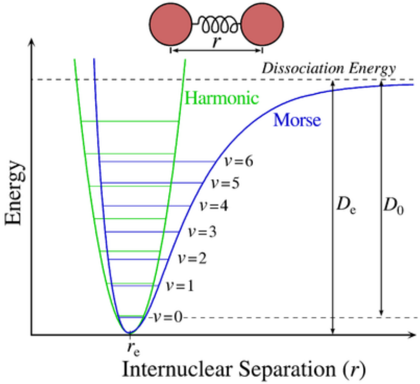
3. For many molecules (and atoms and nuclei) they're potential energies are, sometimes, well described by the harmonic oscillator.



Angular Momentum Conservation