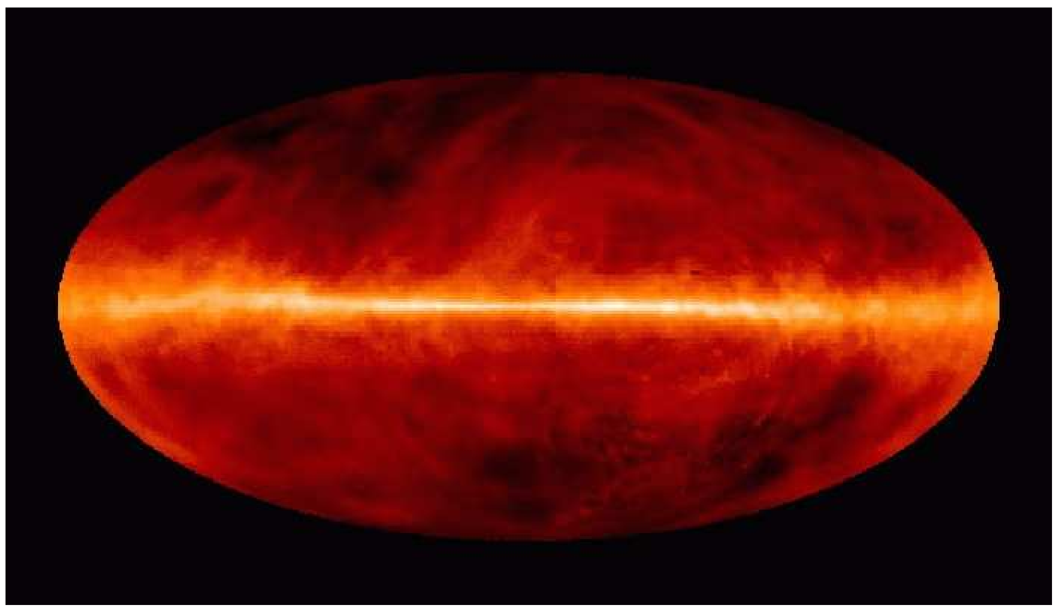
Seeing the Milky Way

The hydrogen in our galaxy has been mapped by the observation of the 21-cm wavelength line of hydrogen gas. At 1420 MHz, this radiation from hydrogen penetrates the dust clouds and gives us a more complete map of the hydrogen than that of the stars themselves since their visible light won't penetrate the dust clouds.



The Energy States of Hydrogen

The optical spectrum of hydrogen can be explained by solving the Schroedinger equation with the Coulomb potential

$$-\frac{\hbar^2}{2\mu}\nabla^2\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) = E\Psi(\vec{r})$$

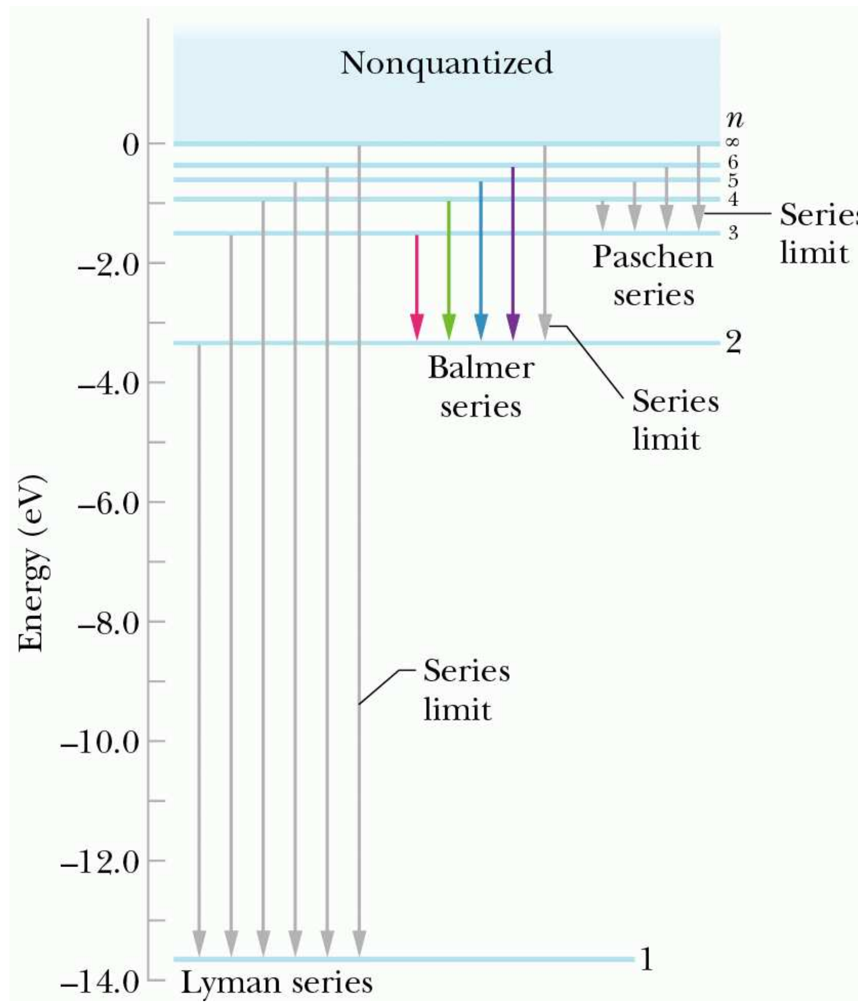
where μ is the reduced mass, $\Psi(\vec{r})$ is the hydrogen wave function, and $V(\vec{r})$ is the potential energy. The Coulomb potential is

$$V_C(\vec{r}) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

where e is the electronic charge and r is the separation. The energy states or eigenvalues are

$$E_n = -\frac{13.6\ eV}{n^2}$$

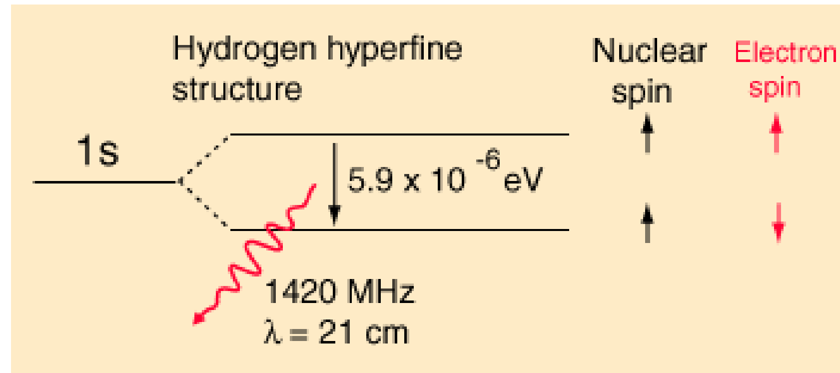
where n is the principle quantum number.



Wait! This is E&M NOT Quantum!

The Hydrogen 21-cm Line

The 1420 MHz radiation comes from the transition between the two levels of the hydrogen 1s ground state, slightly split by the interaction between the electron spin and the nuclear spin. The splitting is known as hyperfine structure. Because of the quantum properties of of radiation, hydrogen in its lower state will absorb 1420 MHz and the observation of 1420 MHz in emission implies a prior excitation to the upper state.



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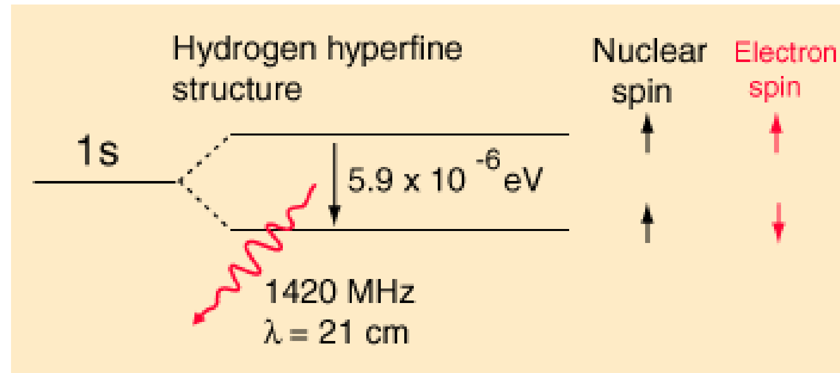


1. What is the energy of this transition? How does it compare with the transitions between the principle quantum states?
2. What is the energy of a magnetic dipole in a magnetic field $\vec{B}$?
3. What is the interaction energy of two magnetic dipoles separated by a displacement $\vec{r}$?
4. Does this model account for the size of the hyperfine interaction in hydrogen?

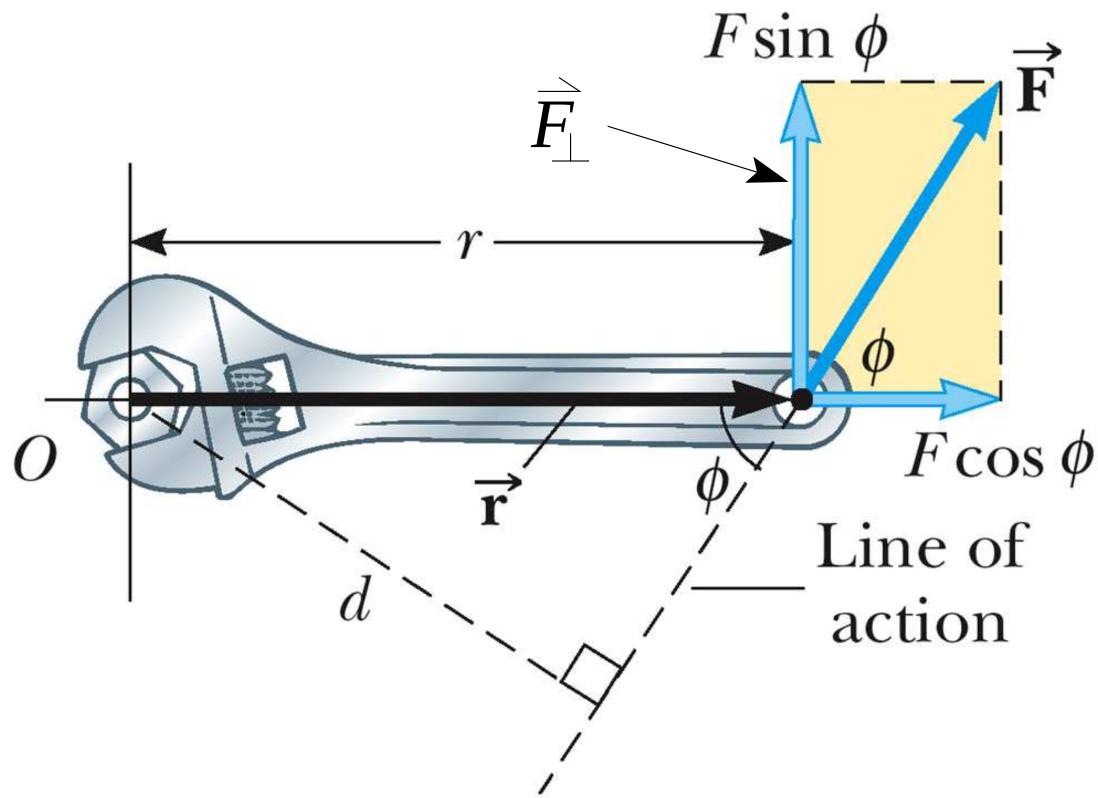
Linear $\rightarrow$ Rotational Quantities

Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v	$v = r\omega$	$\omega = \frac{v}{r} = \frac{d\theta}{dt}$
a	$a = r\alpha$	$\alpha = \frac{a}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$dW = \vec{F} \cdot d\vec{s}$		$dW = \vec{\tau} \cdot d\vec{\theta}$
$\vec{F} = m\vec{a}$	$\vec{N} = \vec{\tau} = \vec{r} \times \vec{F} = rF_{\perp}$	$\vec{\tau} = I\vec{\alpha}$

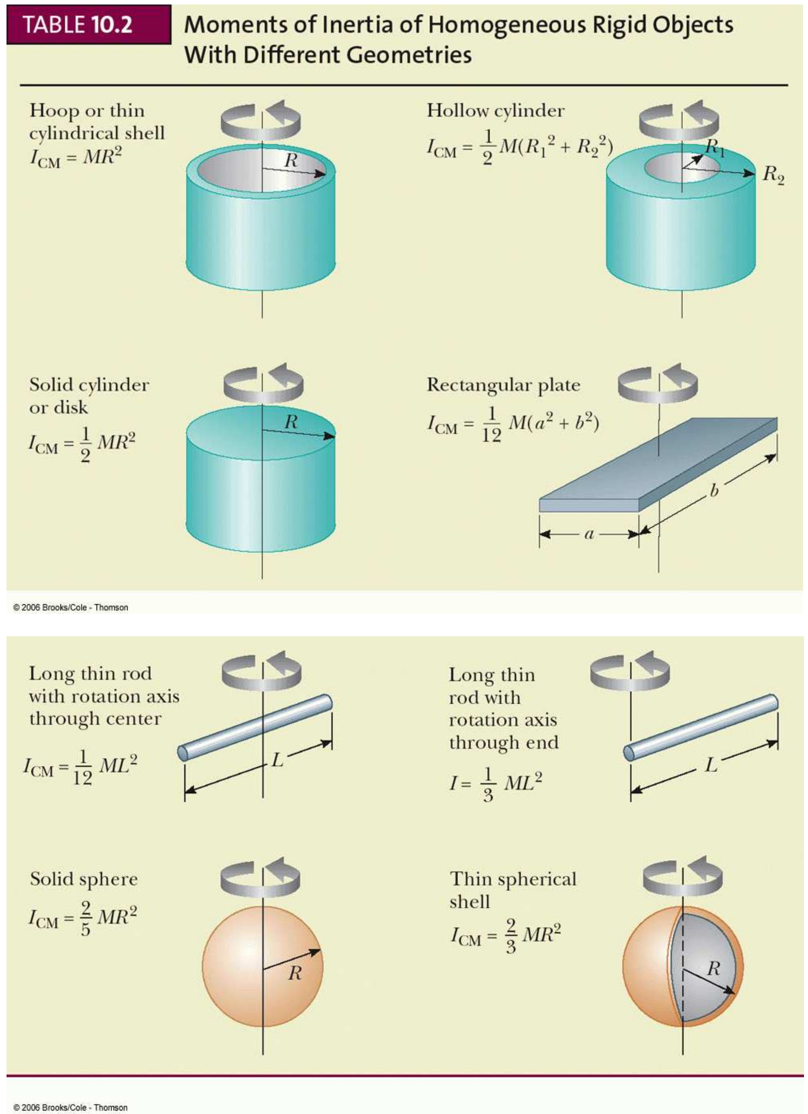
$$I = \int r^2 dm = \int r^2 \rho d\tau$$

Torque - Rotational Equivalent of Force

$$\vec{F} = m\vec{a} \rightarrow \vec{\tau} = \vec{r} \times \vec{F} = r\vec{F}_{\perp}$$



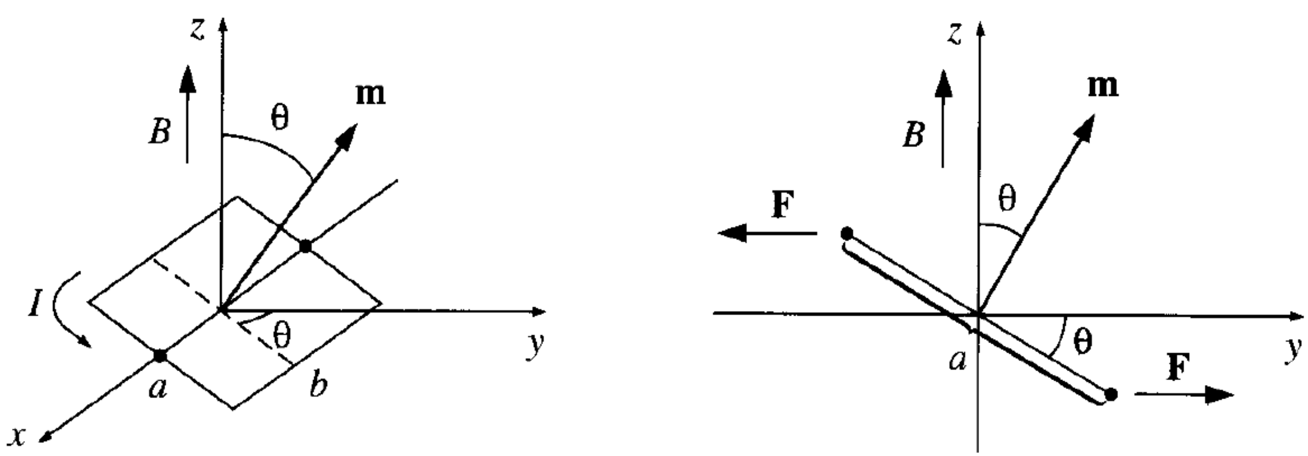
Moments of Inertia



Torque on a Rectangular Current Loop

The rectangular current loop shown below is immersed in a uniform magnetic field $B = B_0\hat{z}$ with current I flowing through it in the direction shown. The loop has width a and length b.

1. What is the force on each straight section of the loop?
2. What is the torque exerted on each section of the loop?
3. What is the net torque?



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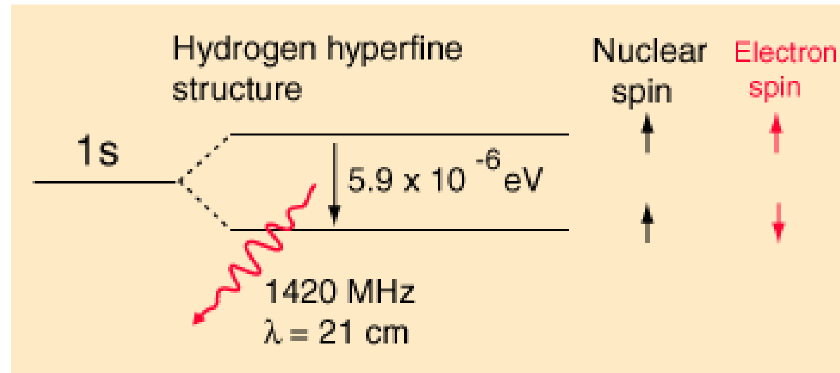

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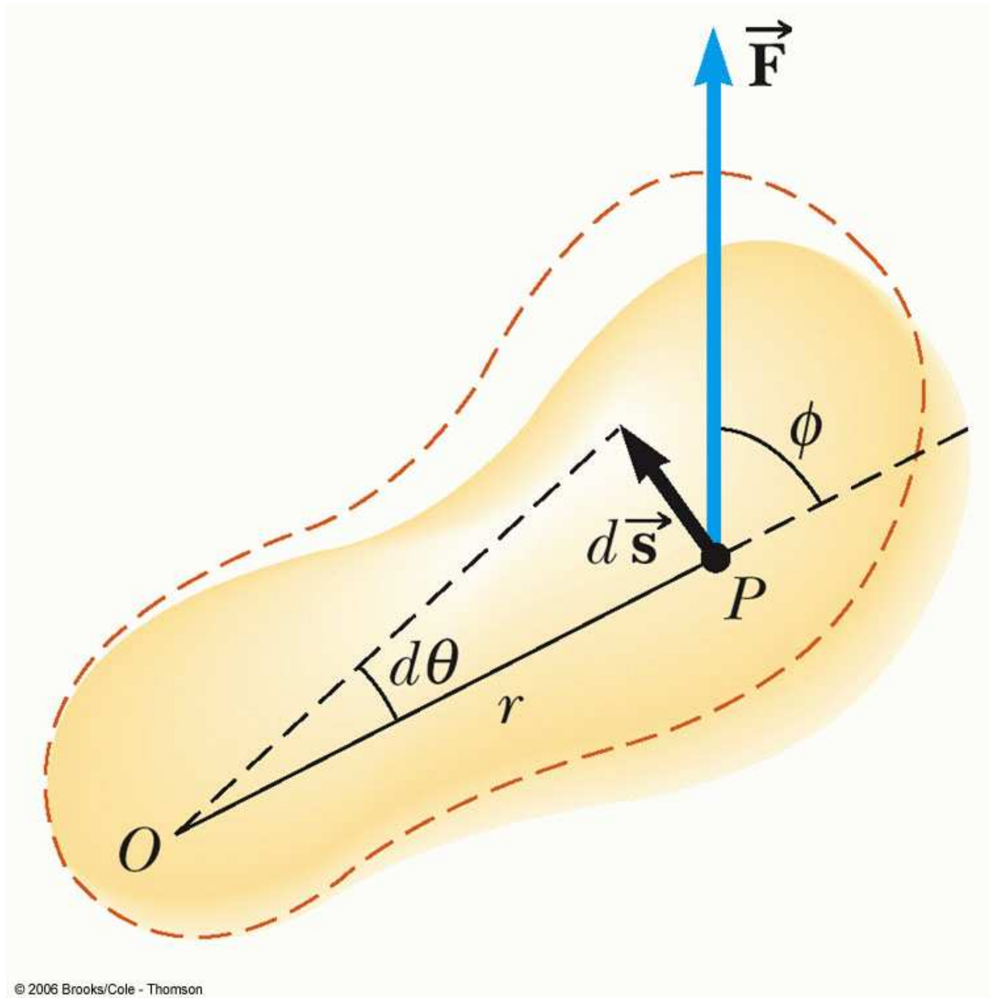
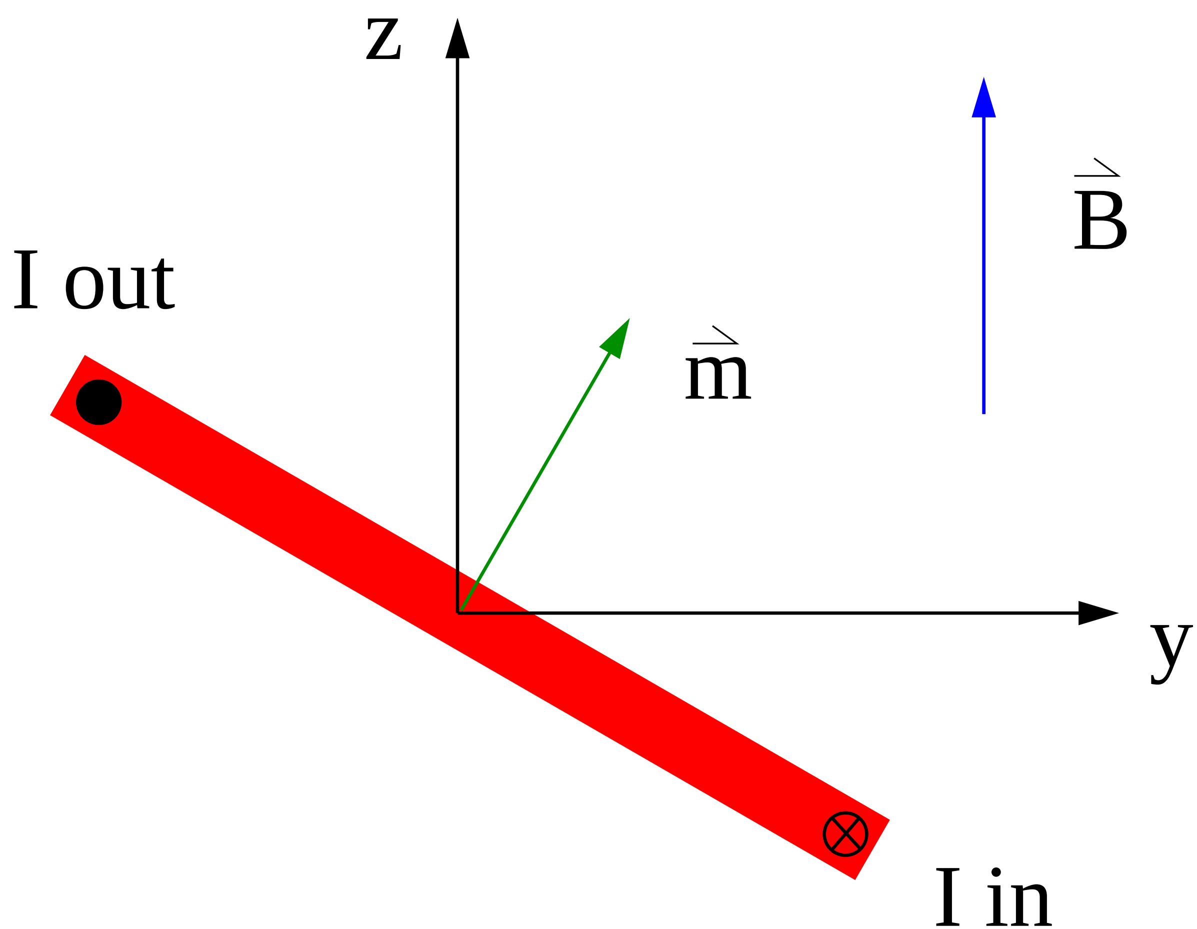
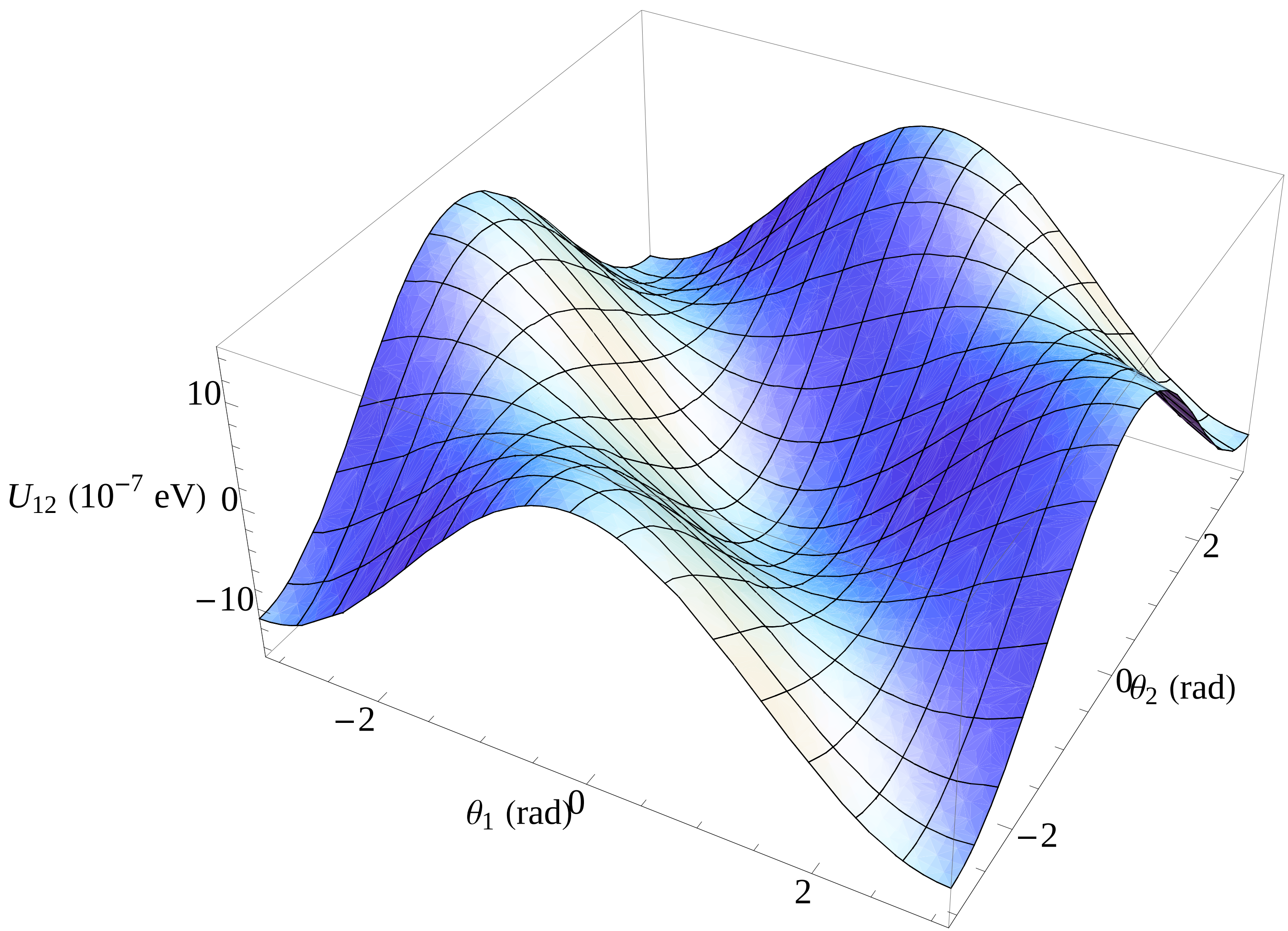
Rotational Work



Energy of a Current Loop in a $\vec{B}$ Field



Potential Energy of Two Magnetic Dipoles



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