# How JLab Makes The Beam

- Jefferson Lab is a US Department of Energy national laboratory and the newest 'crown jewel' of the US.

- The centerpiece is a 7/8-mile-long, racetrack-shaped electron accelerator that produces unrivaled beams.

- The electrons do up to five laps around the Continuous Electron Beam Accelerator Facility (CEBAF) and are then extracted and sent to one of three experimental halls.

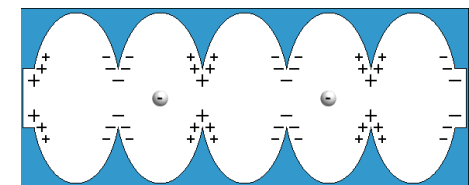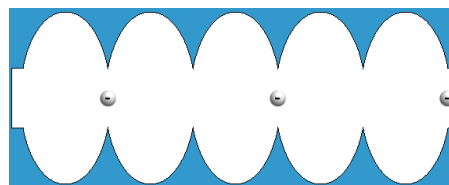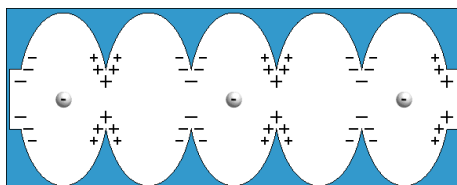- All three halls can run simultaneously.
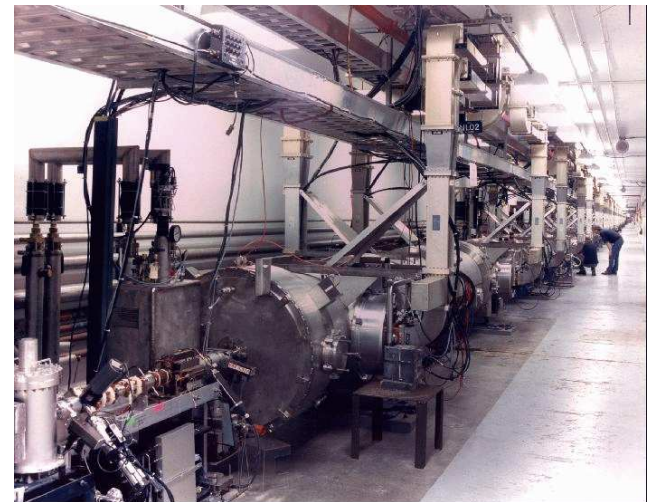
How Does CEBAF Work?
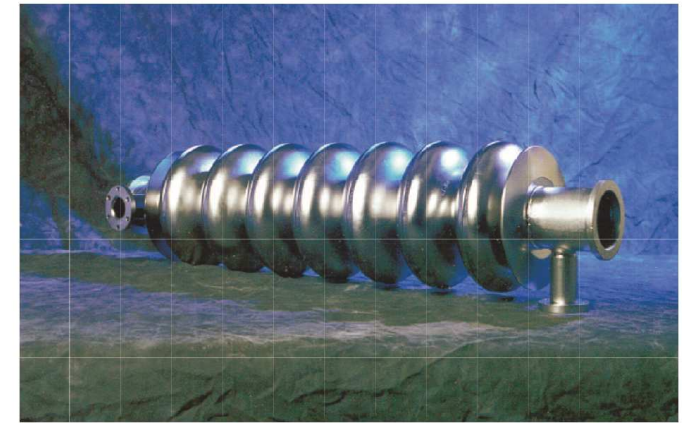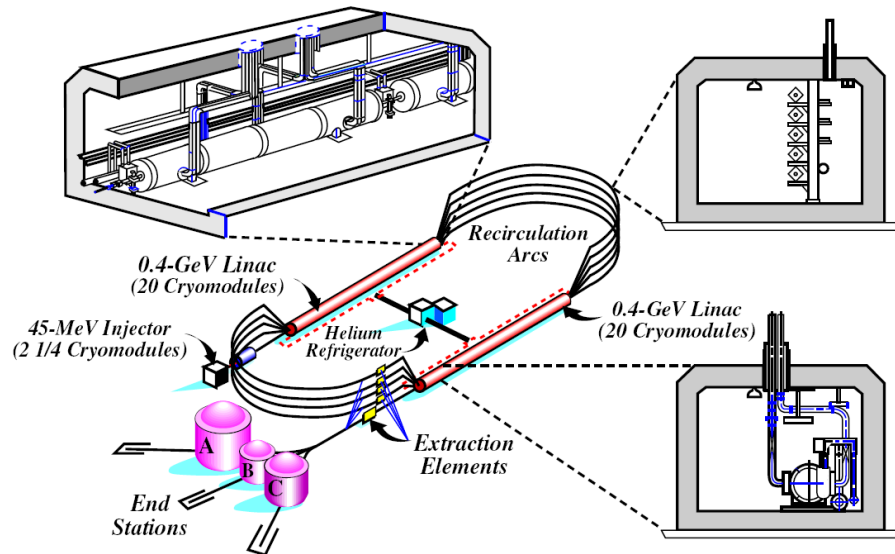
Cavity

What happens inside the cavity? Feed it with oscillating, radio-frequency power at 1.5 GHz! In each hall beam buckets are about 2 picoseconds long and arrive every 2 nanoseconds.
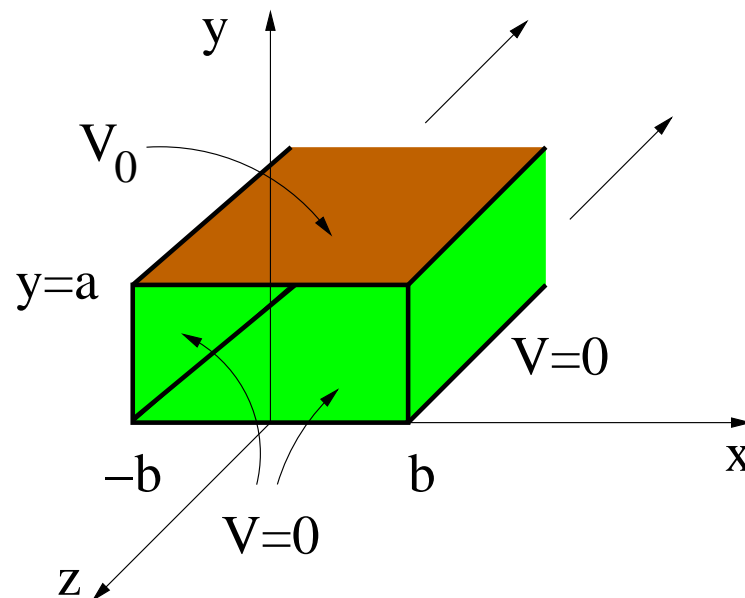

A. → B. → C.

The Potential in a Waveguide

Consider a long, rectangular metal pipe like the one shown in the figure where each side is electrically isolated from its neighbors (a thin layer of insulation at each corner prevents them from shorting out). The upper and lower sides are at $y = 0$ and $y = a$ and the two sides are at $x = \pm b$. The potential on the top side $(y = a)$ is at V_0 and the potential is zero on the other three sides. What is the potential inside the pipe?

The Uniqueness Theorems

1. The solution to Laplace's equation in some volume $\mathcal{V}$ is uniquely determined if V is specified on the boundary $\mathcal{S}$ of $\mathcal{V}$.

2. The solution to Poisson's equation in a volume $\mathcal{V}$ surrounded by conductors and containing a specified charge density ρ is uniquely determined if the total charge on each conductor is is given.

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A license to ~~steal~~ use our imagination!

An example of solving Laplace's equation

Two infinitely long, grounded metal plates are at $y = 0$ and $y = a$ and connected at $x = \pm b$ by metal strips maintained at a constant potential V_0 as shown below (a thin layer of insulation at each corner prevents them from shorting out). What is the potential inside the pipe?
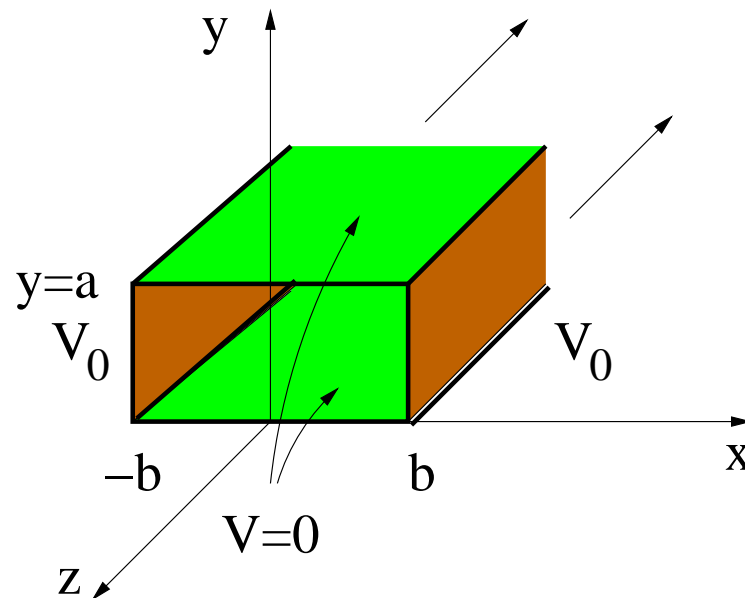
We're Saved!!

- **Completeness** - A set of functions $f_n(y)$ is complete is any other function $g(y)$ can be expressed as a linear combination of the $f_n(y)$'s.

$$g(y) = \sum_{n=1}^{\infty} C_n f_n(y)$$

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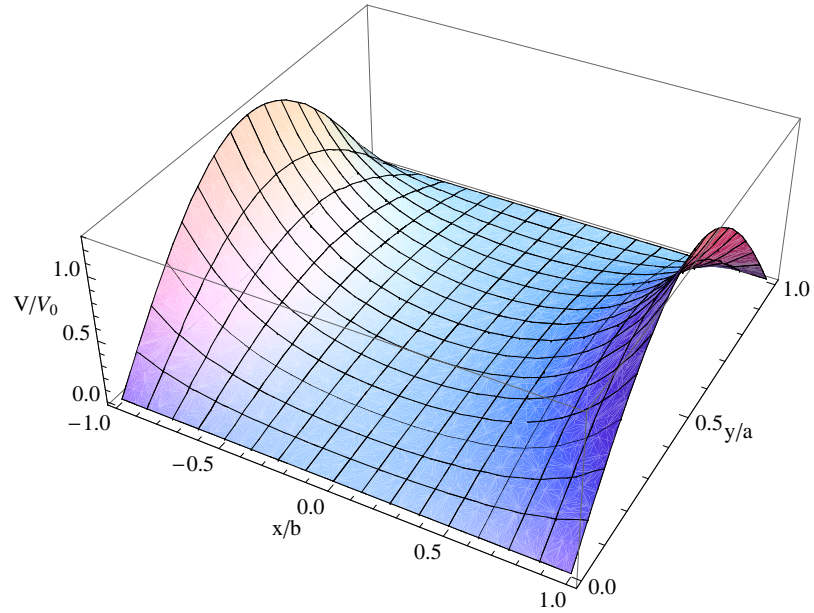- **Orthogonality** - A set of functions is orthogonal if there is an inner product defined for them so that

$$\int_0^a f_n(y) f_n'(y) dy = 0 \qquad n' \neq n$$
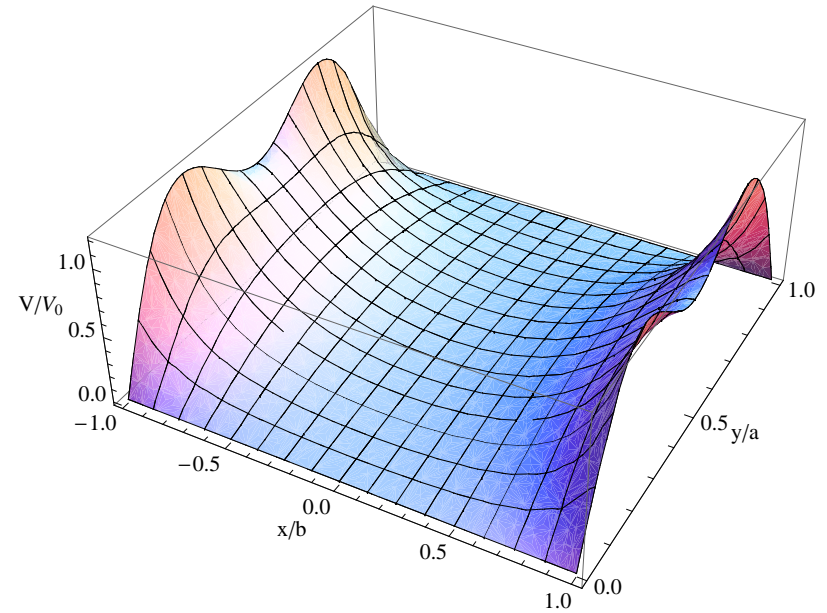
Amazing, but True!
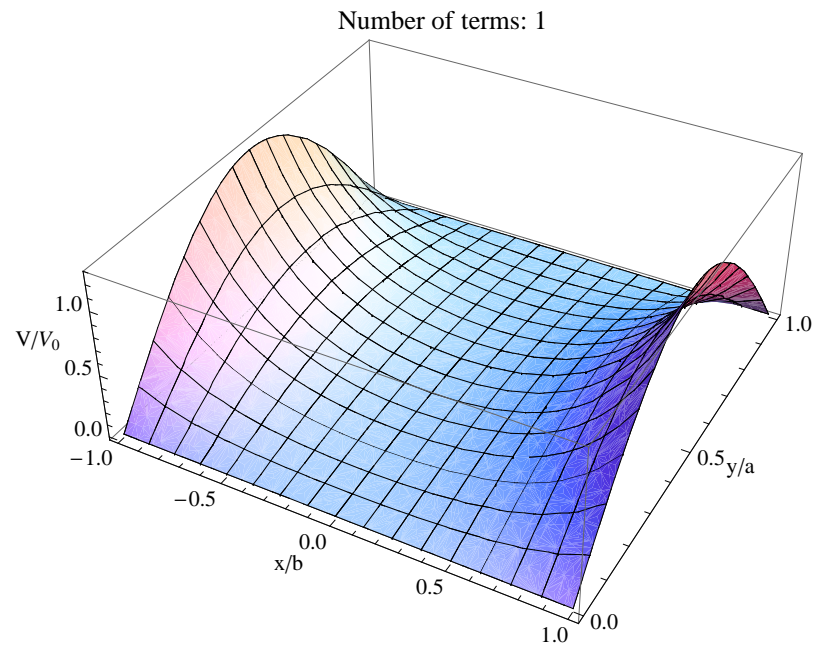
Number of terms: 1
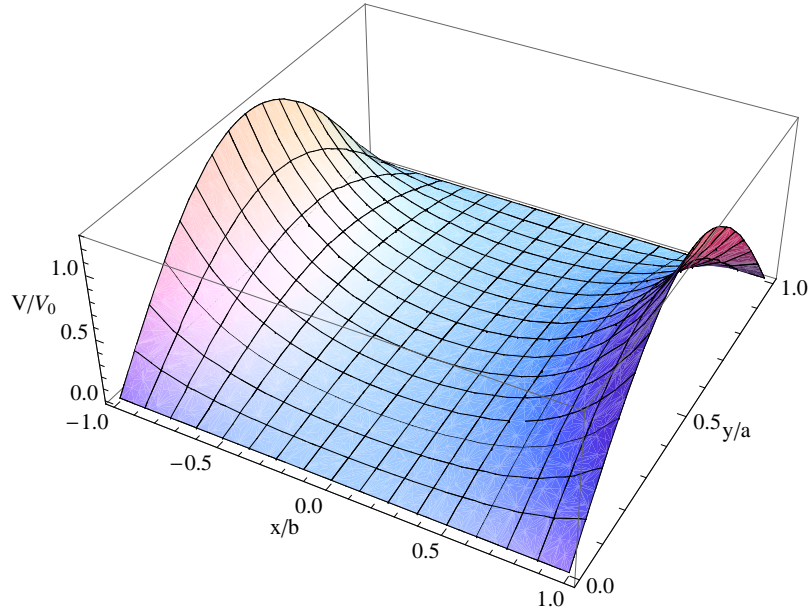
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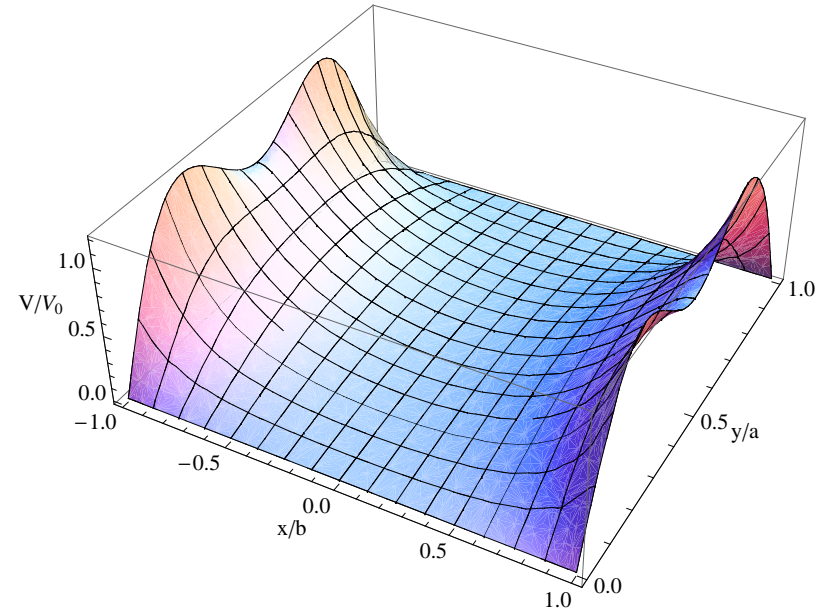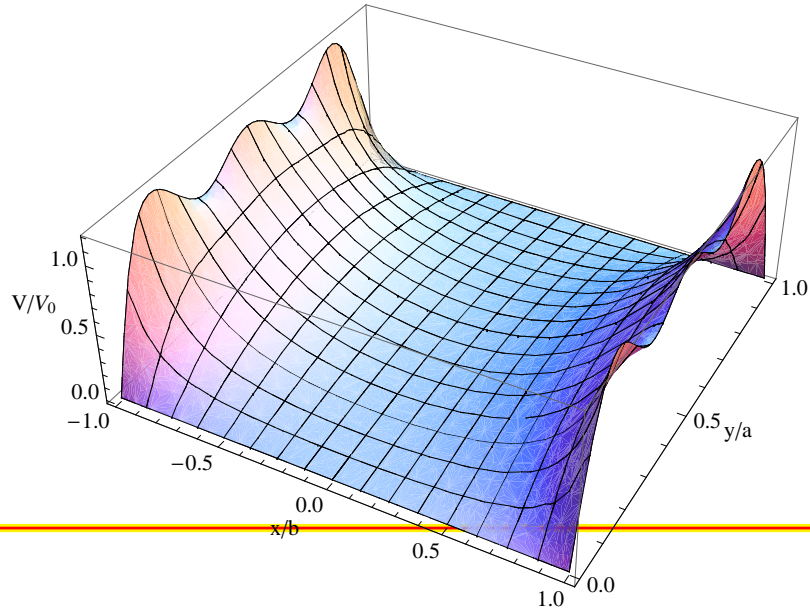Number of terms: 1

Number of terms: 2

Amazing, but True!
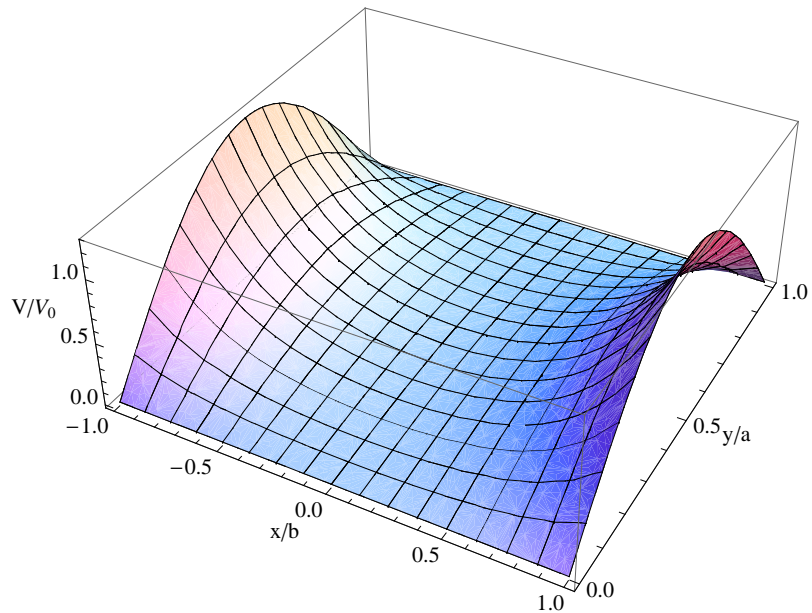

Number of terms: 1

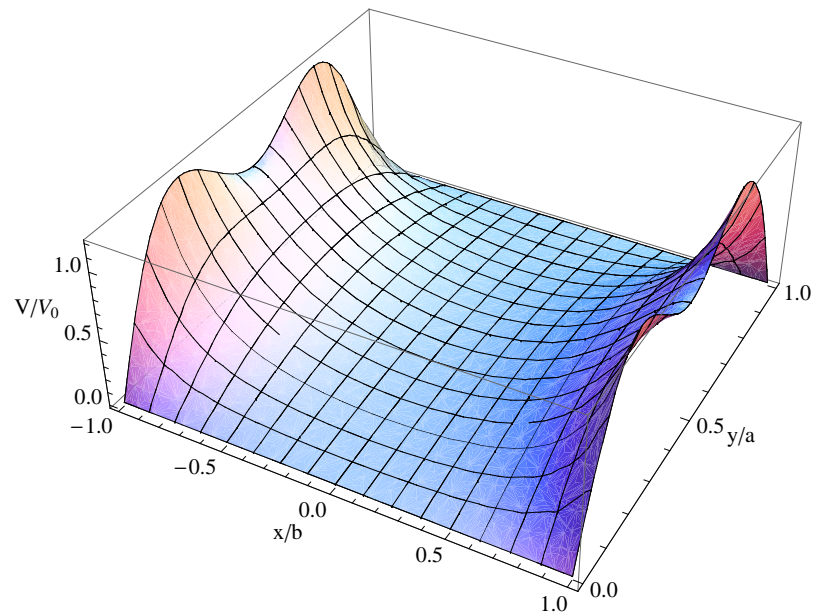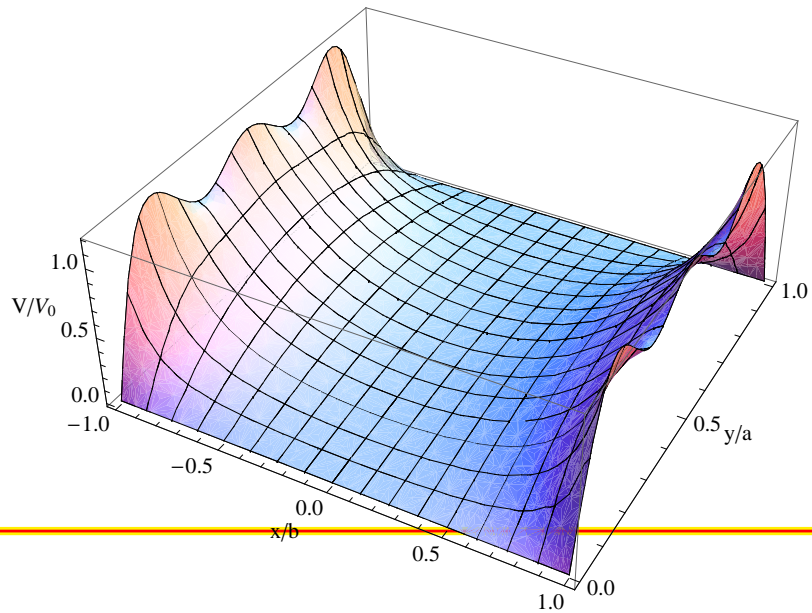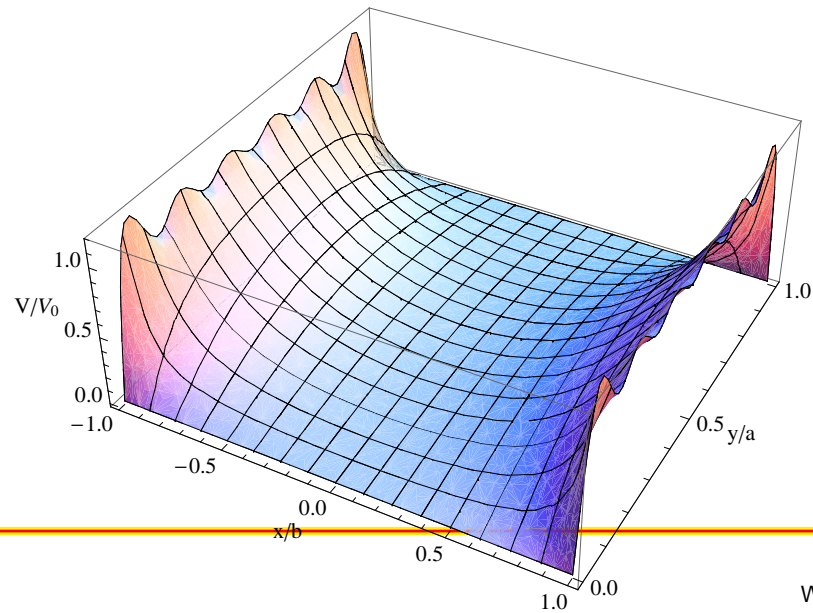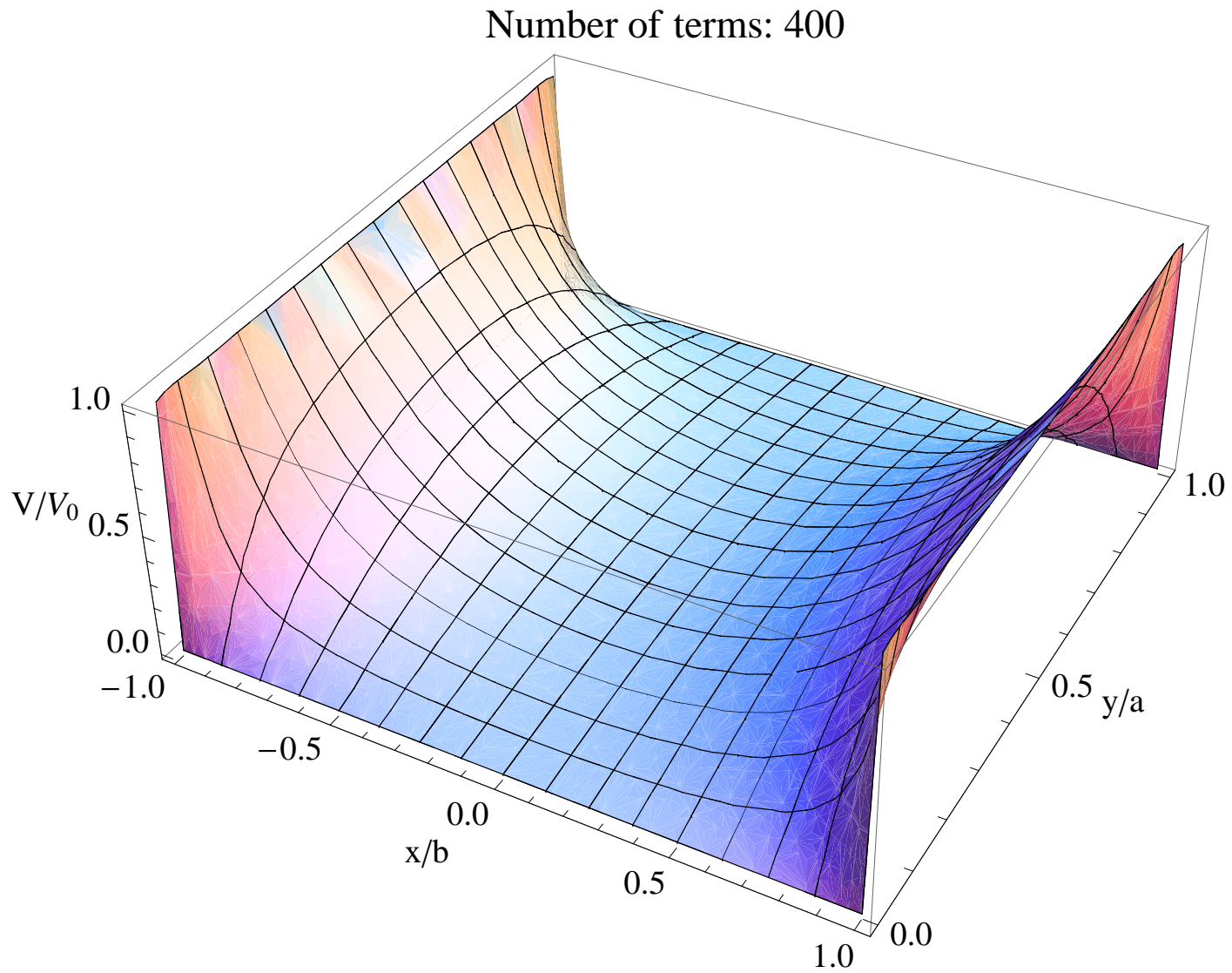Number of terms: 2

Number of terms: 3

Amazing, but True!

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Number of terms: 400
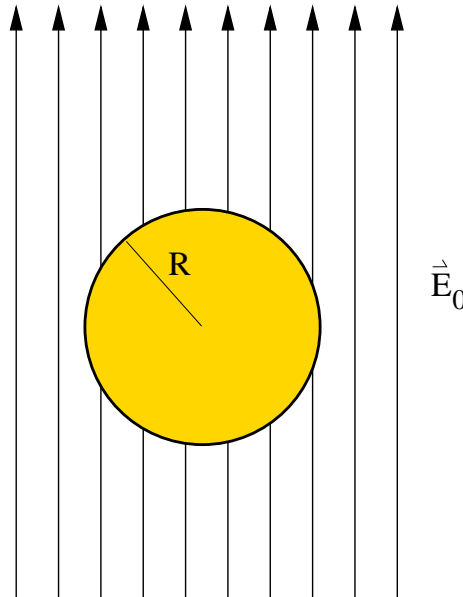
Laplace's Equation in Spherical Coordinates

A metal (and hence conducting) thin, spherical shell of radius R is immersed in an uniform, external electric field $\vec{E}_0 = E_0\hat{z}$. The charge distribution that creates $\vec{E}_0$ is unaffected by the presence of the sphere. What are the potential and field inside and outside the sphere? What is the surface charge density?

Problem 2-7: Field of a Spherical Shell

2-7)

$dq = \nabla dA = \sigma R^2 d\cos\theta \, d\phi$

$\imath^2 = R^2 + z^2 - 2Rz\cos\theta$

$|d\vec{E}| = \dfrac{1}{4\pi\epsilon_0} \dfrac{\sigma^2 R^2 d\cos\theta \, d\phi}{R^2 + z^2 - 2Rz\cos\theta}$

The transverse components will cancel so get the z component

$dE_z = dE\cos\gamma \qquad \cos\gamma = \dfrac{z - R\cos\theta}{\imath}$

for $R > z$

$E_z = \dfrac{1}{4\pi\epsilon_0} 2\pi\sigma R^2 \dfrac{1}{z^2} \left[\dfrac{z-R}{R-z} + \dfrac{z+R}{z+R} \right]$
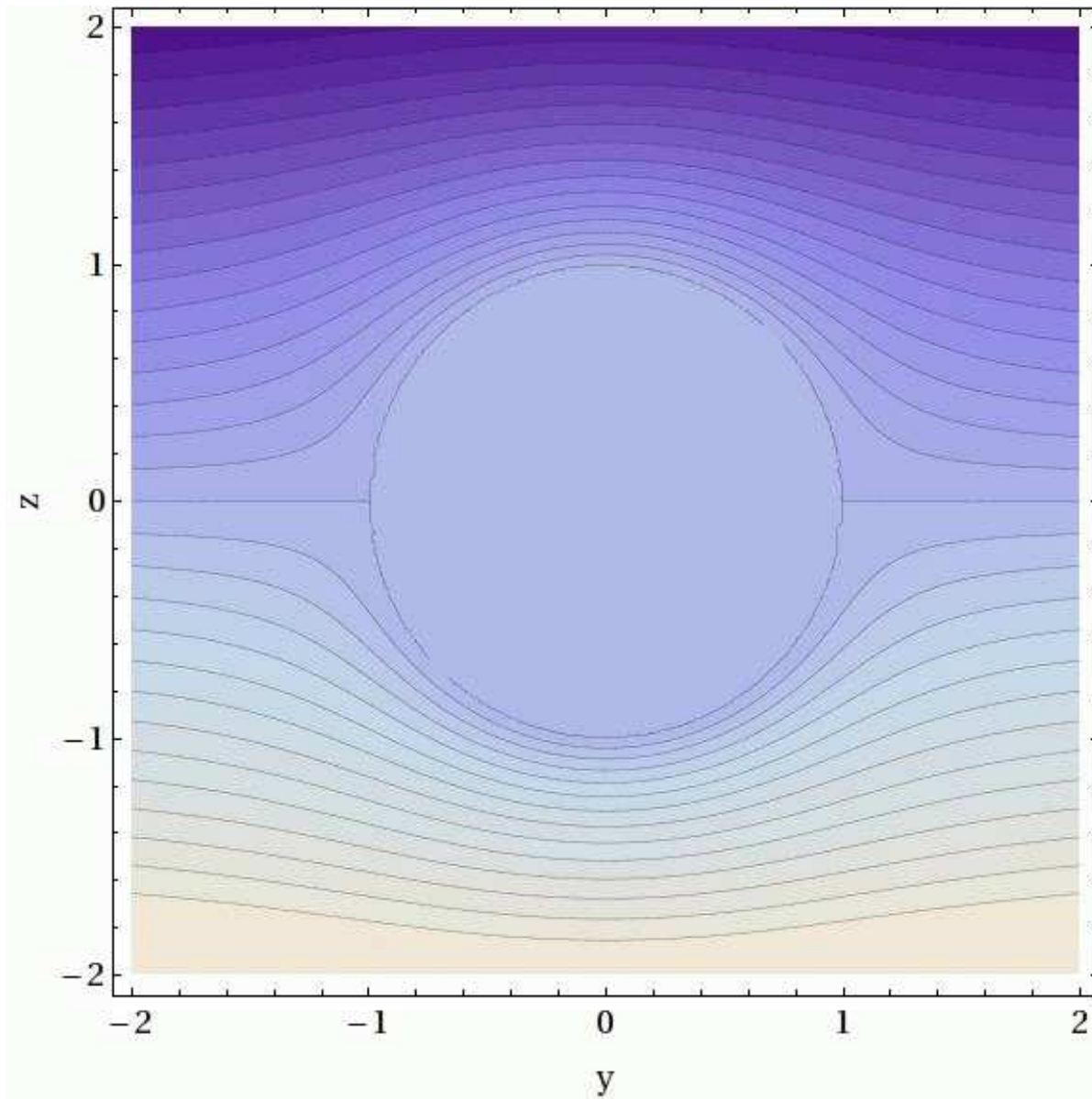
$= 0 \, !$

for $R < z$

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$= \dfrac{1}{4\pi\epsilon_0} \dfrac{4\pi R^2 \sigma}{z^2} \} \text{total charge}$

It looks like a point charge!

# Equipotentials for shell in an external field

Properties of Conductors

1. $\vec{E} = 0$ inside.

2. $\rho = 0$ inside.

3. The charge is all on the surface.

4. A conductor is an equipotential.

5. $\vec{E}$ is perpendicular to the surface at the surface.

Problem 2.6

Find the electric field a distance z above the center of a flat circular disk of radius R (see figure below), which carries a uniform surface charge σ. What does your formula give in the limit $R \to \infty$? Also check the case $z \gg R$.

z

Plane with surface
charge density σ

y

x

R
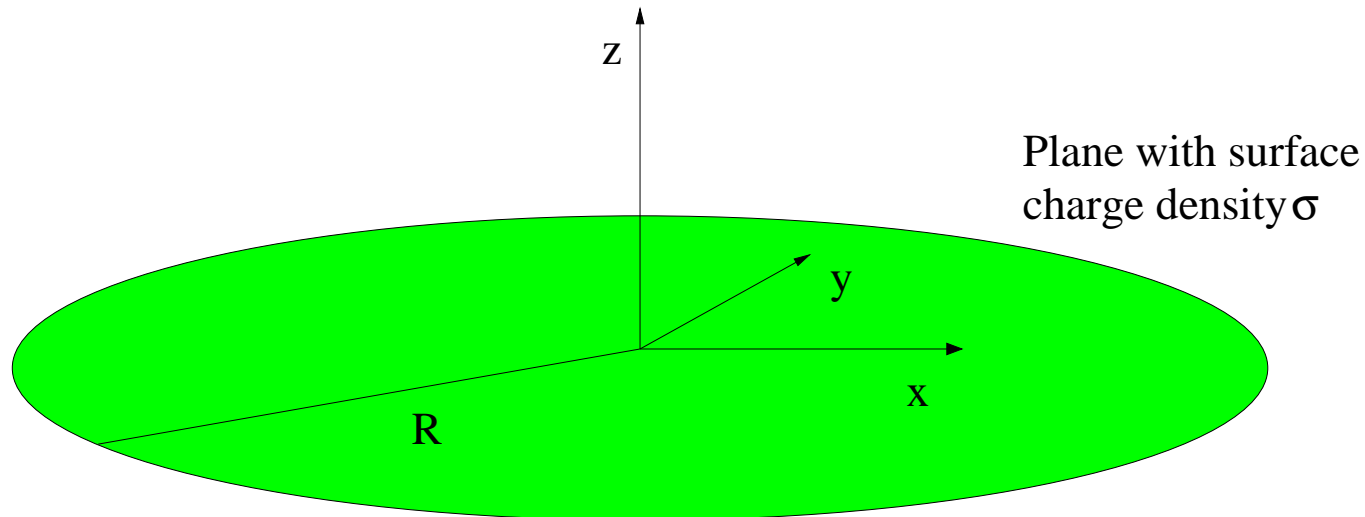
$$\vec{E} = \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left(\frac{1}{z} - \frac{1}{\sqrt{z^2 + R^2}} \right) \hat{z}$$
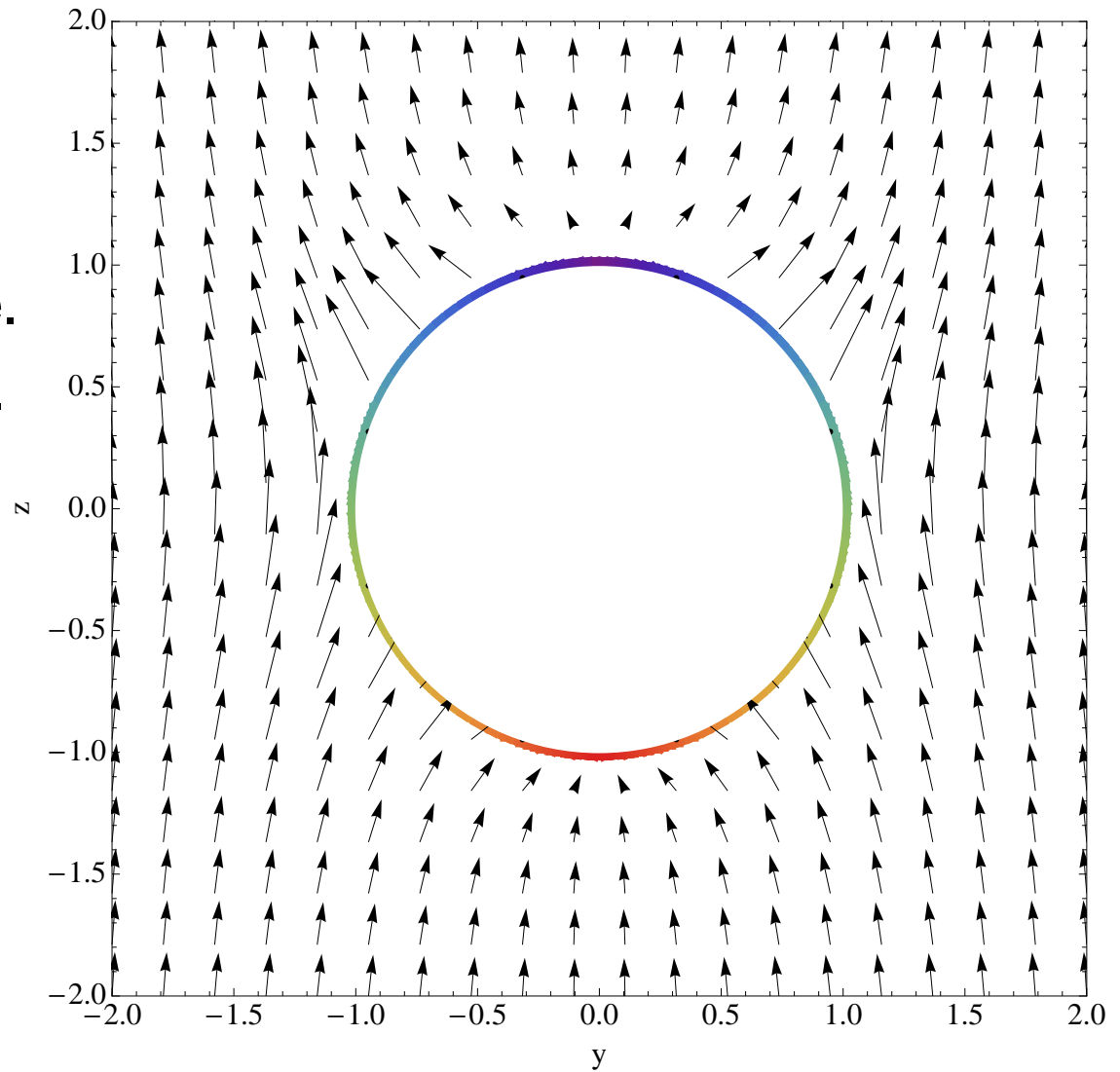
Charge Density

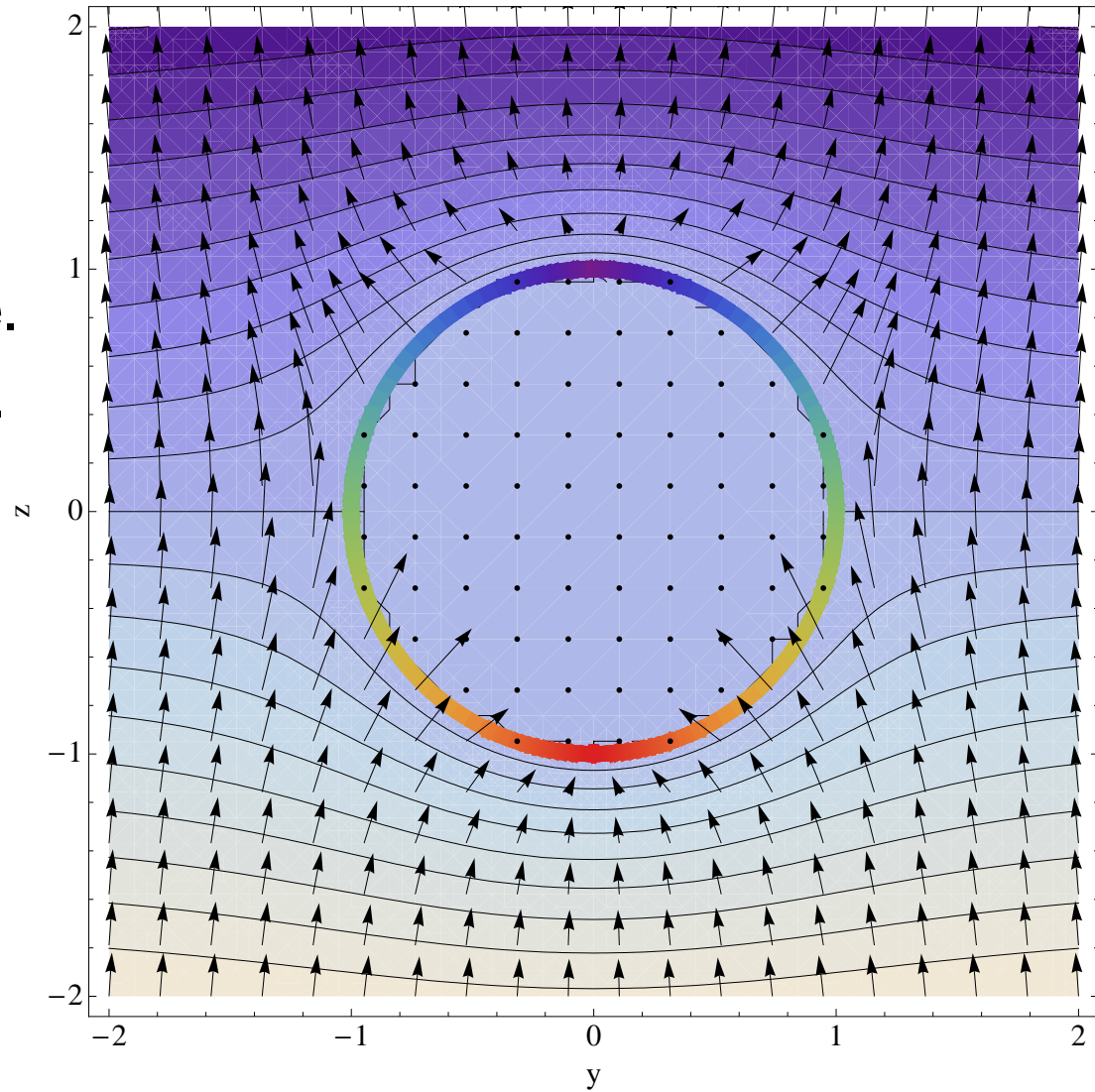Red - negative charge.

Blue - positive charge.
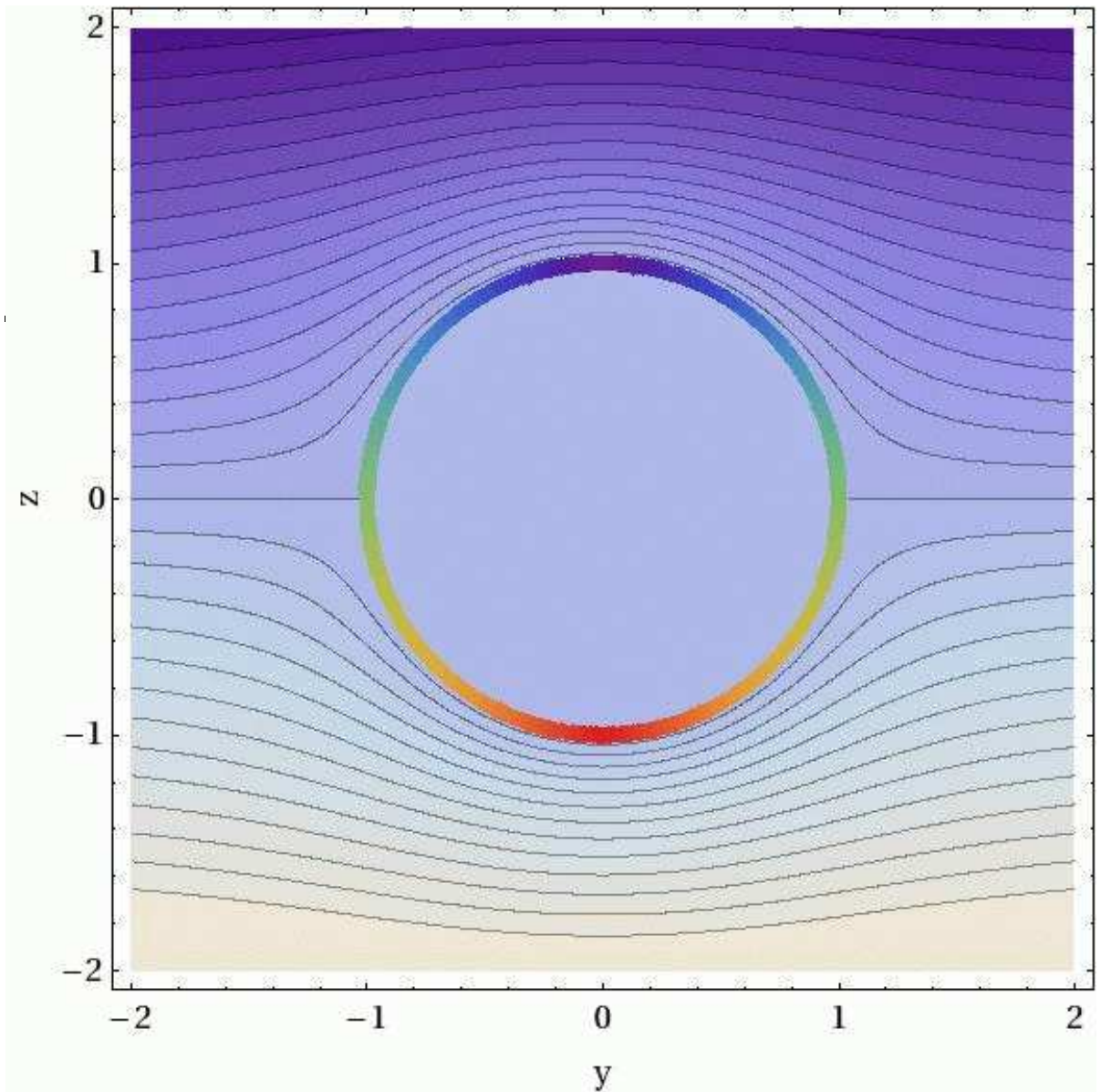
Electric Field

Red - negative charge.
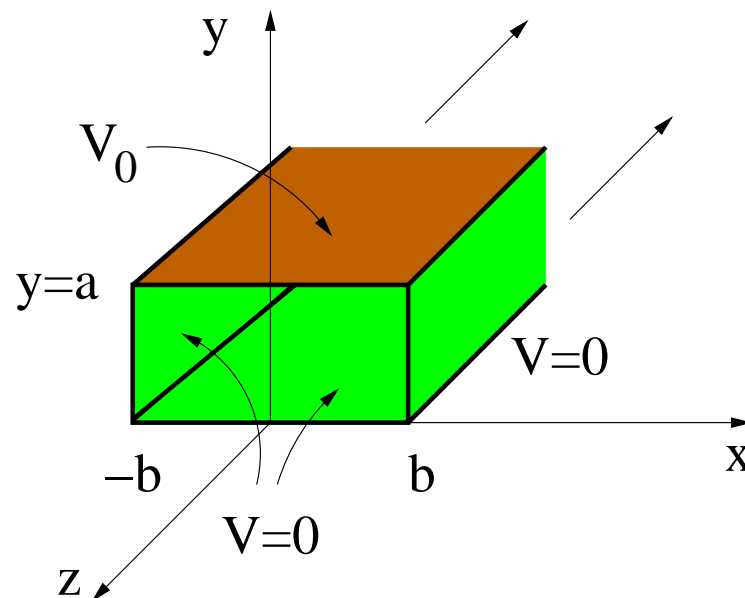
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All of It

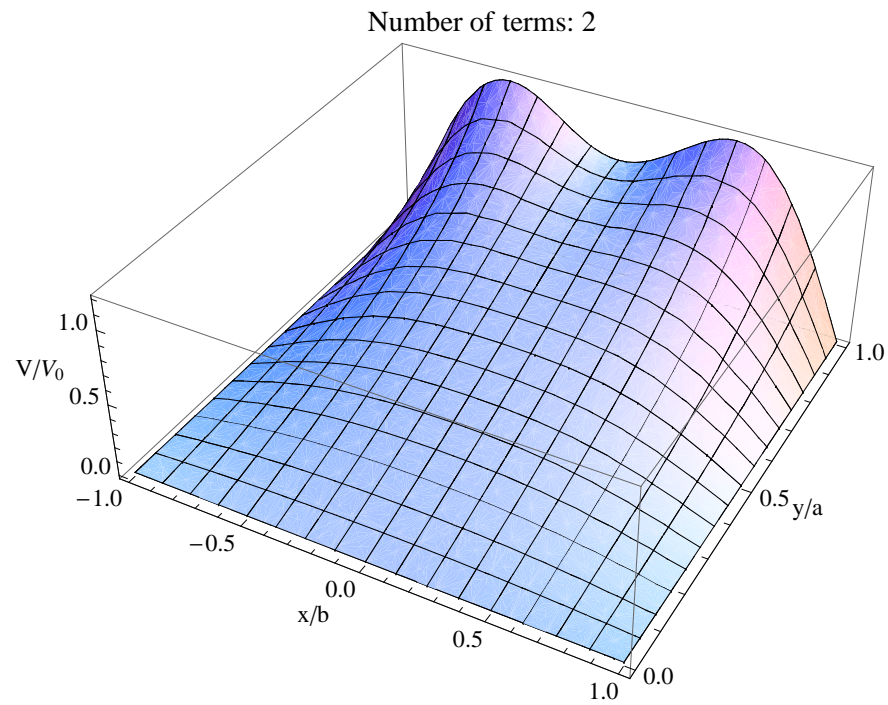Red - negative charge.

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What It Looks Like.

Number of terms: 2

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Number of terms: 300