Physics 215

Second Order Differential Equations

1. Show that the second-order correction to the 3-point formula for the second derivative of a function *f(x)* is

$$-h^2 \frac{f_n^{(iv)}}{12} \quad .$$

2. Generate a recursion relation equation for the following equations using the centered formula for the second derivative. State the order of the error for each result.

$$a. \quad \frac{d^2y}{dt^2} - 2ty = 5 \qquad b. \quad \frac{d^2y}{dt^2} + \frac{dy}{dt} + t^2 = 0 \qquad c. \quad \frac{d^2y}{dt^2} + e^{y^2} + t^2 y = 10$$

3. Corresponding to to any physical pendulum that oscillates about a given suspension point O with period T is a simple pendulum of length L with the same period. The point along the line joining the point of suspension O and the center of mass C of the physical pendulum at this distance L from O is called the center of oscillation. Show that $L = \frac{I}{mL_{cm}}$ where I is the moment of inertia of the physical pendulum.

4. The center of oscillation of a physical pendulum has this interesting property: If an impulsive force (assumed horizontal and in the plane of oscillation) acts at the center of oscillation, then no reaction is felt at the point of support. Chipper Jones knows (like most good hitters in baseball) that unless the ball hits the bat at this point (the 'sweet spot'), the reaction due to the impact will sting their hands. To prove this property, let the stick in the figure below simulate a baseball bat. Suppose a horizontal force $\vec{F}$ (due to the impact of the ball) acts towards the right at P, the center of oscillation. Assume the batter holds the bat at the support point of the stick at O.

   (a) What acceleration does the point O undergo as a result of $\vec{F}$?

   (b) What angular acceleration is produced by $\vec{F}$ about the center of mass of the bat?

   (c) As a result of the angular acceleration in the previous part, what linear acceleration does point O undergo?

   (d) Considering the magnitude and directions of the accelerations in (a) and (c), show that P is indeed the sweet spot.