Physics 215

Simple Harmonic Motion

1. Hooke's Law states that

$$F_r = -kx$$

where F_r is the restoring force of a spring k is the spring constant, and x is the displacement from equilibrium. If this is combined with Newton's Law then one obtains

$$F = ma = m\frac{d^2x}{dt^2} = -kx \qquad .$$

It is claimed that the general solution to the previous equation is

$$x(t) = A\cos(\omega t + \phi)$$

where ω is the angular frequency and ϕ is called the phase angle. Show that this last equation is a solution to Hooke's Law and get an expression for ω in terms of the spring constant k and the mass m.

2. A body oscillates with simple harmonic motion according to the equation

$$x = (6.0\ m)\cos\left((3\pi\ rad/s)t + \pi/3\ rad\right).$$

At $t = 2.0\ s$ what are the displacement, the velocity, and the acceleration?

3. The general solution to Hooke's Law is written in Problem 1. Find the particular solution for the boundary conditions: $x(0) = 0\ m$, $\frac{dx}{dt}(0) = 1.0\ m/s$. Assume $k/m = 1.0\ kg/s^2$.

4. A harmonic oscillator is set in motion such that at $t = 0\ s$ the displacement is $x_0 = 0.2\ m$ and the velocity is $v_0 = 0.0\ m/s$. The mass of the oscillator is $m = 1.0\ kg$ and the spring constant is $k = 0.6\ N/m$. What are the values of the constants that describe the motion? In other words, what are the constants in the expression for $x(t)$?

5. Consider the somewhat strange coincidence between the properties of a mass on a spring and a simple pendulum. Suppose you hang a mass m from a spring of spring constant k. The spring stretches a distance l. Show that the frequency of the block-spring system is the same as the frequency of a simple pendulum whose length is l.