Physics 215

Taylor Series

1. Write out the first four terms of the Taylor series expansion of

$$\tan\theta \quad \text{about } \theta = \frac{\pi}{4} \qquad \text{and} \qquad e^x \quad \text{about } x = 0 \qquad .$$

2. Provide an estimate to third order of the Fresnel integral

$$\int_0^1 \sin(x^2)dx$$

and compare your value with exact one of 0.310268. Note that you are free to choose the value of x about which to expand. It will simplify the problem if you make an astute choice.

3. A body with rest mass m_0 and moving at a speed near that of light, as in the case of an electron close to the nucleus of a heavy atom, has a kinetic energy given by the relativistic formula

$$K = m_0c^2\left(\frac{1}{\sqrt{1-(v^2/c^2)}} - 1\right) \qquad .$$

Express K as a series in v and show that for $v \ll c$ it reduces to the usual nonrelativistic expression.

4. Suppose a constant force F acts on a body, initially at rest, for a time t. Using the relativistic definition of momentum

$$p = \frac{m_0v}{\sqrt{1-(v^2/c^2)}}$$

and the definition of force as the time derivative of the linear momentum show that

$$v = c\left(1 + \left(\frac{m_0c}{Ft}\right)^2\right)^{-1/2} \qquad .$$

(Hint: use force equals the time derivative of the momentum, separate variables and integrate). Then show that if $Ft \ll m_0c$ one has the familiar expression

$$Ft = m_0v$$

or

$$\text{impulse} = \text{momentum} \qquad .$$