Physics 303 Test 2

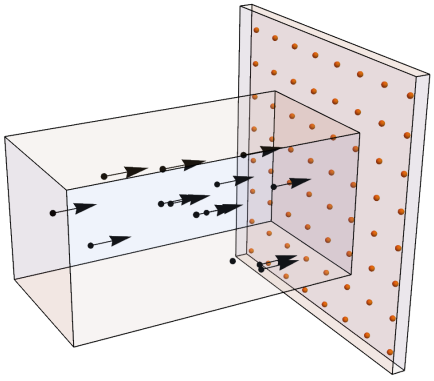
I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name _____ Signature _____

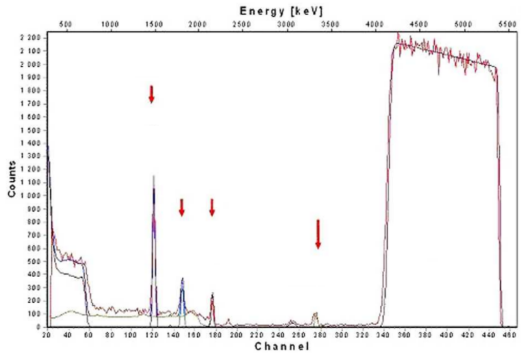
Questions (6 pts. apiece) Answer questions 1-5 in complete, well-written sentences WITHIN the spaces provided.

1. What is an eigenvalue? Why is it special?

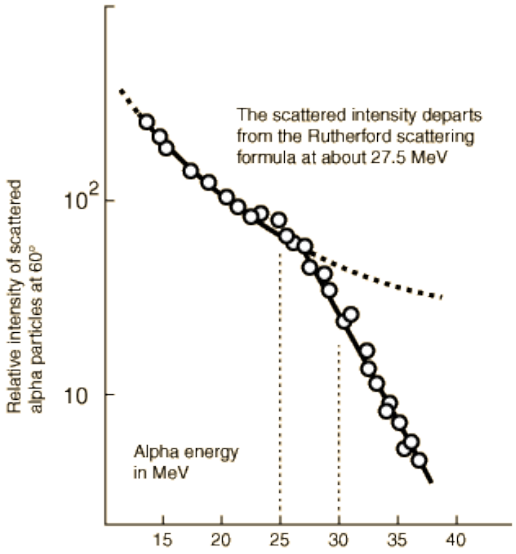
2. In developing the differential cross section $d\sigma/d\Omega$ we used the areal target density as one of the essential parameters of a measurement. How might our calculation change if the cross-section of the incoming beam where shaped not like a right cylinder with a circular endcap, but as an oval or a square endcap? Cross section here does NOT refer to $d\sigma/d\Omega$, but to the intersection of a 'body' (the beam) with a plane (the target).



3. The figure below shows the energy spectrum from Rutherford back scattering with 6-MeV alpha ($^4_2$He) particles on a 2 μm thick gold piece. The energy of the particles is increasing to the right. What does the broad peak at high energy tell us about the sample? What do the narrow peaks marked by the arrows tell us about the sample?



4. The figure below shows the measured differential cross for a scattering experiment of ${}^{4}_{2}\mathrm{He}$ on gold. The curve is the Rutherford scattering calculation for elastic scattering. What does the deviation between the calculated and measured cross sections mean? Is it possible to extract the radius of the ${}^{208}_{82}\mathrm{Pb}$ from this measurement alone? Explain.



5. Suppose two identical balls of Play-Doh hang from massless strings at the same height. They barely touch when both are at rest. One ball is pulled back to 60 *deg* and released. It swings down, collides with the other ball, and they stick together. What laws of physics would you invoke to calculate the final height of the struck ball? Explain.

Problems (1-3). Work out Problems 1-3 on a separate sheet to show your work. Clearly show all reasoning for full credit.

1. 20 pts. Recall our study of the collision of the Earth with an asteroid that led to the demise of most dinosaurs. Consider an asteroid that collides with the Earth and sticks to the surface (a perfectly inelastic collision). How much energy is released in the collision? Some potentially useful data is below. Ignore the effects of potential energy here. Clearly state your reasoning and discuss and justify any approximations you make.

   Asteroid: $m_A = 3.4 \times 10^{15}\ kg \quad v_A = 2.5 \times 10^{3}\ m/s$

   Earth: $m_E = 6.0 \times 10^{24}\ kg \quad v_E = 3.0 \times 10^{4}\ m/s$

Do not write below this line.

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2. 25 pts. In a scattering experiment like the ones Rutherford performed the projectile starts its journey essentially infinitely far away. What is its initial angle θ_i and radius r at that moment? Use this information about r and θ_i and the orbit equation

$$\frac{1}{r} = \frac{\mu\alpha}{\ell^2}\left(1 + \epsilon\cos(\theta - \theta_0)\right)$$

to get an expression relating θ_0 to ϵ and any other known, necessary constants.

3. 25 pts. A typical heavy atom consists of a nucleus of radius 10^{-14} m surrounded by an electronic cloud of radius 10^{-10} m which neutralizes the nuclear electric charge for radii larger than 10^{-10} m. For an alpha particle ($Z_\alpha = 2$, $m_\alpha = 4$ u) striking a stationary magnesium atom ($Z_{Mg} = 12$, $m_{Mg} = 24$ u) with energy $E_{cm} = 3.1$ MeV, what is the minimum scattering angle where the Rutherford formula is valid? The Coulomb constant in Rutherford scattering is $\alpha = -Z_\alpha Z_{Mg} e^2 = -34.5$ MeV $-$ fm.

Do not write below this line.

Equations, Conversions, and Constants

$$\vec{F} = m\vec{a} = \dot{\vec{p}} = -\frac{dV}{dx} \quad \vec{F}_G = -\frac{Gm_1m_2}{r^2}\hat{r} \quad \vec{F}_C = \frac{kq_1q_2}{r^2}\hat{r} \quad \vec{F}_g = -mg\hat{y} \quad \vec{F}_s = -kr\hat{r}$$

$$|\vec{F}_{cent}| = m\frac{v^2}{r} \quad \vec{F}_f = -bv\hat{v} \quad \vec{F}_f = -cv^2\hat{v} \quad x = \frac{a}{2}t^2 + v_0t + x_0 \quad v = at + v_0 \quad \int \frac{df}{dx}dx = \int df$$

$$\ddot{y} + A\dot{y} + By = 0 \Rightarrow y = Ce^{\lambda t} \quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\ddot{y} + \omega_0^2 y = 0 \Rightarrow y = A\sin(\omega_0 t + \phi) = \alpha_1 e^{i\omega_0 t} + \alpha_2 e^{-i\omega_0 t} \quad \omega_0^2 = \frac{k}{m} \quad \omega_0^2 = \frac{g}{l} \quad \mu = \frac{m_1m_2}{m_1+m_2}$$

$$V = -\int_{x_s}^{x} \vec{F}(\vec{r}\,') \cdot d\vec{r}\,' \quad E = K + V \quad V_s = \frac{kx^2}{2} \quad V_g = mgy \quad V_G = -\frac{Gm_1m_2}{r} \quad V_C = \frac{kq_1q_2}{r}$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad \mathscr{L} = K - V \quad \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} = 0 \quad p_q = \frac{\partial \mathscr{L}}{\partial \dot{q}} \quad \underset{\sim}{\mathbf{A}}\vec{C} = -\omega^2 \underset{\sim}{\mathbf{I}}\vec{C}$$

$$\vec{p} = m\vec{v} \quad \vec{\ell} = \vec{r}\times\vec{p} \quad \ell = \mu r^2\dot{\theta} \quad \mu = \frac{m_1m_2}{m_1+m_2} \quad \vec{p}_i = \vec{p}_f \quad K_i = K_f \quad V_{eff}(r) = \frac{\ell^2}{2\mu r^2} + V(r)$$

$$\alpha = -ke^2Z_1Z_2 \qquad ke^2 = \frac{\hbar c}{137} = 1.44\,\text{MeV} - \text{fm} \quad \vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} \quad E_{cm} = \frac{m_{tgt}}{m_{tgt}+m_{proj}}E_{lab}$$

$$\frac{1}{r} = \frac{\mu\alpha}{\ell^2}\left(1 + \epsilon\cos(\theta - \theta_0)\right) \quad \theta_0 = -\arccos\left(\frac{1}{\epsilon}\right) \quad \epsilon = \sqrt{1 + \frac{2E_{cm}\ell^2}{\mu\alpha^2}} = \frac{1}{\sin\left(\frac{\theta_S}{2}\right)} \quad r_{min} = -\frac{\alpha}{2E_{cm}}(1+\epsilon)$$

$$d\sigma = 2\pi b db \quad d\Omega = \sin\theta d\theta d\phi \quad \frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{4E_{cm}}\right)^2 \frac{1}{\sin^4\left(\frac{\theta_s}{2}\right)} \quad \sin A = \cos(A - \pi/2) \quad r_{min} = \frac{\ell^2}{\mu\alpha}\frac{1}{1-\epsilon}$$

$$\cos A = -\sin(A - \pi/2) \quad \arccos(-x) = \pi - \arccos(x) \quad \cos(\arcsin(x)) = \sin(\arccos x) = \sqrt{1-x^2}$$

$$\cos(\arctan(x)) = \frac{1}{\sqrt{1-x^2}} \quad \tan(\arccos x) = \frac{\sqrt{1-x^2}}{x} \quad \tan(\arcsin x) = \sin(\arctan x) = \frac{x}{\sqrt{1-x^2}}$$

More Equations, Conversions, and Constants

Speed of light (c)	$3.0 \times 10^8\ m/s$	g	$9.8\ m/s^2$
Gravitation constant (G)	$6.67 \times 10^{-11}\ N - m^2/kg^2$	Earth's radius	$6.37 \times 10^6\ m$
Coulomb constant (k_e)	$8.99 \times 10^9 \frac{N-m^2}{C^2}$	Earth's mass	$5.97 \times 10^{24}\ kg$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	Proton/Neutron mass	$1.67 \times 10^{-27}\ kg$
Planck's constant (h)	$6.626 \times 10^{-34}\ J - s$	Proton/Neutron mass	$932 \times 10^6\ eV/c^2$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} \frac{kg^2}{N-m^2}$	Electron mass	$9.11 \times 10^{-31}\ kg$
Permeability constant (μ_0)	$4\pi \times 10^{-7} N/A^2$	Electron mass	$0.55 \times 10^6\ MeV/c^2$
1 MeV	$10^6\ eV$	1.0 eV	$1.6 \times 10^{-19}\ J$
1 u	$931.5\ MeV/c^2$	1 u	$1.67 \times 10^{-27}\ kg$
Earth-Moon distance	$3.84 \times 10^8\ m$	Moon's radius	$1.74 \times 10^6\ m$
Earth-Sun distance	$1.50 \times 10^{11} m$	Moon's mass	$7.36 \times 10^{22}\ kg$

$$\int \frac{dx}{x^2 - a^2} = -\frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \qquad \int \tanh x dx = \ln\left[\cosh x\right] \qquad \int \coth x dx = \ln[\sinh x]$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} \qquad \int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \sinh^{-1}\frac{x}{a} \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\ln ax = \frac{1}{x} \qquad \frac{d}{dx}\cot x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{1 + x^2}$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) \qquad \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2 - a^2}}{2} + \frac{a^2}{2}\ln\left[x + \sqrt{x^2 - a^2}\right]$$

$$\int \tanh^2(x) dx = x - \tanh x \qquad \int \tanh^3(x) dx = \ln\left[\cosh x\right] + \frac{\operatorname{sech}^2(x)}{2}$$

$$\int \sqrt{\tanh x} dx = -\tan^{-1}\left[\sqrt{\tanh x}\right] - \frac{1}{2}\ln\left[1 - \sqrt{\tanh x}\right] + \frac{1}{2}\ln\left[1 + \sqrt{\tanh x}\right]$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x \qquad \frac{d}{dx}\sinh x = \cosh x$$

$$\int \frac{1}{\sqrt{ar^4 + br^3 - r^2}} dr = \frac{r\sqrt{-1 + br + ar^2}\arctan\left(\frac{-2+br}{2\sqrt{-1+br+ar^2}}\right)}{\sqrt{r^2(-1 + br + ar^2)}}$$

$$\int \frac{1}{\sqrt{ax^2 + bx + c}} dx = -\frac{1}{\sqrt{-a}}\arcsin\left(\frac{2ax + b}{\sqrt{b^2 - 4ac}}\right)$$

$$\int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}}\arctan\left(\frac{2ax + b}{\sqrt{4ac - b^2}}\right)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Even More Equations, Conversions, and Constants

Pythagorean identities :

$$\sin^2\theta+\cos^2\theta=1$$

$$\tan^2\theta+1=\sec^2\theta$$

$$1+\cot^2\theta=\csc^2\theta$$

Reciprocal identities :

$$\csc x=\frac{1}{\sin x}$$

$$\sec x=\frac{1}{\cos x}$$

$$\cot x=\frac{1}{\tan x}$$

Even - odd identities :

$$\sin(-x)=-\sin x$$

$$\cos(-x)=\cos x$$

$$\tan(-x)=-\tan x$$

Product to sum formulas :

$$\sin x\cdot\sin y=\frac{1}{2}\left[\cos(x-y)-\cos(x+y)\right]$$

$$\cos x\cdot\cos y=\frac{1}{2}\left[\cos(x-y)+\cos(x+y)\right]$$

$$\sin x\cdot\cos y=\frac{1}{2}\left[\sin(x+y)+\sin(x-y)\right]$$

$$\cos x\cdot\sin y=\frac{1}{2}\left[\sin(x+y)-\sin(x-y)\right]$$

Sum to product :

$$\sin x\pm\sin y=2\sin\left(\frac{x\pm y}{2}\right)\cos\left(\frac{x\mp y}{2}\right)$$

$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Double - angle formulas :

$$\sin 2\theta=2\cdot\sin\theta\cos\theta$$

$$\cos 2\theta=\cos^2\theta-\sin^2\theta=1-2\sin^2\theta=2\cos^2\theta-1$$

$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$$

Co - function identities :

$$\cos\left(\frac{\pi}{2}-x\right)=\sin x$$

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x$$

$$\tan\left(\frac{\pi}{2}-x\right)=\cot x$$

$$\cot\left(\frac{\pi}{2}-x\right)=\tan x$$

$$\csc\left(\frac{\pi}{2}-x\right)=\sec x$$

$$\sec\left(\frac{\pi}{2}-x\right)=\csc x$$

Periodicity identities :

$$\sin(x\pm 2\pi)=\sin x$$

$$\cos(x\pm 2\pi)=\cos x$$

$$\tan(x\pm\pi)=\tan x$$

$$\cot(x\pm\pi)=\cot x$$

$$\sec(x\pm 2\pi)=\sec x$$

$$\csc(x\pm 2\pi)=\csc x$$

Sum and difference formulas :

$$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$$

$$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$$

$$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$$

Half - angle formulas :

$$\sin\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right)=\frac{(1-\cos x)}{\sin x}$$

Law of sines :

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Law of cosines :

$$a^2=b^2+c^2-2bc\cos A$$

$$A=\cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

Area of triangle :

$$\frac{1}{2}ab\sin C$$

$$\sqrt{s(s-a)(s-b)(s-c)},$$

where $s=\frac{1}{2}(a+b+c)$

Periodic Chart

hydrogen 1 H 1.0079																	helium 2 He 4.0026
lithium 3 Li 6.941	beryllium 4 Be 9.0122											boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305											aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078	scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
rubidium 37 Rb 85.468	strontium 38 Sr 87.62	yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 ✶	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]
francium 87 Fr [223]	radium 88 Ra [226]	89-102 ✶ ✶	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	unununium 111 Uuu [272]	ununbium 112 Uub [277]		ununquadium 114 Uuq [289]			

*Lanthanide series	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
**Actinide series	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}.$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \\ = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}.$$

- Hyperbolic cotangent: $x \neq 0$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \\ = \frac{e^{2x} + 1}{e^{2x} - 1} = \frac{1 + e^{-2x}}{1 - e^{-2x}}$$

- Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \\ = \frac{2e^x}{e^{2x} + 1} = \frac{2e^{-x}}{1 + e^{-2x}}$$

- Hyperbolic cosecant: $x \neq 0$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} = \\ = \frac{2e^x}{e^{2x} - 1} = \frac{2e^{-x}}{1 - e^{-2x}}$$

$$\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right); x \geq 1$$

$$\operatorname{artanh}(x) = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right); |x| < 1$$

$$\operatorname{arcoth}(x) = \frac{1}{2}\ln\left(\frac{x + 1}{x - 1}\right); |x| > 1$$

Odd and even functions:

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$

Hence:

$$\tanh(-x) = -\tanh x$$
$$\coth(-x) = -\coth x$$
$$\operatorname{sech}(-x) = \operatorname{sech} x$$
$$\operatorname{csch}(-x) = -\operatorname{csch} x$$