Physics 303

One-Dimensional Oscillators

1. An object of mass $m = 7.0\ kg$ is hung from the bottom end of a vertical spring fastened to an overhead beam. The object is set into vertical oscillations of period $T = 2.6\ s$. What is the force constant of the spring?

2. A particle of mass $m = 0.50\ kg$ is attached to a horizontal spring with a spring constant $k = 50\ N/m$. At the time $t = 0$, the particle has its maximum speed $v_{max} = 20\ m/s$ and is moving to the left. What is the particle's equation of motion? What is the minimum time interval required for the particle to move from $x = 0$ to $x = 1.0\ m$?

3. For the damped oscillator we showed the general solution to the differential equation

$$\ddot{y} + 2\gamma\dot{y} + \omega_0^2 y = 0 \tag{1}$$

is

$$y = c_1 e^{\lambda_+ t} + c_2 e^{\lambda_- t} \tag{2}$$

where

$$\lambda_\pm = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2} = -\gamma \pm \Omega$$

and Ω is real for $\gamma^2 > \omega_0^2$. For the damped oscillator with $\gamma^2 < \omega_o^2$ show the general solution is

$$y(t) = c_1 e^{(-\gamma + i\Omega')t} + c_2 e^{-(\gamma + i\Omega')t} \tag{3}$$

where $\Omega' = \sqrt{\omega_0^2 - \gamma^2}$. This step makes the imaginary component of the solution explicit.

4. Apply the following boundary conditions

$$\text{for } t = 0 \Longrightarrow\ y = y_0 \text{ and } \dot{y} = 0 \tag{4}$$

to Equation 3 and show

$$c_1 = y_0 \frac{\Omega' - i\gamma}{2\Omega'} \tag{5}$$

and

$$c_2 = y_0 \frac{\Omega' + i\gamma}{2\Omega'} \tag{6}$$

5. Now insert the results in Equations 5-6 into Equation 3 and show the following equation is true.

$$y(t) = \frac{y_0}{\Omega'}\ e^{-\gamma t}\left(\Omega' \cos\Omega' t + \gamma \sin\Omega' t\right) \tag{7}$$

6. Consider the function.

$$f(x) = \frac{1}{\sqrt{1+x}} \tag{8}$$

What is the Taylor series for this function for the first four terms? What does the n^{th} term look like? When can we approximate the function with the first two terms in the series? Explain.

7. For a damped oscillator, Newton's second law gives us

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \tag{9}$$

in one dimension. Show that the expression

$$x = Ae^{-bt/2m}\cos(\omega t + \phi) \tag{10}$$

is a solution as long as $b^2 < 4mk$.